THE ANALYSIS AND PREDICTION OF TIDES IN SHALLOW WATER

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The harmonic method of analysis and prediction of tides initiated by Lord Kelvin quickly established itself as necessary and invaluable in areas where the diurnal tides were comparable in magnitude with the semidiurnal tides, and where, also, these tidal oscillations were of the deep-water type in which there were no oscillations associated with the product terms in the equations of motion. But when the method was applied to tides in the waters surrounding the British Isles it was found to be much less accurate than the results obtained from the older non-harmonic methods. This was so even when the quarter-diurnal and sixth-diurnal tides were included. These are the main shallow water constituents which are of importance only when the tidal amplitude is not small compared with the mean depth of the sea or channel. Even to this day the tides for many of the ports overseas are still predicted by the non-harmonic methods because of the failure of the direct harmonic method.

When the Tidal Institute was founded in the year 1919 this subject received much attention, and it is no exaggeration to say that the problem has been the greatest one that has confronted the predictor of tides.

The considerations which have led to the development of practicable methods of overcoming the difficulties will now be explained. It is necessary to discuss the mode of generation of the shallow water tides and the inter-relations of the harmonic constituents, and then to proceed to the special problem of predicting the times and heights of high and low water.

It cannot be expected that all those who may desire to use these methods will be able to follow the mathematical parts but it is necessary to justify the methods. Only the minimum of mathematical argument has been used but a great deal of work has been done on this method which cannot be dealt with in a paper of this kind.

Special attention has therefore been paid to the need for giving instructions for analysis and prediction which can be followed by computers who are little concerned with theory.

The methods which have been evolved at the Tidal Institute have been used since the year 1926 so that much varied experience has been gained as to their efficiency and value. The most intricate problems have been found to be susceptible to treatment by them, so that it has come to be regarded as the most powerful and valuable method in use at the Tidal Institute.

2. *The generation of shallow-water tides.*

It will be sufficient to consider the indications of the equations of the motion of water in a uniform channel, with the following notation:

ζ the elevation of the sea surface above the mean level.
t the time.
x the distance along the channel.
h the mean depth of a cross section, below mean level.
b the breadth of the surface at time t.
b_0 the breadth at the mean level.
A the cross-sectional area at time t.
b_0h the cross-sectional area below the mean level.
μ the mean velocity of stream at time t over a section.
g the acceleration of gravity.
σ the speed of a harmonic motion.

We shall use the following notation for the differential operators:

D_t the partial differential operator with respect to t. (1)

D_x the partial differential operator with respect to x. (2)

It will be convenient to consider a special case where the sides of the channel between low water and high water are inclined at an acute angle to the horizon, so that we may take:

$$b = b_0 (1 + 2 b_1 \zeta) \tag{3}$$

$$A = b_0h + b_0 (1 + b_1 \zeta) \zeta \tag{4}$$

Here b is the cotangent of the angle of inclination of the sides of the channel with the horizontal.

The tidal equations then give:

$$D_tu + gD_x\zeta = - uD_xu \qquad = U \tag{5}$$

$$D_t\zeta + hD_xu = - D_x(u\zeta) - b_1 D_t(\zeta^2) - b_1 D_x(u\zeta^2) \; = \; V \tag{6}$$

These equations can be combined to give:

$$D_t^2\zeta - ghD_x^2\zeta = D_tV - hD_xU \tag{7}$$

$$D_t^2u - ghD_x^2u = D_tU - gD_xV \tag{8}$$

In deep water the elevations and currents may be regarded normally as small quantities of the first order, so that the terms of U and V may be determined by iteration, theoretically. If the first-order part of the elevation has an amplitude R and a speed σ then the second order terms are derivable from the square of the first-order term and introduce terms with speed 2σ. The third-order terms have speeds 3σ and σ, while the fourth-order terms have speeds 4σ and 2σ. Thus the composite term with speed 2σ is compounded of two terms with amplitudes proportional to the second and fourth powers of the amplitude of the primary term. Consequently it is not an easy solution of the problem to consider the shallow-water terms as functions of the amplitude of the primary term, though such a method has been used where the terms were not large. The complications increase rapidly with the range of the tide.

