COMPENSATION OF EUROPEAN LEVELLING NETWORKS

International Union of Geodesy and Geophysics
INTERNATIONAL ASSOCIATION OF GEODESY
Florence Meeting (23rd - 28th May, 1955)

RESOLUTIONS ADOPTED

I. — *General*

During its General Meeting in Rome in September 1954, the International Association of Geodesy (I.A.G.), took important decisions concerning the scientific definition of altitudes, allowing for the variations of separation of the level surfaces (geo-potential surfaces), by means of the values of gravity at ground level.

Each of these level surfaces can be characterized by a quantity, to be called the ***geo-potential number***. This quantity represents the gravity work of unit mass moving in a homogeneous gravity field from a standard level surface (Geoid) to the surface considered.

This geo-potential number c of any point can thus be represented by the following quantity

$$c = \int g.\, dh$$

It is independent of the path travelled between the standard surface and the level surface through the point considered.

It can be computed satisfactorily by replacing the above integral by a sum of finite terms, provided the sub-division is properly made.

In this case, the above formula will read as follows :

$$c = \Sigma \left[\frac{g_A + g_B}{2} \times \Delta h \right]_A^B$$

where A and B represent the ends of each successive segment considered, the elementary quantities Δh being uncorrected differences of level.

The values of g to be used at A and B should be the *best* values available, i. e. those as near as possible to the actual values of g at the points A and B. When available, directly measured values should be used. Otherwise values should be, derived, according to the circumstances, either from gravimetry charts, or from ellipsoidal values of γ_o, corrected insofar as possible for both altitude and relief.

The *orthometric* altitude H of any point will be derived from its geo-potential number c by dividing the latter by the *mean value* of g throughout the vertical of this point to the Geoid.

$$H = \frac{c}{g}$$

This mean value of g can be obtained only exceptionally and in a very localized way from direct measures. Approximative values of this mean value can be obtained by diverse methods that take into account both the surrounding relief and specific gravity values of the neighbouring and underlying ground.

The I.A.G., in its General Assembly in Rome, decided to leave each Country free to compute and publish so-called « official » altitudes in its own way for the different points of its levelling network.

On the other hand, at this Meeting, it insisted upon the geo-potential number being computed according to the established rules, at least for the main network junction points.

The I.A.G. decided that comparisons and adjustments of an *international* character should be made exclusively through values of the geo-potential numbers.

Lastly, it decided that a comprehensive adjustment of the main European levelling networks should be carried out on these principles as soon as possible, and that a Committee comprising representatives of all the interested European countries should meet early in the year 1955 to organize the details of this comprehensive adjustment and, at the same time, to develop the lines on which the successive geo-potential numbers should be computed. Some points of those levelling networks directly connected to *recording tide gauges* should be included in this comprehensive network, so that the relation between positions of the different mean sea levels observed on the European coasts can be obtained by differences.

Such was the object of the European Meeting organized in Florence from the 23rd to 28th May 1955, the major decisions of which are now reported.

The meeting's scope was limited to the conditions for making out a *Unified European Levelling Network* (U.E.L.N.), depending exclusively on the geo-potential numbers — without reexamining any of the decisions taken during the General Meeting in Rome — and deferring until later reunions all other studies, especially:

— the method of computation to be adopted for the mean values of g along a vertical and the orthometric altitudes which can be derived from geo-potential numbers ;

— the study of the local disturbances of tides at the different tide-gauges connected to the U.E.L.N., in order to derive the probable level of the Geoid with reference to the mean sea level as found from very longs periods of observations;

— the study of the ground movements with reference to the mean sea levels along the coasts, and also inside the continents. The existence of an extensive European network, adjusted as a whole and forming one unit, will no doubt in the future facilitate the study of these relative ground movements as a whole and not only at isolated points as formerly; etc...

II. — *Terminology adopted and units of measure*

a) *Definitions.* The International Committee was unanimous in adopting the term *geo-potential number* (in French: cote géo-potentielle) as the characteristic of any terrestrial equipotential surface. It recommended that, when this expression has to be translated into other languages, the words chosen should be distinct from, and not liable to confusion with, those which signify altitude.

b) *Unit. The difference of the geo-potential number* Δ c *of two neighbouring points* is the difference of their uncorrected levels dh *expressed in metres* multiplied by the mean value of g *expressed in kilogals* (i.e. by taking g = 0.98...), for the corresponding segment.

