NEW METHODS OF SHIP POSITION FINDING FROM CELESTIAL OBSERVATIONS

by Stjepo M. KOTLARIC
Technical Assistant, Hydrographic Institute of the Yugoslav Navy.

In this article I have tried to show in a few lines my short tabular methods for computation of altitude and azimuth (Tables K_I), as well as for direct computation of the observed position (Tables K_{II}); the latter is independent of the conventional Marcq de Saint-Hilaire method, and Longitude and Latitude methods.

In order to enable the reader to get a clear idea of the computation of a position line and the observed position respectively, I have shown extracts of these tables and examples of work too.

TABLES K_I

The main principle of computing the altitude and azimuth by these tables consists of the following:

Instead of solving the astronomical triangle directly or dividing it into two right-angled spherical triangles by dropping a perpendicular from the zenith or the celestial body, three right-angled spherical triangles formed by the intersection of the celestial equator, the celestial horizon, the lower branch of the celestial meridian, the vertical circle and the hour circle of the celestial body are used. These three triangles have one common pole at the point where the hour circle of a celestial body intersects the celestial horizon. This point is marked by letter g in Figure 1.

From the first triangle Wfg (see Figure 1) with known elements $90°-s$ (complement of the local hour angle) and $90°-\varphi$ (complement of the latitude) the angle C and angular distance M are to be computed. Then from the second triangle Jhg with known C and $M+\delta$ (algebraic sum of M and declination) the computation of the altitude (V) and angular distance F is to be made. The angular distance $\omega+F$ (azimuth angle plus F) is to be found from the third triangle PngN with known φ and $180°-s$. The value F subtracted from $\omega+F$ gives the azimuth angle (ω), and in this way the whole problem is completely solved.

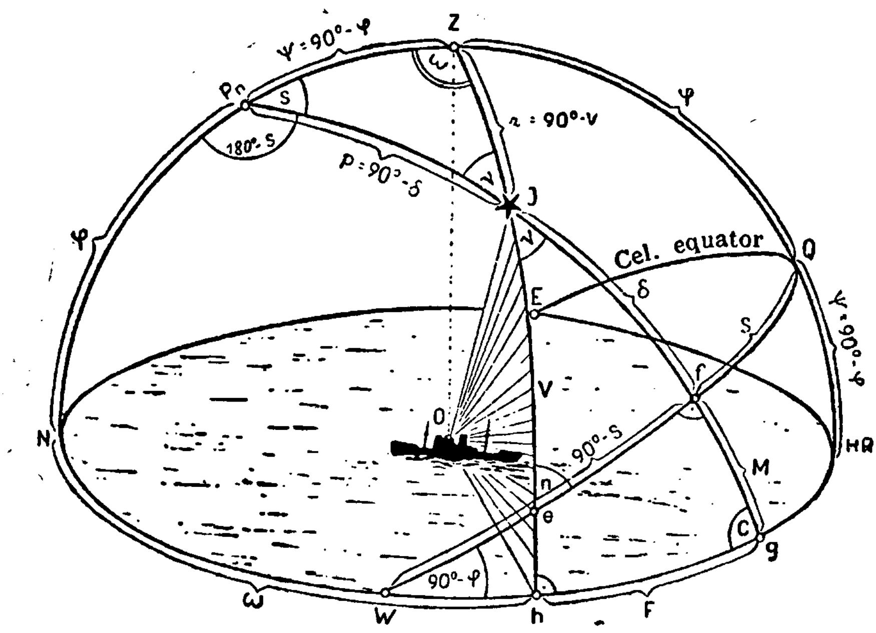


Fig. 1.

FORMULAE FOR COMPUTATION OF THE ALTITUDE

Applying the Napier rules to the following triangles it is found:

From the triangle Wfg (Figure 1):

$$\cos C = \sin (90° - \varphi) \cdot \sin [90° - (90° - s)]$$

$$\cos C = \cos \varphi \cdot \sin s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 1$$

$$\cos [90° - (90° - s)] = \text{ctg} (90° - M) \cdot \text{ctg} (90° - \varphi)$$

$$\cos s = \text{tg} M \cdot \text{tg} \varphi$$

$$\text{tg} M = \text{ctg} \varphi \cdot \cos s \quad . \; . \; . \; . \; . \; . \; . \; . \quad 2$$

or

$$\cos (90° - \varphi) = \sin C \cdot \sin (90° - M)$$

$$\sin \varphi = \sin C \cdot \cos M$$

$$\cos M = \frac{\sin \varphi}{\sin C} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 2a$$

From the triangle Jgh:

$$\cos (90° - V) = \sin C \cdot \sin (M + \delta)$$

$$\sin V = \sin C \cdot \sin (M + \delta) \quad . \; . \; . \; . \; . \; . \; . \; . \quad 3$$

$$\cos (M + \delta) = \text{ctg} \cdot \nu \cdot \text{ctg} \cdot C$$

$$\text{ctg} \nu = \cos (M + \delta) \cdot \text{tg} C \quad . \; . \; . \; . \; . \; . \; . \; . \quad 4$$

or

$$\cos C = \sin \nu \cdot \sin (90° - V) = \sin \nu \cdot \cos V$$

$$\sin \nu = \frac{\cos C}{\cos V} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 4a$$

Figure 2 shows the arguments M, C and altitude (V) for two celestial bodies (J and J_2) whose declinations (δ and δ_2) are of the same name as latitude (φ) but local hour angle of the first body (s) is less and local hour angle of the second body (s_2) is greater than 90°.

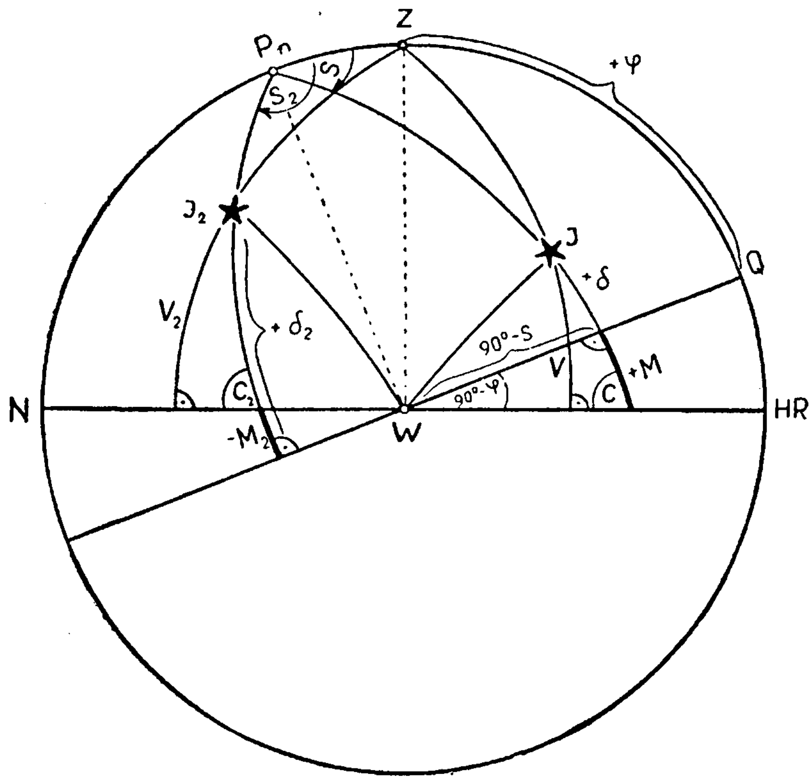


Fig. 2.