The harmonic expression of shallow water constituents may be considered by taking three terms with amplitudes M, S, N and speeds m, s, n. Then if these express the primary tide the square of this will give terms with amplitudes and speeds as follows:

$$\begin{array}{cccccc} M^2, & S^2, & N^2, & 2MS, & 2MN, & 2SN, \\ 2m, & 2s, & 2n, & m+s, & m+n, & s+n, \end{array}$$

and similarly the cube of the primary oscillation will yield terms with amplitudes and speeds as follows:

$$\begin{array}{cccccccc} M^3, & S^3, & N^3, & 3M^2S, & 3M^2N, & 3S^2N, & 6MSN, & \ldots\ldots \\ 3m, & 3s, & 3n, & 2m+s, & 2m+n, & 2s+n, & m+s+n, & \ldots\ldots \end{array}$$

Very elaborate tables would be required to include all possible terms, but these serve to bring to light a very important point that though M may be much more important than S or N yet the term with amplitude proportional to M may be less important than the terms with amplitudes proportional to $3M^2S$ or 6MSN. A numerical illustration will show this clearly. Taking M, S, N, as proportional to 1.0, 0.5, 0.2 then the amplitudes M^3, $3M^2S$, $3M^2N$, 6MSN are 1.00, 1.50, 0.60, 0.60 respectively.

It is therefore necessary to include several constituents of each species and the necessity for this increases with the species number, even though the total amplitude of the constituents of any species may be decreasing with the species number. This may be seen also by a direct method of estimation. Suppose that the neap and spring ranges of the semidiurnal tide have the ratios 0.5 and 1.5 with the mean tide. Then the ratios of ranges of the higher species of tides relatively to their mean ranges are 0.25 and 2.25 for the quarter-diurnal tide, 0.12 and 3.38 for the sixth-diurnal tide, and 0.06 and 5.06 for the eighth-diurnal tide. Some old predicting machines included only M_6 and M_8 to represent two species, but the above ratios show the futility of this. At neap tides, for instance, the sixth and eighth-diurnal tides might be so grossly over-estimated as to show sixth and eighth-diurnal oscillations in the tide curve, and these would yield large errors in the times of high and low water.

In an infinitely long channel mathematical theory shows that the ranges of the shallow water tides decrease steadily at any one place with the species number, but in an actual channel it is not a safe policy to assume this law. Equation (6) shows that if the channel has sloping sides then as compared with a channel of rectangular section there are additional terms, those with coefficients b_1, and the second of these will give sixth-diurnal tides by a different law of generation than that indicated above. This may mean that the sixth-diurnal tides may be either increased or decreased relatively to those that might otherwise have been expected. Similarly the second-order terms may reinforce or annul one another so that the quarter-diurnal tide may even be small relatively to the sixth-diurnal tide. Further, the shape of the sides of the estuary may so affect the tide that at or near the entrance where there is a bar there may be large quarter-diurnal tides (or sixth-diurnal ones) while further up the river, in deeper water, the shallow water tides may be less prominent. This phenomenon has been observed in some rivers.

3. *The failure of the harmonic method.*

Though the harmonic method had been in use for very many years and it was known that it failed to give accurate predictions for large tides in shallow water, little had been done by the year 1919 to explore the causes of the failure. An intensive analysis of the tides, at Newlyn and Liverpool, and syntheses of the harmonic constituents, did much to reveal these causes and it was realized that there was only a very slow rate of convergence of the amplitudes of the shallow-water tides, those whose periods were sub-multiples (1/2, 1/3, 1/4...) of the period of the semi-diurnal tide. Though tides for most of the ports in the world could be represented to a satisfactory degree by the inclusion of the quarter-diurnal tide this was not the case for many ports in estuaries, for which it was necessary to include the sixth-diurnal tide. But the problem had been so imperfectly studied that the predicting machines at the time only included one constituent M_6 to represent the sixth-diurnal tide. Similarly the eighth-diurnal tide was represented by only one constituent.