The geo-potential number c *of any point* is equal to the sum of the partial differences Δ c along a levelling route, starting from a standard surface and subdivided according to rules given later.

Both these quantities are said to be expressed in *geo-potential units* (g.p.u.).

c) *System of values of* g *used.* Unless a contrary decision is taken by the International Association of Geodesy, the values of g used shall remain in terms of the so-called « Potsdam » conventional system (in which the basic value accepted for Potsdam is g = 0.981274 kilogal).

If, later on, the I.A.G. decides to modify this basic value, all the geo-potential numbers will be altered by small amounts directly proportional to their values.

It is easy to see that any variation Δg_0 affecting the Potsdam basic value g_0 brings about variations Δ c, given by the following formula, in the geo-potential elevation c of any point :

$$\frac{\Delta c}{c} = \frac{\Delta g_0}{g_0}$$

It is known that the ratio $\frac{\Delta g_0}{g_0}$ will be of an order of magnitude of 15×10^{-6} in our time. Then for c = 1,000 g.p.u., corrections in the order of magnitude of Δ c = 0,015 g.p.u. may have to be made.

Remark. The geo-potential number c of any point in g.p.u.'s is roughly 2 per cent less than the number *h* that represents its altitude in metres.

III. — *Composition of the Unified European Levelling Network (U.E.L.N.)*

The International Committee decided that the U.E.L.N. should be composed of a set of large *polygons* whose perimeters may vary as a rule between 500 and 1200 km. (excepting some areas where separate lines or smaller polygons exist).

After considering the proposals made by the different Countries represented in Florence, a draft on the U.E.L.N. was made out and the drawing of the final pattern was entrusted to a Sub-Committee, presided over by Dr. O. SIMONSEN, Secretary of the I.A.G. 's Levelling Section.

Accordingly, Dr. O. SIMONSEN will have the small-scale (1 :6,000,000) map of the U.E.L.N. drafted at Florence reproduced, and forward a copy of it to each of the participating European countries to obtain their final accord. The different countries will return a larger scale (1 :1,000,000) map of the parts of the U.E.L.N. that concern their national territory to Dr. O. SIMONSEN, indicating any necessary corrections.

Each participating Country should examine thoroughly the choice of its national lines pertaining to the U.E.L.N., taking into account what is said in section IV hereafter.

A final intermediate scale (1 :2,500,000) map will then be drawn in four sheets, printed, and widely distributed by the I.A.G. 's office.

The International Committee requires that these preliminary works be completed *before the end of the year* 1955.

The same method of work will be used for the *tide-gauges* that are to be included in the U.E.L.N. and the *international junction points,* tentative lists of which were adopted at the Florence Meeting.

After the final levelling network has thus been settled, the different *junction points, tide-gauges and international junction* points of this network will be designated by numbers and international abbreviations, corresponding to the country to which they belong. (Such abbreviations have been adopted as conform to the registration numbers of automobiles). The different segments (1) of the U.E.L.N. are thus designated without ambiguity by the appellations of their terminals.

The tide-gauges will also be designated by their numbers preceded by the letter M. — For instance : M. 17, M. 83, etc...

Detailed rules deciding the designation of the different junction points, tide-gauges and international junction points will be laid down by the Sub-Committee, commissioned to decide upon the structure of the U.E.L.N. (Dr. O. SIMONSEN). A descriptive list of these points, with all the necessary explanations, will be submitted to the different countries concerned, as well as the draft of the 1 :6,000,000 map as mentioned above.

In principle, no change shall be made in the network thus adopted without a previous agreement between the countries concerned and the I.A.G.

Prior to December 31, 1956, the countries concerned should forward *three copies* of all the documents necessary for the calculation of the network adjustment to the Central Bureau of the I.A.G. in Paris.

IV. — *Values of* g *used for computing the successive differences of geo-potential numbers*

As has been said, there can be no question at present of using values of g other than those corresponding to the Potsdam conventional system.

If, in some particular cases, a first computation is effected from *normal* values of gravity (should no sufficiently accurate values of g be available), it must be understood that any final computation of a scientific nature must depend exclusively upon true values of g, directly measured or obtained by interpolations from accurate gravimetric maps.