From this figure it is seen that:

a) M is of the same name as latitude (above the horizon) if the local hour angle is less than 90°; but M is of the contrary name (below the horizon) if the local hour angle is greater than 90°.

b) C never exceeds 90°.

FORMULAE FOR COMPUTATION OF THE AZIMUTH

Applying the Napier rules to the triangles mentioned below it is found:

From the triangle PngN (Figure 1):

$$\cos(90° - \varphi) = \text{ctg}(180° - s) \cdot \text{ctg}[90° - (\omega + F)]$$

$$\sin\varphi = \text{ctg}(180° - s) \cdot \text{tg}(\omega + F)$$

$$\text{tg}(\omega + F) = \sin\varphi \cdot \text{tg}(180° - s)$$

$$\text{tg}(\omega + F) = -\sin\varphi \cdot \text{tg}\, s \quad \ldots\ldots\ldots\ldots \quad 5$$

From the triangle Jgh (Figure 1):

$\cos C = \text{ctg}\,(90^\circ - F)\ .\ \text{ctg}\,(M+\delta)$

$\cos C = \text{tg}\,F\ .\ \text{ctg}\,(M+\delta)$

$\text{tg}\,F = \cos C\ .\ \text{tg}\,(M+\delta)$ 6

From the triangle PngN (Figure 1):

$\omega = (\omega + F) - F$ 7

The azimuth angle is reckoned from the elevated pole of the observer to 180°, E or W depending on the local hour angle.

Formulae 1 to 7 developed from these three triangles render a very simple tabulation of the values necessary for the computation of the altitude and azimuth.

INFLUENCE OF VARIOUS ARGUMENTS IN THE FINAL RESULT OF THE COMPUTED ALTITUDE

The changes in altitude due to a change of various arguments are determined by formulae 8 to 14b developed from Figures 3, 4 and 1.

The change in altitude (dV) *due to a small change in hour angle* (ds) is derived from Figure 3.

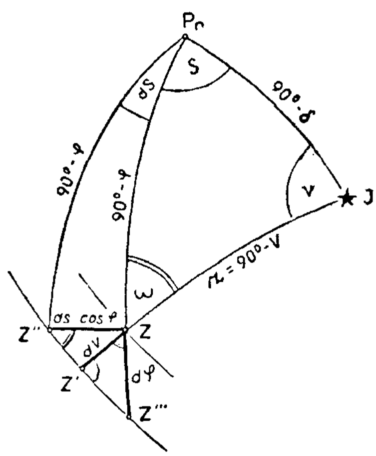


Figure 3

Fig. 3.

$$\sin \omega = \frac{dV}{ds\ .\ \cos \varphi}$$

$dV = ds\ .\ \cos \varphi\ .\ \sin \omega$ 8

In this case dV represents the correction to the altitude due to a small change (minutes of arc) in hour angle and is written as ks.

$ks = ds\ .\ \cos \varphi\ .\ \sin \omega$ 8a

$ds\ .\ \sin \omega = U1$

$ks = U1\ .\ \cos \varphi$ 8b

In the similar way the other altitude corrections are determined. kC, *i.e. the correction to the altitude due to a small change (minutes of arc) of* C is developed from Figure 1.

$dV = dC \cdot \cos(90^\circ - F) \cdot \sin 90^\circ$

$dV = dC \cdot \cos(90^\circ - F)$ 9

$\cos(90^\circ - F) = iC$, i.e. index of altitude correction due to a change of 1 minute of arc of C.

$dV = dC \cdot iC$ 9a

$kC = dC \cdot iC$ 9b

$k\varphi$, *i.e. the correction to the altitude due to a small change (minutes of arc) of latitude* ($d\varphi$) is developed from Figure 3.

$dV = d\varphi \cdot \cos\omega$ 10

$\omega = i\varphi$. i.e. index ol altitude correction due to a change of latitude ($d\varphi$).

$dV = d\varphi \cdot \cos i\varphi$ 10a

$k\varphi = d\varphi \cdot \cos i\varphi$ 10b

$kM\delta$, *i.e. the correction to the altitude due to a small change (minutes of arc) of* $M+\delta$ is developed from Figures 4 and 1.

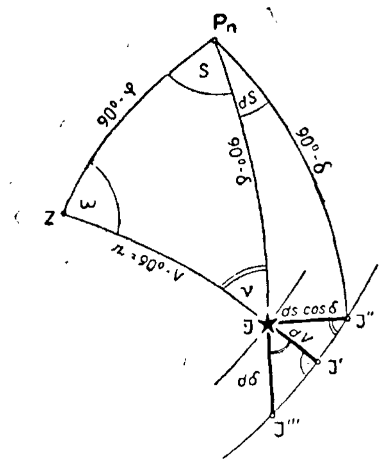


Figure 4

Fig. 4.

$d\delta$ is small change in declination δ.

$$\cos\nu = \frac{dV}{d\delta}$$

$dV = d\delta \cdot \cos\nu$ 13

$d(M+\delta)$ is a small change in the value $(M+\delta)$. From Figure 1 it is seen that small change in altitude due to a small change in $(M+\delta)$ is influenced by $\cos\nu$ in the same way as already seen in the formula 13 for declination.

$dV = d(M+\delta) \cdot \cos\nu$ 14

$\cos\nu = iM\delta$, i.e. index of altitude correction due to a change of 1 minute of arc of $(M+\delta)$.

$$dV = d(M+\delta) \, . \, iM\delta \quad \ldots\ldots \quad 14a$$
$$kM\delta = d(M+\delta) \, . \, iM\delta \quad \ldots\ldots \quad 14b$$

Tables K_1 are based on these formulae (1 to 14b) and give the computed values of altitude and azimuth for all latitudes and all celestial bodies. The interpolations, rules for signs and logarithmic operations are avoided when the sight is worked using these tables.