The result of these early researches at the Tidal Institute was to indicate that if the eighth-diurnal tide was appreciable it was futile to expect a satisfactory prediction. The rate of convergence of the amplitudes of the shallow-water tides at springs at Avonmouth is shown by the figures 2.50, 0.75, 0.40, 0.20, 0.13 ft. for the quarter-diurnal tides, sixth-diurnals... twelfth-diurnals.

Thus two reasons could be given for the failure of the harmonic method:

(a) the slow convergence of the harmonic expressions;

(b) the necessity of including a large number of constituents for each species of tide.

It was evident that no machine then in existence, or which was likely to be built, or even which could possibly be built, could give a direct prediction of tides for places such as Liverpool, Bristol, London, Quebec, and Shanghai.

4. *Non-harmonic and direct harmonic methods.*

The simplest means of correcting the predictions made by the harmonic method seemed to be by means of corrections which depended simply upon the amplitude of the semi-diurnal tide, seeing that the amplitudes of the shallow-water tides depended upon the amplitude of the semi-diurnal tide. This meant that the machine predictions had to be made without the use of the quarter-diurnal and sixth-diurnal constituents as the shallow-water tides could only be dealt with as a whole on this principle. The method was applied for some years to the predictions for Liverpool with fair success but it was not satisfactory for Avonmouth. The reason may be that indicated earlier, when it was shown that under some conditions for each species there were contributions following different laws of generation. Whatever may have been the cause it was evident that a more elaborate method was needed, and a fresh examination of the problem of treating tides in shallow-water was undertaken. The main problem was that of predicting the times and heights of high and low water, for ports where the diurnal tide was not large.

In the year 1919, when this problem of predicting tides in shallow-water was seen to be acute because of the lack of convergence of the amplitudes of the higher species of tides an investigation had been made as to the possibility of predicting high water heights by a direct method without using the normal harmonic constituents. It was evident that in a sequence of high-water heights at intervals of a lunar day, approximately, there were variations with periods of approximately a month and submultiples thereof. Each of these variations involved contributions of slightly different periods. Reference should be made to table I, which gives the increments of phase, in degrees per mean lunar day, for all the principal tidal constituents. This table shows clearly the groups of constituents with nearly equal phase-increments. It may be noted, however, that the monthly variations due to the diurnal constituents differ so little from those for the semidiurnal constituents that they would not change relatively to one another by 360° in a year but would take 19 years. In other words the two could not be analysed by means of a year of observation but would require 19 years. But in most ports there are no 19 years of observation available, and even if there were the expense of analysis would be 19 times greater than what most harbour authorities would care to pay. Moreover, as conditions may change in an estuary, by siltation or dredging, it is desirable to make fresh analyses from time to time, and constants depending upon old observations to any degree might well fail to represent with sufficient accuracy the conditions that had come into being.

It is shown later that this difficulty can be overcome by treating the high-water heights in two sequences, each at intervals of a lunar day, but one following the lower transit of the moon and the other following the upper transit.

But even if this problem did not arise because the diurnal tide might be considered to be negligible there would still remain the problem of the degree of convergence of the amplitudes of the variations as their periods diminished. A simple examination of this problem can be readily made by considering only two constituents M_2 and S_2, with amplitudes proportional to 1 and r. Then if S_2 changes in phase by the angle θ relative to the phase of M_2 then the conjoint amplitude is approximately proportional to:

$$R = (1 + 2\,r \cos\theta + r\,2)\,\tfrac{1}{2}$$

Now it can be readily shown by direct harmonic analysis, say with $r = 0.5$, that:

$$R = 1.064 + 0.484 \cos\theta - 0.058 \cos 2\theta + 0.014 \cos 3\theta - 0.005 \cos 4\theta + \ldots$$

The harmonic terms in θ, 2θ, 3θ, ... have periods of approximately 14, 7, 3.5, ... days. It would be necessary to include at least the first three of these, even if only M_2 and S_2 were considered but when other constituents conspire with S_2 then the rate of convergence is much slower. Also, of course, in each of the variations there will be many contributory terms. Thus the direct analysis and prediction of high-water heights involves the same kind of problem as in the ordinary methods of tidal prediction.