The root-mean-square errors arising in the computation from the adoption of doubtful values of g and the sub-division of the lines into finite, non-negligible segments, ought not to exceed 10^{-4} g.p.u. for a levelled length of 1 kilometre. However, errors double this limiting value will be admissible when the values of g are derived from gravimetric maps and if the ground is moderately or very hilly.

In order to obtain such high accuracy, the following rules must be adhered to :

(1) From one junction point to another or to an international junction point.

a) Besides the gravity determinations made in the vicinity of benchmarks, extra determinations should be made at any points where there are significant variations of slope or direction of the levelling lines.

b) The gravity determinations should be the closer to each other as the differences of level, gravity anomalies and topographical corrections are greater and more irregular.

c) Generally speaking, where good evaluations of gravity have been made under normal conditions, the following may be considered as admissible intervals between the gravity determination points:

in a flat area	2 to 3 km.
in a moderately hilly area	1 to 2 km.
in a very hilly area	0.3 to 1.5 km.

To reach the required degree of accuracy an analysis of the precision is recommended on the lines set forth in Professor RAMSAYERS's report entitled : « Contribution to the Theory of Errors on the Gravity Corrections applied to Levelling Operations ». (Special Publication I of Zeitschrift für Vermessungswesen, 1954.)

Such an analysis is particularly necessary in high mountains, and if gravimetric maps are used.

It will be wise to check that the required degree of accuracy has been effectively attained. To this end, some typical lines should be chosen, where the gravity value will be measured at very short intervals. The root-mean-square errors of interpolation of gravity and altitude values will be experimentally determined on these lines, in terms of the spacings of the gravimetric stations and the root-mean-square errors entailed by the use of gravimetric maps.

V. — *Documents to be supplied for the adjustment computation*

For *all the segments* of the U.E.L.N. each country will have to supply the following data to the Central Bureau:

a) the differences of the geo-potential numbers in g.p.u., for the whole segment;

b) the length L of the segment (to the nearest 1 km.) ;

c) the total root-mean-square error per kilometre, *t*, as given by the formulae internationally adopted, based on the closures of the polygons of the national general network of the 1st order

$$t^2 = \left[\text{mean} \frac{\varphi^2}{F} \right] \text{lim.}$$

(See Bull. Geod. No. 18.)

Other equivalent formulae may be used for computing *t*, provided such use is clearly mentioned in the document.

d) the *weight* of the segment, expressed in the following form

$$P = \frac{200}{L} \cdot \frac{1}{t^2} \quad (L \text{ in km})$$

e) Sufficiently precise particulars regarding the way in which both the *subdivision* of each segment has been done and the successive values of g used in this calculation have been determined.

VI. — *Computation of adjustment*

As regards Central, Southern and Southwestern Europe, adjustment operations will be carried out simultaneously according to three different (though equivalent) methods which will be used by the three following Organizations:

— Deutsche Geodätische Kommission (method by successive approximations) ;

— Institut Géographique National Français (method of indirect observations and method of groups) ;

— Technische Hogeschool voor Geodäsie, Delft (Mr. Waalewijn's method).

The compensation of the North European network, which is connected to the Centre block by a single line, will be computed by the Finnish Institute of Geodesy.

All the geo-potential numbers will be computed from the data of the Amsterdam basic point. A general correction may later be applied to all the results obtained.

Remark. It is recommended that each Country calculate later the geo-potential numbers of all the benchmarks of the U.E.L.N.

VII. — *Elements of all tide-gauges*

Each participating Country is asked to supply a dossier as complete as possible, for each tide-gauge in the U.E.L.N., which will allow discussion not only of the comparative data from the fixed benchmarks adjacent to the tide-gauge and those given by the tide-gauge concerning the mean sea level, but also of the fluctuations of these differences of level — as well as the conditions of local production of the tides (with a study of the known local disturbances). The question of *dates* will play a significant role in the general comparisons that will be made between the different mean sea levels.

The Central Bureau of the I.A.G. is invited to set up a *joint* Committee, in concert with the International Association of Physical Oceanography, commissioned with the study of these problems, and which will meet in principle in connection with the General Meeting of the I.U.G.G., in 1957.