The use of Tables K_1 consists of the following:

(A) SOLUTION FOR A POSITION LINE FROM THE ASSUMED POSITION

1. Enter Table I with the local hour angle (s) and assumed latitude (φi), rounded to whole or half degrees, and pick out the values: $\omega + F$ (azimuth angle plus F), M and C (see extracts of these Tables on Fig. 7).

2. Label M with the sign according to the short remarks printed in the head of Table I (M is of the same name as latitude when the local hour angle is less than 90°; if the local hour angle is greater than 90°, M is of the contrary name).

3. Add algebraically M and declination (δ).

4. With the value $M+\delta$ thus found and the value C found in step 1 enter Table II, using the next lower whole or half degrees of these values, and find: F, V (altitude), $iM\delta$ and iC.

The Tables are constructed for every whole degree and half degree only, and for remaining minutes of arc for the values $M+\delta$ and C, Table IV will be used as interpolation Table, from which $kM\delta$ and kC can be taken out as corrections respectively.

The values $iM\delta$ and iC respectively represent the indices of corrections serving as the entering argument in Table IV. Signs for $kM\delta$ and kC are given at the head of Table II above $iM\delta$ and iC.

5. The azimuth angle (ω) is always found by subtracting F from $\omega + F$. The azimuth angle is measured through 180°; label it N or S to agree with the latitude, and E or W to agree with the local hour angle.

If an extreme accuracy of the azimuth is desired, F may be interpolated by inspection. That will only be necessary for very high altitudes.

6. Enter Table IV with $iM\delta$ at the side, using the nearest tabulated value, and the remainder of minutes of arc of $M+\delta$ at the top and take out $kM\delta$ (the correction to the altitude for minutes of arc of $M+\delta$). The Table is so arranged that it may be entered at the top with both whole numbers and tenths.

7. kC, i.e. the correction to the altitude for the remainder of minutes of arc of C, is to be found in the same way as $kM\delta$ in step 6, but entering Table IV with iC at the side and with minutes of arc of C at the top.

8. The computed altitude (Vr) is found by applying the correction $kM\delta$ and kC to the tabulated altitude (V) found in step 4. The signs for these corrections are printed at the head of Table II above $iM\delta$ and iC (already mentioned in step 4).

9. Find the intercept (ΔV) by taking the difference between the true altitude (Vp) and the computed altitude (Vr). If the true altitude is the larger, the intercept is towards (+). If the true altitude is the smaller, the intercept is away (—).

EXAMPLE 1

Finding the Ship's Observed Position from Simultaneous Observations of Two Celestial Bodies.
Solution from the Assumed Position by Tables K_1.

21st February 1950, the GMT 1726 dead reckoning position of a ship was $\varphi z = 34° 51,5'N$, $\lambda z = 38° 06,4'W$. About this time the navigator observed Sun's lower limb with a marine sextant, as follows: Greenwich Mean Time $Ts = 17^h 26^m 40,8^s$, true altitude $Vp = 30° 51,6'$. 21 s. later on, i.e. at $Ts = 17^h 27^m 01,8^s$ the true altitude of the Moon was taken $Vp = 65° 27,3'$. Find the observed position (Pp), i.e. the observed latitude (φp) and the observed longitude (λp).

NOTE: Extracts of Tables K_1 which are necessary for solution of this example are shown in Figure 7.

Solution of the Sun Sight:

$Vp = 30°51,6'$

$Ts = 17^h 26^m 40,8^s$

$\delta = -\ 10°36,0'$
$+\ 1,3$
$\delta = -\ 10°34,7'$

$S = 78°13,9'$
$\lambda i = 38\ 13,9W$
$s = 40°W$
$\varphi i = 35°N$

$\omega + F = 154,3°$
$F = 21,4$
$\omega = N\ 132,9°W$

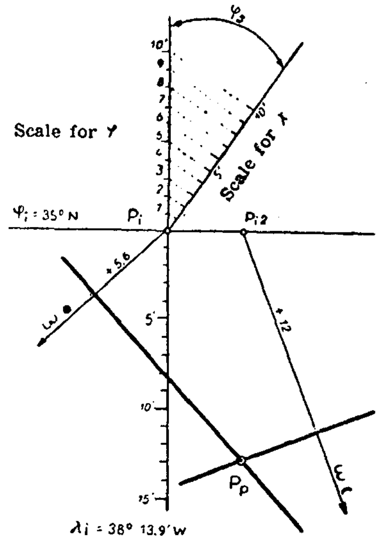
$M = +\ 47°34,3'$
$M + \delta = 36°59,6'$

$C = 58°13,7'$

$V = 30°17,6'$
$78,9\ kM\delta = +\ 23,4$
$36,3\ kC = +\ 5,0$
$-\ Vr = 30°46,0'$
$Vp = 30\ 51,6$
$\Delta V = +\ 5,6'$

Solution of the Moon Sight:

Vp = 65° 27,3'	Ts = 17^h 27^m $01{,}8^s$	S = 29° 08,3'
		+360
	δ = + 11° 19,6'	λi2 = − 38 08,8 W
	+ 19,1	s = 351 00,0
	δ = + 11° 38,7'	s = 9° E
		φi = 35° N
ω+F = 174,8°	M = + 54° 39,9'	C = 82° 38,3'
	M+δ = 66° 18,6'	
F = 16,3	V = 64° 55,3'	
ω = N 158,5°E	95,1 kMδ = + 17,7	
	27,3 kC = + 2,3	
	—Vr = 65° 15,3'	
	Vp = 65 27,3	
	ΔV = + 12,0'	



φi = 35°00,0'N	λi = 38°13,9'W
Δφ = 12,9 S	Δλ = 5,1 E
φp = 34°47,1'N	λp = 38°08,8'W

Fig. 6.