5. *Further considerations regarding harmonic corrections.*

The weakness of the method just considered lies essentially in endeavouring to regard all constituents such as S_2 and N_2 as perturbations of M_2, and the method failed because the perturbations were too large. The next process of thought was to consider small perturbations of the semidiurnal tide as would be obtained from a machine prediction. An examination of the residues of an observed tide after subtracting this basic prediction showed that the main variations in the residual height from the high water heights had monthly, fortnightly, and other periods exactly the same as would have been evident if M_2 had been predominant and all other constituents very small. This would hardly have been expected, normally, because in the latter case the residues are taken at exact intervals of a lunar day, while in the former case they are taken at times which vary appreciably from the mean interval of a lunar day; these variations, of course, are harmonic.

Questions then arose as to:

(a) the number of special harmonic terms which would express the variations in the residues;

(b) the methods of analysis of the residues;

(c) the practicability of using these in actual predictions, whether by a special machine or by the adaptation of existing machines.

It may be said at once that the investigation of these matters revealed no insuperable difficulties, but they are discussed in the following articles, in order.

6. *Choice of harmonic shallow-water constituents.*

If the basic tide were given by M_2 only, and all other constituents were small, then the tide could be represented by constituents indicated in table I. The terms in the first two columns would need to be treated separately from those in the last two columns because of the small differences in speeds, but terms with negative values of μ would coalesce with terms with positive values of μ. If, however, we take the basic tide as that part of the semidiurnal tide which may be obtained on the machine we can still expect to have constituents with speeds indicated in table I, but with large tides many more shallow-water constituents would need to be included. Some of these are unfamiliar so that it may be well to explain the notation. Thus MSN6 refers to a constituent whose speed is m+s+n, where m, s, n, are the speeds of M_2, S_2, and N_2 respectively; also MSN_2 has a speed of m+s—n, while MNS_2 has a speed of m+n—s. A number of general considerations, supplemented by analyses, have led to the adoption of the constituents defined in table II. This list of constituents which combine together can be greatly extended for each of the shallow-water constituents named, as is indicated by the dots in each list, but it is evident that the 11 compounds, represented by C(00) to C(52), represent at least 42 normal tidal constituents derived from semidiurnal constituents, and that 7 other compounds, denoted by C'(11) to C'(40), derived from diurnal constituents and combinations of diurnals and semidiurnals, represent at least 18 more normal tidal constituents.

Seeing that tidal predictions are normally based upon fewer than 40 normal tidal constituents, it is evident that the representation of an even greater number of ordinary constituents by a few shallow-water constituents indicates that we have a powerful method of dealing with an intricate problem.

7. *Mathematical expansions.*

Consider the elevation of tide given by:

$$\zeta = Z_0 \cos(\sigma_0 t - \varepsilon_0) + \Sigma\, Z \cos(\sigma t - \varepsilon) \qquad (9)$$

where zero suffix denotes M_2, regarded as predominant.

Then the times of high or low water for M_2 and for the compound tide may be denoted by t_0, t respectively, if we restrict t in this way and we may write:

$$\sigma_0 t_0 = \varepsilon_0 + m\,\pi,\quad \sigma_0 t = \varepsilon_0 + m\,\pi + \eta \qquad (10)$$

where m is an integer, and η/σ is the difference in time of the high or low waters of M_2 and the compound tide. Also write:

$$\left.\begin{aligned} \theta &= (\sigma-\sigma_0)\,t - (\varepsilon - \varepsilon_0), & Y &= Z/Z_0 \\ p &= \Sigma\, Y \cos\theta, & q &= \Sigma\, Y \sin\theta \\ \sigma_0 p_1 &= \Sigma\, \sigma\, Y \cos\theta, & \sigma_0 q_1 &= \Sigma\, \sigma\, Y \sin\theta \end{aligned}\right\} \qquad (11)$$

where the expressions for the elevation and for its maxima or minima become:

$$\left.\begin{aligned}\zeta &= Z_0 \cos(\eta + m\pi) + \Sigma\, Z \cos(\eta + m\pi + \theta)\\ o &= \sigma_0 Z_0 \sin(\eta + m\pi) + \Sigma\, \sigma Z \sin(\eta + m\pi + \theta)\end{aligned}\right\} \quad (12)$$

whence:

$$\tan\eta = -\frac{q_1}{1+p_1}, \quad \frac{\zeta}{Z_0 \cos m\pi} = (1 + p)\cos\eta - q\cos\eta \quad (13)$$