VIII. — *Reiteration of the surveys of the U.E.L.N.*

It is quite evident that the scientific value of the results obtained through this first general computation of the U.E.L.N. will be strongly impaired by the heterogeneity of the European levelling networks. It is very likely that the accuracy of measurements is much the same for different countries owing to the long-established unification of instruments and methods; but it is another question as regards the dates on which the different levelling networks were surveyed and the places where the different tide-gauges used were operated. These surveys have extended over a period of about 60 years, and ground movements have certainly occurred in these intervals of time, at least in some areas.

While admitting that the present general computation may be carried out, as a first stage, using the data now available, the International Committee expressed the following wishes :

a) that the U.E.L.N. be regarded in any Country as a major levelling network of an international nature, of which any links, deemed defective or too old, should be revised at such times as could be arranged internationally under the International Association of Geodesy's patronage ;

b) that, at the time of these new surveying operations, the siting of the benchmarks should be made with special care, taking into account both phenomena of a geologic nature and mechanical movements of the ground;

c) that methods be studied and developed to define under what conditions the U.E.L.N.'s general results could be used to exhibit the general movements of the ground relative to the mean sea level in certain areas, notably the Netherlands and Scandinavian peninsula. If the necessity arises, special operations could be undertaken internationally under the I.A.G. 's patronage to provide data for these problems, which are of vital importance to several countries;

d) that the installations existing at the different tidal stations of the U.E.L.N. be improved as far as possible, and that observations be made there with extreme regularity. These operations should be, whenever possible, supplemented by studies of the local conditions of production of the tides in the neighbourhood of the tidal stations.

LIST OF DELEGATES TAKING PART IN MEETINGS

A. — *Delegates from the International Association of Geodesy's Central Organizations :*

Dr. J. de GRAAFF-HUNTER, President ;

Mr. G. CASSINIS, First Vice-President;

Mr. P. DORE, President of the Levelling Section ;

Mr. C. F. BAESCHLIN, Honorary President of the Association;

Mr. R.P. LEJAY, Director of the International Gravimetric Bureau ;

Mr. J. VIGNAL, Former President of the Levelling Section;

Mr. P. TARDI, Director of the Central Bureau;

Mr. J.J. LEVALLOIS, Assistant Secretary of the Association;

Mlle S. CORON, Secretary of the Gravimetric Section.

B. — *Invited to take part in proceedings :*

General BENEDETTI, Director of the Italian Military Geographic Institute.

C. — *National Delegates* (The countries are listed in the order in which the applications were booked) :

Switzerland ..... Mr. de RAEMY

Italy ..... Mr. BOAGA
Mr. SOLAINI
Mr. SALVIONI

West Germany	Mr. KNEISSL Dr. RAMSAYER Dr. WERNTHAELER Dr. WOLF
Belgium	Mr. JONES
Netherlands	Mr. WAALEWIJN Mr. BAARDA
Denmark	Mr. SIMONSEN
Sweden	Mr. ASPLUND
Norway	Mr. TROVAAG
Finland	Mr. KUKKAMÄKI
Great Britain	Mr. de GRAAFF-HUNTER (on record, see above).
France	Mr. JACQUINET Mr. DESCOSSY Mr. CAHIERRE
Spain	Mr. RAMON DORDA DE VALENZUELA Mr. GARCIA DE ARANGOA
Turkey	Mr. SOYDAN
Hungary	Mr. TARCZY HORNOCH
Austria	Mr. MADER Mr. LEDERSTEGER
Yugoslavia	Mr. MITIC Mr. PROSEN

D. — *Countries that presented excuses for not sending delegates.*

Soviet Union
Czechoslovakia
Greece
Portugal
Ireland

E. — *Countries that did not answer the* 29.12.1954 *circular.*

East Germany
Poland

Abbreviations to be adopted for the different « junction points » of the Unified European Levelling Network.

A	- Austria	L	- Luxembourg
B	- Belgium	N	- Norway
CH	- Switzerland	NL	- Netherlands
CS	- Czechoslovakia	P	- Portugal
D	- Germany	PL	- Poland
DK	- Denmark	S	- Sweden
E	- Spain	SF	- Finland
F	- France	SU	- Soviet Union
GR	- Greece	TR	- Turkey
H	- Hungary	YU	- Yugoslavia
I	- Italy		