EXTRACTS FROM TABLES K_1

Tabl. I. *S*

φ	9° ω+F	contr. φ / same φ M	C	171° ω+F	…	40° ω+F	contr. φ / same φ M	C	140° ω+F	φ
° ′	°	° ′	° ′	°		°	° ′	° ′	°	° ′
0										0
⋮										⋮
34						154,9	48 38,1	57 47,9	25,1	34
34 30						154,6	48 06,1	58 00,7	25,4	34 30
35	174,8	54 39,9	82 38,3	5,2		154,3	47 34,3	58 13,7	25,7	35
⋮										⋮
90										90

Tabl. II. $M+\delta$

C	36°30′ F	V	− + iMδ	+ + iC	143°30′ F	…	66° F	V	− + iMδ	+ + iC	114° F	C
° ′	°	° ′			°		°	° ′			°	° ′
1												1
⋮												⋮
58	21,4	30 17,6	78,9	36,3	158,6		50,0	50 46,8	54,2	76,3	130,0	58
58.30	21,1	30 28,5	79,4	35,8	158,9		49,6	51 09,7	54,9	75,9	130,4	58 30
⋮												⋮
82 30	5,5	36 08,3	98,7	9,3	174,5		16,3	64 55,3	95,1	27,3	163,7	82 30
83	5,2	36 11,1	98,8	8,7	174,8		15,3	65 03,5	95,7	25,5	164,7	83
⋮												⋮
90												90

Tabl. IV.

iMδ or iC	ω = iφ	(M+δ)′ or C′ or φ′ 1′	…	8′	…	13′	…	18′	…	29′	30′	0,1′	0,2′	0,3′	…	0,6′	0,7′	0,8′	0,9′
0	90°	0′		0′		0′		0′		0′	0′	0′	0′	0′		0′	0′	0′	0′
1	+89,4 - 90,6																		
2	88,9 91,1																		
⋮	⋮ ⋮																		
27				2,2										0,1					
28				2,3										0,1					
⋮																			
36						4,7											0,3		
37						4,8											0,3		
⋮																			
78										22,6						0,5			
79										22,9						0,5			
80										23,2						0,5			
⋮																			
95								17,1								0,6			
96								17,3								0,6			
⋮	⋮ ⋮																		
100	+ 0° - 180°	1	…	8	…	13	…	18	…	29	30	0,1	0,2	0,3	…	0,6	0,7	0,8	0,9

Fig. 7.

(B) SOLUTION FOR A POSITION LINE FROM THE DEAD RECKONING POSITION

The computed altitude and azimuth for the dead reckoning position (Pz) are found in the same way as for the assumed position (Pi) mentioned before under heading A, except for the following:

In step A_1: The local hour angle and the latitude are not to be rounded to a whole or half degree, and Table I is entered with the next lower whole or half degrees of these entering arguments.

In addition to step A8: The application of corrections $kM\delta$ and kC to the tabulated altitude found in Table II is not enough to find the computed altitude in this case. Corrections to the altitude for the remainders of minutes of arc of the local hour angle and the dead reckoning latitude are still to be applied. Table IV gives correction to the altitude ($k\varphi$) for minutes of arc of latitude when entering with the azimuth angle (ω) at the side, using the nearest tabulated value, and the minutes of arc of latitude (φ') at the top. The sign for this correction is printed near the entering azimuth angle. The correction for the remainder of minutes of arc of the local hour angle is found in Table III in the following way: enter the Table with the azimuth angle (ω) at the left hand side (using the nearest tabulated value) and the minutes of arc (whole numbers and tenths) of the local hour angle (s') at the top and pick out the value U1; then enter the same Table with U1 (whole numbers and tenths) at the foot and the latitude (φ) at the right hand (using the nearest tabulated value) and take out ks, i.e. the correction to the altitude for minutes of arc of the local hour angle. The sign for this correction is always minus and is printed at the foot of the Table.

EXAMPLE 2

Working the Sight from the Dead Reckoning Position by Tables K_1

In order to enable easier comparison between procedures of working sights from the assumed position and the dead reckoning position, in this example are used the same data (date and GMT of observation of the Sun, dead reckoning position and true altitude) as in example 1.

NOTE: Extracts of Tables K_1 which are necessary for solution of this example are shown on Figure 9.

Solution of the Sun Sight:

$Vp = 30^0\ 51{,}6'$

$Ts = 17^h\ 26^m\ 40{,}8^s$
$\delta = -10^0\ 36{,}0'$
$+\ 1{,}3$
$\delta = -10^0\ 34{,}7'$

$S = 78^0\ 13{,}9'$
$\lambda z = 38\ \ 06{,}4\ W$
$s = 40^0\ 07{,}5'W$
$\varphi z = 34^0\ 51{,}5'N$

$\omega + F = 154{,}6^0$
$F = 22{,}1$
$\omega = N\ 132{,}5^0 W$

$M = +\ 48^0\ 06{,}1'$
$M + \delta = 37^0\ 31{,}4'$
$V = 31^0\ 04{,}9'$
$78{,}5\ kM\delta = +\ 1{,}1$
$37{,}5\ kC = +\ 0{,}3$
$k\varphi = -\ 14{,}6$
$ks = -\ 4{,}6$
$-Vr = 30^0\ 47{,}1'$
$Vp = 30\ \ 51{,}6$
$\Delta V = +\ 4{,}5'$

$C = 58^0\ 00{,}7'$

From Figure 8 it is seen, that the same position line is obtained with the intercept + 5,6 found in example 1 and laid off from the assumed position (Pi), and the intercept + 4,5 found in this example and laid off from the dead reckoning position (Pz).

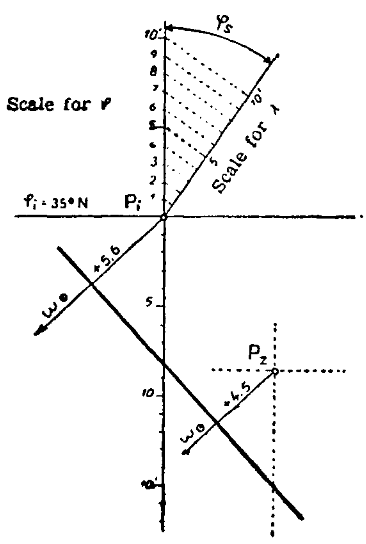


Fig. 8.

EXTRACTS FROM TABLES K_1

Tabl. I. S

φ	40°	contr. φ		140°
		same φ		
	ω + F	M	C	ω+F
0° ′	°	° ′	° ′	°
⋮				
34 30	154,6	48 06,1	58 00,7	25,4
⋮				
90				

Tabl. II. M + δ

C	37°30′		−	+	142°30′
			+	+	
	F	V	i Mδ	i C	F
1° ′	°	°			°
⋮					
58	22,1	31 04,9	78,5	37,5	157,9
⋮					
90					

Tabl. III.