It is proposed to regard p, q, p_1, q_1, η as small quantities and to expand η and ζ/Z_0 throughout to the third order of small quantities. These restrictions provide criteria for the degree of predominance of M_2. Then, since

$$\eta = \tan\eta - {}^1/_3 \tan^3\eta + \dots$$

we obtain:

$$\left.\begin{aligned}\eta &= -q_1 + p_1 q_1 - p_1^2 q_1 + {}^1/_3\, q_1^3 + \dots\\ \frac{\zeta}{Z_0 \cos m\pi} &= 1 + p + q\, q_1 - \tfrac{1}{2} q_1^2 + p_1 q_1^2 - \tfrac{1}{2} p\, q_1^2 - p_1 q_1 q + \dots\end{aligned}\right\} \quad (14)$$

The function θ is not uniformly variable with time, and it is therefore desirable to replace it by a function φ which will increase in phase at a uniform rate per mean lunar day. Such a function is given by:

$$\left.\begin{aligned}\varphi &= \frac{\sigma - \sigma_0}{\sigma_0} m\pi - (\varepsilon - \frac{\sigma}{\sigma_0}\varepsilon_0)\\ \text{with } \theta &= \varphi + \frac{\sigma - \sigma_0}{o}\eta = \varphi + \xi, \text{ say}\end{aligned}\right\} \quad (15)$$

which may be derived by using (10) to eliminate t from θ in (11).

Since $\cos\theta$ and $\sin\theta$ occur only with small factors it is sufficient to expand them to the second order of small quantities whence:

$$\left.\begin{aligned}\cos\theta &= (1 - \tfrac{1}{2}\xi^2)\cos\varphi - \xi\sin\varphi\\ \sin\theta &= (1 - \tfrac{1}{2}\xi^2)\sin\varphi + \xi\cos\varphi\end{aligned}\right\} \quad (16)$$

It is now convenient to write:

$$\left.\begin{aligned}P &= \Sigma\, Y\cos\varphi, \quad Q = \Sigma\, Y\sin\varphi\\ P_\eta &= \Sigma\left(\frac{\sigma}{\sigma_o}\right)^n Y\cos\varphi, \quad Q_\eta = \Sigma\left(\frac{\sigma}{\sigma_c}\right)^n Y\sin\varphi\end{aligned}\right\} \quad (17)$$

and thence:

$$\left.\begin{aligned}p &= P - \tfrac{1}{2}\eta^2 (P - 2P_1 + P_2) + \eta (Q - Q_1)\\ q &= Q - \tfrac{1}{2}\eta^2 (Q - 2Q_1 + Q_2) - \eta (P - P_1)\\ p_1 &= P_1 - \tfrac{1}{2}\eta^2 (P_1 - 2P_2 + P_3) + \eta (Q_1 - Q_2)\\ q_1 &= Q_1 - \tfrac{1}{2}\eta^2 (Q_1 - 2Q_2 + Q_3) - \eta (P_1 - P_2)\end{aligned}\right\} \quad (18)$$

Substituing into (14) gives to the second order:

$$\eta = -Q_1 + P_2 Q_1$$

and on substitution of this into (18) we get:

$$\left.\begin{aligned}p_1 q_1 &= P_1 Q_1 - Q_1^2 (Q_1 - Q_2) + P_1 Q_1 (P_1 - P_2)\\ q\, q_1 &= Q\, Q_1 + Q_1^2 (P - P_1) + Q\, Q_1 (P_1 - P_2)\\ -\tfrac{1}{2} q_1^2 &= -\tfrac{1}{2} Q_1^2 - Q_1^2 (P_1 - P_2)\end{aligned}\right\} \quad (19)$$

whence:

$$\left.\begin{array}{l} \eta = -Q_1 + P_2 Q_1 - P_2^2 Q_1 - {}^1/_6\, Q_1^3 + \tfrac{1}{2} Q_1^2 Q_3 \\ \dfrac{\zeta}{Z_0 \cos m\pi} = 1 + P + \tfrac{1}{2} Q_1^2 - \tfrac{1}{2} P_2 Q_1^2 \end{array}\right\} \qquad (20)$$