ω		S′														
		1′	2′	…	5′	6′	7′	…	30′	0,1′	…	0,5′	0,6′	…	0,9′	
		Ul′	Ul′	…	Ul′	Ul′	Ul′	…	Ul′	Ul′	…	Ul′	Ul′	…	Ul′	
0°	180°															90°
0,6	179,4															89,4
⋮	⋮															⋮
46,9	133,1						5,1					0,4				
47,7	132,3						5,2					0,4				
⋮	⋮															⋮
					4,1								0,5			35,9
					4,1								0,5			34,9
																⋮
90°																0°
		- ks	- ks	…	- ks	- ks	- ks	…	- ks	-ks	…	-ks	-ks	…	-ks	
		1′	2′	…	5′	6′	7′	…	30′	0,1′	…	0,5′	0,6′	…	0,9′	φ
		Ul′														

Tabl. IV.

iMδ or iC	ω = i φ	(M + δ)′ or C′ or φ′															
		1′	…	7′	8′	…	21′	…	30′	0,1′	…	0,4′	0,5′	0,6′	0,7′	0,8′	0,9′
0	90°	0′		0′	0′		0′		0′	0′		0′	0′	0′	0′	0′	0′
1	+89,4 - 90,6																
⋮	⋮ ⋮																
37	⋮ ⋮														0,3		
38	⋮ ⋮														0,3		
⋮																	
67	+47,9 - 132,1				5,4		14,1						0,3				
68	47,2 132,8				5,4		14,3						0,3				
⋮	⋮ ⋮																
78	⋮ ⋮	0,8										0,3					
79	⋮ ⋮	0,8										0,3					
⋮																	
100	+ 0° - 180°	1	…	7	8	…	21	…	30	0,1	…	0,4	0,5	0,6	0,7	0,8	0,9

Fig. 9.

CONCLUSION ABOUT TABLES K_1

We are thus briefly acquainted with Tables K_1. In order to determine which is the best to use in actual practice, a comparison was made between Tables K_1 and a series of more than twenty other Tables of Celestial Navigation. The use of Tables K_1 is found to be the least involved, except American Tables H.O. 214, in the amount of work in turning pages, entering Tables, additions and subtractions, rules for signs, interpolation of Tables and interpretation of diagrams etc. Tables K_1 have, however, a greater advantage in having only 200 pages in place of 2400 used in Tables H.O. 214.

DIRECT SOLUTION OF THE SHIP'S OBSERVED POSITION

The method mostly used in the navigational practice of today is the Marcq de St. Hilaire Method. This method is eighty years old and during this relatively long period of time nothing of it has been changed except the way of computing the altitude and azimuth. However, the method of Marcq de St. Hilaire is an indirect one because when the altitudes and azimuths of two celestial bodies have been computed, it is necessary to make one additional procedure, either graphical or arithmetical, in order to obtain the coordinates of the observed position; therefore a direct method of computing the observed position would be welcome.

From simultaneous observations of two celestial bodies we obtain their Greenwich hour angles, declinations and true altitudes. From the spherical triangle $PnJ_1 J_2$, with the difference of Greenwich hour angles (ΔS) of the observed two bodies (see Figure 10) and their declinations (δ_1 and δ_2) the angular distance between these two bodies ($90^\circ - Vx$) and the angle A can be computed in the same way as the zenith distance and azimuth angle are computed from the astronomical triangle. Then from the spherical triangle $ZJ_1 J_2$, with true altitudes V_1 and V_2 and angular distance $90^\circ - Vx$ the angle B can be computed. This value B subtracted from A gives the angle ν (called the parallactical angle). Finally, from the spherical triangle $PnZJ_2$ with the declination δ_2, true altitude V_2 and the angle ν, the local hour angle (s_2) and the latitude (φ) of the observer can be computed; combining the local hour angle with the Greenwich hour angle the observed longitude is determined.

Accordingly, the true position of a ship is determined without using a dead reckoning or assumed position. Attention is drawn to the fact that the coordinates of the observed position can be computed also by means of the parallactical angle of the first (eastern) body J_1. For the simplicity of the procedure it is advisable to always name the western body as J_2 and regarding this one to compute angles A and B, and furthermore with its parallactical angle to find the final result of the latitude and local hour angle. Exception of this occurs only in the case when western body is in the vicinity of the meridian; in this case eastern body has to be named as J_2. Using this principle the formulae and the scheme of work remain always the same.

Sights of one celestial body taken at different times may be also used for direct computation of the observed position. In this case the main principle is the same as already mentioned before for the simultaneous observations of two celestial bodies. The only difference would be that the true altitude from the first observation has to be corrected for the ship's run between the first and the second observation, i.e. the correction of the true atlitude from the first observation for the difference of latitude and the difference of longitude of the ship's run.

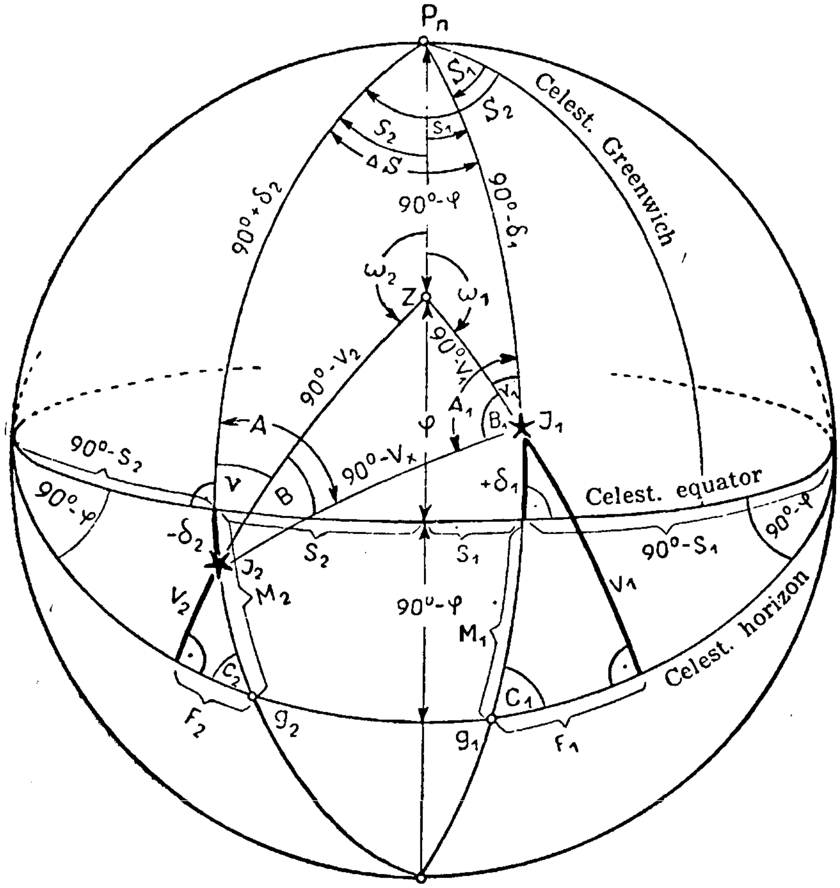


Fig. 10.

The procedure of logarithmic computation of five unknown elements (Vx, A, B, φ and s_2) in two spherical triangles is long and unsuitable for use in navigation, because up-to-date practical work on board requires fast working and simple methods of finding ship's position at sea. With the aim of making direct computation of the ship's observed position shorter and simpler I devised Tables K_{11}.

TABLES K_{11}

(Tables of Computed Latitudes and Local Hour Angles)

By this Table method the direct solution of the ship's observed position can be carried out without computation of the auxiliary values Vx, A, B and ν. The observed latitude and local hour angle are obtained directly from these Tables (see extracts in Figures 11 and 12) using true altitudes of two stars as entering arguments. Comparing the local hour angle with the Greenwich hour angle the observed longitude is determined.

Tabulation of these results was possible because the constant values of declination and difference of Greenwich hour angles (or difference of sideral hour angles) are used. For changes in minutes of altitudes, declinations and difference of Greenwich hour angles the corrections are applied by Multiplication Table using indices of corrections (ik φ and ik s) given in the main Tables. Signs for these indices are given in the main Tables for the positive changes of the following arguments: true altitudes, declinations and difference of Greenwich hour angles. When changes in these arguments are negative, signs are opposite. Multiplication of the mentioned indices and changes in minutes of the corresponding arguments (with their signs) gives the corrections to the latitude and local hour angle. This multiplication is made by Multiplication Table which is similar to Table IV in Tables K_1.

Tables K_{11} can be applied in the first place to the selected pair of stars because their relative positions differ only slightly. It is also possible that the Tables be compiled for two observations of the Sun or Moon (see extract in Figure 13, but the second observation must be made after a certain interval of time which is equal to that tabulated, e.g. 45 degrees of Greenwich hour angle difference. For small differences of this interval the correction of latitude and local hour angle will be made by Multiplication Table in the same way as for the differences in other arguments.

It would be necessary to tabulate two results for latitude and local hour angle, because two circles of position intersect in two points, but as they are usually very far apart it would not be difficult to decide which is the one in question.

It is quite clear that the disadvantage of these Tables is the bulk of the volume, but by the following arrangements it is made possible for it to be divided into several volumes for certain zones of latitudes similar to Tables H.O. 214.

At the beginning of Tables K_{11} one short Table comprehending only two sheets is supplied giving for each zone of 5° of latitude and for each 15° of local hour angle of the First Point of Aries the names of selected pairs of stars which are the most convenient for observation and computation, as well as the number of the page where these pairs of stars are tabulated. In this way for a certain volume it is not necessary to tabulate two results for latitudes and local hour angles, and tabulation can be reduced to only those altitudes which come into consideration for the selected volume, i.e. the selected zone of latitudes. Selection of stars which have to be used for observation for the given latitude and local hour angle of the First Point of Aries is already applied in the American Tables H.O. 249, Vol. I.

Similar short Table will be also introduced at the beginning of main Tables for the Sun or Moon tabulated for a certain zone of latitudes. From that short Table the difference of Greenwich hour angle used for tabulation and number of page where data are tabulated are taken out.

EXAMPLE 3

Direct Computation of the Ship's Observed Position from Observations of Two Stars; Solution by Tables K_{11}.

On 6th February 1939, the GMT 1648 dead reckoning position of a ship was φz = 42° 13,2'N, λz = 18° 19,0'E. About this time the navigator observed Rigel and Markab as follows:

Rigel : Greenwich Mean Time Ts = $16^h 45^m 23{,}3^s$, true altitude V_1p = 31° 32,8', true azimuth ωp = 142°. The ship was on course Kp = 310°, speed 7 knots.

Markab: Ts = $16^h 48^m 11{,}8^s$, true altitude V_2 p = 31° 28,3'.

Find the latitude and longitude of the observed position (φp and λp) at the moment of the second observation.

NOTE:

1. Extracts of Tables K_{11} necessary for solution of this example are shown in Figures 11 and 12.

2. Explanations of additional abbreviations used in this example are listed below:

Kp	=	True course.
D_1	=	Rhumb line distance.
Nm	=	Nautical mile.
N	=	North.
E	=	East.
R	=	Departure.
$\Delta\varphi$	=	Difference of latitude.
$\Delta\lambda$	=	Difference of longitude.
Δs	=	Difference of local hour angle.
$\Delta S \gamma$	=	Difference of Greenwich hour angles of the First Point of Aries.
ΔTs	=	Difference of Greenwich Mean Time.
ωp	=	True azimuth.
$V_1 p$	=	True altitude of the first celestial body.
$V_2 p$	=	True altitude of the second celestial body.
$k\varphi$	=	Correction to the altitude for minutes of arc of latitude, $k\varphi = \Delta\varphi \cdot \cos\omega$.
ks	=	Correction to the altitude for minutes of arc of hour angle, $ks = \Delta s \cdot \cos\varphi \cdot \sin\omega$.
$V_1 k$	=	True altitude from the first observation corrected to the moment of the second observation.
V_1	=	Altitude of the first celestial body.
V_2	=	Altitude of the second celestial body.
$360°-\alpha$	=	Sideral hour angle.
$\Delta(360°-\alpha)$	=	Difference of sideral hour angles.
φN	=	North latitude.
S	=	Greenwich hour angle; South.
s	=	Local hour angle; Seconds (of time).
W	=	West.

Correction of the first observation altitude to the moment of the second observation

		Rigel : $\omega p = 142°$
$Kp = 310°$,		$V_1p = 31° 32,8'$
$D_1 = 0,3$ Nm :	$\Delta\varphi = 0,2'$ N	$k\varphi = -\ 0,2$
	$R = 0,2'$ W	
	$\Delta\lambda = \Delta s = +\ 0,3'$	
$Ts_1 = 16^h45^m23,3^s$		
$Ts_2 = 16^h48^m11,8^s$		
$\Delta Ts = 2^m48,5^s$:	$\Delta S_\gamma = -\ 42,3$	
	$\Delta s = -\ 42,0'$	$ks = +\ 19,1$
		$V_1k = 31° 51,7'$

Taking out data for both stars *from the Nautical Almanac* we obtain the following data necessary for the solution by Tables K_{11}.