The perturbations of M_2 can thus be resolved into true harmonic terms in φ.

8. *Deductions from the mathematical expansions.*

The first deduction from the mathematical expansions is that in P and Q the increments of phase in a lunar day are the same as for the ordinary tidal constituents in (9), and that the relative amplitudes are also the same as in (9). The relative phases may not be the same at any given time but this does not affect the matter as we are principally interested in possible speeds and relative amplitudes in order to ascertain whether methods of analysis and prediction are possible.

Then the speeds of the terms contributing to $P_2 Q_1$, and $\frac{1}{2} Q_1^2$ are the same as those contributing to $(\zeta - M)^2$, but the relative amplitudes are somewhat different because of the factors $(\sigma/\sigma_0)^n$, though in any species these factors will be much the same for all constituents. Thus, apart from the phase-changes, the speeds are exactly the same as could be derived from the normal shallow-water constituents derived from ζ^2, while the relative amplitudes within a species are much the same.

Similar conclusions apply to the terms of the third order in (20).

The next deduction is that if we have a partial tide composed of principal constituents, then, relative to M_2, this partial tide will give expressions η' and ζ', with expansions in dashed variables according to (20). The perturbations of this partial tide by the remaining constituents of the full tide are thus equal to $(\eta - \eta')$ and $(\zeta - \zeta')$. It is again true that no new speeds are introduced, and previous conclusions as to the amplitudes to be expected are substantially unchanged, save that the major shallow-water constituents will have been included in the partial tide. The residual constituents will have only small amplitudes and it may be concluded that the expansions for $(\eta - \eta')$ will be much more rapidly convergent than those for either η or η', and so also for $(\zeta - \zeta')$.

Hence the remarkable fact emerges that the differences in time and height of the full tide and the partial tide at high or low water at non-uniformly spaced times may be expressed by true harmonic variations at constant intervals of time.

A third deduction is that exactly the same principles may be applied to the derivation of a tide at place B from the tide at place A, if the two places are so close together that the tides do not vary greatly as between the two places. The tidal prediction for A may be regarded as a partial or basic tide for predictions for B and corrections may be applied as though the basic tide had been taken from constituents at B. Any variations in the major constituents for the two places are automatically included. Again, the tide for A may itself require predictions to be based upon a partial tide and corrections, but the principles of application remain.

This last deduction is of very great importance, for it means that in an estuary tidal predictions for many places may be obtained by corrections applied to the predictions for the tide at the mouth. There is great flexibility in the method because the same partial tide may be taken for many ports, which mak es for cheaper predictions, or each port may be systematically related to the next in order towards the mouth. Again, if the tide at B differs by a few hours from that at A, and has a smaller range, then appropriate modifications of the predictions for A, by factors and phase shifts, may be made to give the partial or basic tide for B.

Experience has shown that this method is very powerful and has many varied applications. Without it, predictions for many important ports would be very inaccurate, and for some places almost impossible, particularly where the diurnal constituents are large and comparable in amplitude with the semidiurnal constituents. Examples of maximum corrections (irrespective of sign) for the year 1955 are shown below:

	Liverpool	Avonmouth	Antwerp
High water time (mins)	9	17	40
Low water time (mins)	16	61	37
High water height (feet)	0.9	3.0	1.6
Low water height (feet)	0.6	3.1	2.4

9. *Preliminary notes on the analysis.*

In (15) it is convenient to write

$$m = s + 4L$$

where L = number of mean lunar days elapsed from the origin of time,

s = the sequence number, 0, 1, 2, or 3.