Rigel: $V_1k = 31°51,7'$, $\delta_1 = -\ 8°16,5'$, $360° - \alpha_1 = 282°05,5'$
Markab: $V_2p = 31°28,3'$, $\delta_2 = +14°52,7'$, $360° - \alpha_2 = 14°34,2'$, $S_2 = 42°32,4'$

$\Delta = 267\ 31,3$
360

$\Delta\,(360° - \alpha) = 92°28,7'$

Difference between given and tabulated coordinates δ_1, δ_2 and $\Delta(360° - \alpha)$ is found only for δ_1 in the amount of $-0,1'$. That means, from Tables K_{11} with whole or half degrees of known altitudes of Rigel and Markab (V_1k and V_2p) as the entering arguments we shall pick out tabulated latitude (φ) and local hour angle (s_2), and adding to these values the corrections for the remainders of minutes of arc of these two altitudes and for $-0,1'$ difference of declination of Rigel we obtain the observed latitude (φp) and local hour angle or Markab (s_2). Comparing the local hour angle s_2 with the Greenwich hour angle S_2 the observed longitude (λp) is determined.

From Tables K_{11} (see extracts in figures 11 and 12) we obtain:

For V_1 32° and V_2 31° 30':	$\varphi = 41° 57,5'$N	$s_2 = 60° 47,1'$
For differences:		
$V_1 - 8,3'$ x	$-113 = +\ 9,3$	$+\ 24 = -\ 2,0$
$V_2 - 1,7'$ x	$-\ 71 = +\ 1,2$	$-122 = +\ 2,1$
$\delta_1 - 0,1'$ x	$+101 = -\ 0,1$	$-\ 19 = \ 0$
	$\varphi p = 42° 07,9'$N	$s_2 = 60° 47,2'$W
		$S_2 = 42° 32,4$
		$\lambda p = 18° 14,8'$E

Solution of the same example by the Marcq de St- Hilaire method gives the following result of the observed position: $\varphi p = 42° 07,8'$N, $\lambda p = 18° 14,6'$E.

EXTRACT FROM TABLES K_{11}

V_2 MARKAB [$\delta_2 = +14°52,7'$; $\Delta(360° - \alpha) = 92°28,7'$]

V_1 RIGEL ($\delta_1 = -8°16,4'$)	31° φ_N	ikφ V_1	ikφ V_2	ikφ δ_1	ikφ δ_2	ikφ $\Delta\alpha$	s_2	iks V_1	iks V_2	iks δ_1	iks δ_2	iks $\Delta\alpha$	31° 30′ φ_N	ikφ V_1	ikφ V_2	ikφ δ_1	ikφ δ_2	ikφ $\Delta\alpha$	s_2	iks V_1	iks V_2	iks δ_1	iks δ_2	iks $\Delta\alpha$
° ′	°						° ′						° ′						° ′					
0																								
31 30	42 51,1	112	69				61 17,0	23	123				42 31,3	113	70				60 40,0	24	122			
32	42 18,4	−112	−70				61 23,9	+23	−122				41 57,5	−113	−71	+101	+46	−52	60 47,1	+24	−122	−19	+78	+8

Fig. 11.

NOTE. — 1. This extract from Tables K_{11} is computed for every whole degree and half degree of altitudes V_1 and V_2 of the mentioned bodies for 1st January 1939, i.e. for the given values of declinations δ_1 and δ_2 and difference of sideral hour angles $\Delta(360° - \alpha)$. For difference in minutes of arc between given and tabulated altitudes and changes in minutes of arc of δ_1, δ_2 and $\Delta(360° - \alpha)$ in later months and years, the corrections of the tabulated results φ and s_2 (observed latitude and local hour angle) are obtained from the Multiplication Table using indices of corrections ik_φ and iks (see Figure 12).

2. The corrections being taken out from Multiplication Table entering with the indices of corrections $ik\varphi$ and iks at the left side and with minutes of arc of the difference between given and tabulated values of the arguments V_1, V_2, δ_1, δ_2 and $\Delta(360° - \alpha)$ at the top. When these differences (given minus tabulated) are positive the signs of corrections are the same as those for indices of corrections given in the main table; when these differences are negative the signs are contrary.

EXTRACT FROM TABLES K_{11}

Multiplication Table

i k φ or	V_1' or V_2' or δ_1' or δ_2' or $\Delta(360^\circ - \alpha)'$															
i k s	1'	2'	3'	4'	....	8'	....	30'	0,1'	0,2'	0,3'	....	0,7'	0,8'	0,9'
1	0,0'	0,0'	0,0'	0,0'		0,1'		0,3'	0,0'	0,0'	0,0'		0,0'	0,0'	0,0'
2															
⋮															
8											0,0				
⋮															
19	0,2								0,0		0,1				
⋮															
24						1,9					0,1				
⋮															
46				1,8										0,4	
⋮															
52											0,2				
⋮															
71	0,7												0,5		
⋮															
78				3,1										0,6	
⋮															
101	1,0								0,1		0,3				
⋮															
113						9,0					0,3				
⋮															
122	1,2												0,9		
⋮															

Figure 12

EXAMPLE 4

Direct Computation of the Ship's Observed Position from Observations of Two Stars; Solution by Tables K_{11}

In this example the same data are taken as in example 3 with the only difference that the date of observation is 15 years later, i.e. 6th February 1954. In connection with this the coordinates of stars are changed and now read as follows:

Rigel: $\delta_1 = -\ 8^\circ\ 15{,}2'$ (diff. $+\ 1{,}3'$), $360^\circ - \alpha_1 = 281^\circ\ 54{,}6'$

Markab: $\delta_2 = +\ 14^\circ\ 57{,}5'$ (diff. $+\ 4{,}8'$), $360^\circ - \alpha_2 = 14\ \ 23{,}0$, $S_2 = 42^\circ 43{,}0'$

$$\Delta = 267\ \ 31{,}6$$
$$360$$

$$\Delta(360^\circ - \alpha) = 92^\circ\ 28{,}4'$$

(diff. — 0,3')

Altitudes remain unchanged: V_1k = 31° 51,7' and V_2p = 31° 28,3'.

From Tables K_{11} (see extracts in Figures 11 and 12) we obtain:

For V_1 32° and V_2 31° 30':	φ = 41° 57,5'N	s_2 = 60° 47,1'
For differences:		
V_1 — 8,3' x	—113 = + 9,3	+ 24 = — 2,0
V_2 — 1,7' x	— 71 = + 1,2	—122 = + 2,1
δ_1 + 1,3' x	+101 = + 1,3	— 19 = — 0,3
δ_2 + 4,8' x	+ 46 = + 2,2	+ 78 = + 3,7
Δ (360°—α) — 0,3' x	— 52 = + 0,2	+ 8 = 0
	φp = 42° 11,7'N	s_2 = 60° 50,6'W
		S_2 = 42 43,0
		λp = 18° 07,6'E

Solution of the same example by the Marcq de St. Hilaire method gives the following result of the observed position: φp = 42° 11,6'N, λp = 18° 07,6'E.

EXAMPLE 5

Direct Computation of the Ship's Observed Position from Sun Sights Taken at Different Times; Solution by Tables K_{11}.

31st January 1954, the GMT 0631 dead reckoning position of a ship was φz = 33° 16,6'N, λz = 27° 40,5'E. About this time the navigator observed the Sun's lower limb, as follows: Greenwich Mean Time Ts = $06^h31^m16^s$, true altitude V_1p = 15° 01,9'. Declination δ_1 = —17° 30,8', Greenwich hour angle S_1 = 274° 27,2', true azimuth ωp = 123,3°.

The ship made good a distance of 52,7 nautical miles on course Kp = 141° and at Ts = $10^h02^m04^s$ (Greenwich Mean Time) the Sun's lower limb was observed again, as follows : true altitude V_2p = 39° 44,1' ; declination δ_2 = —17° 28,5', Greenwich hour angle S_2 = 327° 08,9'.

Find the latitude and longitude of the observed position (φp and λp) at the moment of the second observation.

NOTE: Extracts of Tables K_{11} necessary for the solution of this example are shown in Figure 13.

EXTRACT FROM TABLES K_{11}

(for Sun Sights taken at Different Times)

$\delta = -17^{\circ}30'$; $\Delta S = 52^{\circ}30'$

V_1k	$V_2 = 39^{\circ}30'$												$V_2 = 40^{\circ}$	
	φ_N	$ik\varphi$					s_1	iks					φ_N	
		V_1k	V_2	δ_1	δ_2	ΔS		V_1k	V_2	δ_1	δ_2	ΔS		V_1k
° '	° '						° '						° '	
⋮														
15 30	32 45,9	+13	−109	−10	+108	+10	57 34,8	−154	+84	+109	−86	−8	32 13,3	+14
16	32 49,8		108				56 48,7		85				32 17,5	
⋮														

Multiplication Table

ikφ or iks	v_1' or v_2' or δ_1' or δ_2' or $\Delta(360^{\circ}-\alpha)'$																
	1'	...	11'	...	14'	...	22'	...	30'	0,1'	...	0,5'	0,6'	0,7'	0,8'	0,9'
1	0,0'		0,0'		0,0'		0,0'		0,0'	0,0'		0,0'	0,0'	0,0'	0,0'	0,0'
⋮																
8			0,9											0,1		
⋮																
10			1,1											0,1	0,1	
⋮																
13							2,9									
⋮																
84					11,8					0,1						
⋮																
86	0,9											0,4				
⋮																
103															0,8	
⋮																
108	1,1											0,5				
109					15,3					0,1						
⋮																
154							33,9									

Fig. 13.

Correction of the first observation altitude to the moment of the second observation

$S_1 = 274° 27{,}2'$
$S_2 = 327\ 08{,}9$

$\Delta S = 52° 41{,}7'$ $\omega p = 123{,}3°$
$Kp = 141°, Dl = 52{,}7$ Nm : $V_1p = 15° 01{,}9'$

$\Delta\varphi = 41{,}0'S$ .... $k\varphi = +\ 22{,}5$
$R = 33{,}2'E$
$\Delta\lambda = \Delta s = -\ 39{,}5'$ .... $ks = +\ 27{,}6$

$V_1k = 15° 52{,}0'$

From Tables K_{11} (see extracts in Figure 13) we obtain :

For V_1k 15°30' and V_2 39°30' : $\varphi = 32° 45{,}9'N$ $s_1 = 57° 34{,}8'$

For differences :

V_1k + 22' x	+ 13 = + 2,9	—154 = — 33,9
V_2 + 14,1' x	—109 = — 15,4	+ 84 = + 11,9
δ_1 — 0,8' x	— 10 = + 0,1	+103 = — 0,8
δ_2 + 1,5' x	+108 = + 1,6	— 86 = — 1,3
ΔS + 11,7' x	+ 10 = + 1,2	— 8 = — 1,0
	$\varphi p = 32° 36{,}3'N$	$s_1 = 57° 09{,}7'E$
		$S_1 = 274\ 27{,}2$
		331° 36,9'
		$\lambda p = 28° 23{,}1'E$

In this case the hour angle of the first observation (s_1) is tabulated in Tables K_{11} because the local hour angle of the second observation is small (in the vicinity of the observer's meridian).

Solution of the same example by the Marcq de St. Hilaire method gives the following result of the observed position: $\varphi p = 32° 36{,}5'N$, $\lambda p = 28° 23{,}1'E$.

COMPARISON BETWEEN THE METHODS OF MARCQ DE ST. HILAIRE AND DIRECT SOLUTION OF THE SHIP'S OBSERVED POSITION

Taking into consideration that the position lines can be reckoned by the Short Method Tables and that there is a possibility of determining the observed position by plotting the position lines on the Mercator Plotting Sheet or directly on the chart, we may generally say that for the present, however, Marcq de St. Hilaire's method is shorter.

When Tables K_{11} for finding ship's observed position are used, the computation is so much reduced that finding results of the observed latitude and local hour angle becomes in some measure similar to the method of finding results of computed altitudes and azimuths from the dead reckoning position by Tables H.O. 214. The reason for making a comparison between these Tables is that they represent the maximum of achievement in efforts to make the procedure of computing ship's observed position by the direct method and Marcq de St. Hilaire shorter and simpler It is also taken into consideration that whereas Tables K_{11} are tabulated only for selected celestial bodies, Tables H.O. 214 enable the computation of altitude and azimuth of all bodies. Furthermore the work by Tables K_{11} requires more corrections than by Tables H.O. 214, although Tables K_{11} need no special graphical or numerical solution for obtaining the coordinates of the observed position from the computed position lines. Moreover, in case two stars are observed, in computing the observed position by Tables K_{11}, only Greenwich hour angle of one star need to be found.

The methods of computing the intercept and azimuth have been improved over a period of eighty years. Here also an attempt at the improvement of the computation of a position line is given with the aim to make it faster and simpler. Tables K_1, which might be classed among Tables of Computed Altitude and Azimuth, are outstanding in this respect. Whether it will be possible to carry out further improvement of the direct method of solution of the ship's observed position, remains to be seen.

NOTE: More details about these Tables and also some modifications of Tables K_1 (e.g. Tables K_5 reducing 200 pages of Tables K_1 to 14 sheets only) may be found in my book published at the end of May 1955 by the Hydrographic Institute of the Yugoslav Navy under the same title, which in Yugoslav language reads: « Nove metode astronomskog odredjivanja pozicije broda ».

Tables K_1 will be published at the end of 1957 (200 pages, hard cover, size over all 9,4 by 11,8 inches) containing extensive explanation in English and so enabling it to be used by the mariners of other countries.