There are thus four sequences of high and low waters, following one another at approximate intervals of 6 hours. Sequence $s=0$ pertains to the first high water of M_2 following the lower transit of the mean moon. Sequences $s=1$, 3 are low-water sequences. It is important that the order of these sequences be strictly observed, and for precision we define the exact time of lower transit of the mean moon by $(s-h)/14°.492$ mean solar hours, where *s* and *h* are the normal symbols for the mean longitudes of moon and sun. The origin of time is taken at this value on the central day of the observations.

In each sequence we have variations expressed by

$$\Sigma\, R \cos (\mu L - \delta)$$

where μ is derived from the variable part of φ in (15); that is, from $m\,\sigma\,\pi/\sigma_0$. For any constituent this is given by:

$$\mu = (24.8410 \times \sigma) - \text{multiples of } 2\pi.$$

But we take the value of μ as irrespective of its sign.

Analogously to the methods normally used in tidal analyses we shall write:

$$\mu L - \delta = W - \chi$$

where W is defined in terms of the elements of the lunar and solar orbits (s, h, p), taken at the precise origin of time, and χ is the phase-lag. The elements of orbits for the constituents are given on the last of the forms in the example of analysis, tables 7 and 8.

It is not considered necessary to enter into the theory or justification of the methods of analysis, which are very similar to those used in the analysis of hourly heights of tides as are commonly used at the Tidal Institute. A simple introduction and explanation of these methods is found in the Admiralty Manual of Tides, but the complete theory is given in the following paper: « The analysis of tidal observations », by A. T. DOODSON, Phil. Trans. Roy. Soc., Vol. 127, pp. 223-279, 1928.

See also the following: « The analysis of tidal observations for 29 days », by A. T. DOODSON, *Int. Hydr. Review*, Vol. XXXI, pp. 63-91, 1954.

10. ***Instructions for analysis.***

General instructions, with application to an example are given below. The example is for Rosyth, for which observations of high and low water are compared with a primary prediction for Dunbar, a few miles away from Rosyth.

(a) The observations (O) and primary predictions (P) are given as in table I, for which only a few illustrations are given here, so as to illustrate the next procedure. It will be presumed that the observations are reasonably complete, so that any gaps may be filled by inference, as below.

(b) The first procedure is to fill in details of g of M_2 for the basic prediction, *s, h, p,* and $(s—h)/14°.492$ as in tables 7 and 8, from which the mean high water interval and origin of time on the central day may be obtained. It is convenient to verify from a nautical almanac that this time is approximately the same as is given for the time of the lower transit of the moon at Greenwich.

(c) Having obtained the origin of time, then we fix the sequences $s=0$, 1, 2, 3, on the central day. The high water for $s=0$ on that day will be nearly the same as the origin of time plus the mean high water interval (g of M_2)/28°.984. Put a ring in black round the values of time and height of high water and mark it $s=0$. Then the next high water, approximately 12 hours later, should be ringed and marked $s=2$. The low water data approximately 6 and 18 hours later than for $s=0$ should be similarly distinguished.

(d) Prepare 8 forms similar to those in tables 2 and 3, but with 29 rows, —14 to 14 in each column, and mark each column with dates as follows.

If the central day is July 1, and the year is not a leap year, then the dates are: Jan. 5, Feb. 4 (Mar. 5), Apr. 4 (May 3), Jun. 2 (Jul. 1), Jul. 31 (Aug. 29), Sep. 28 (Oct. 27), Nov. 26.

And where the dates are given in brackets mark off the first entry in the top of the column in the example; similarly mark off the bottom entry in the preceding column. The last day in the last column is Dec. 25. If the central day is July 1, and the year is a leap-year, then the first two days are replaced by Jan. 6 and Feb. 5. If the central day of analysis is not July 1, then the dates must be computed from the following table which gives the number of solar days from the central day:
—177, —147, (—118), —88, (—59), —29, (0), 30, (59), 89, (118), 148 and the last entry is on day 177.

By this choice of initial days for the columns we get 29 entries for a month, and in each month the principal constituent C(25) approximately repeats itself but at intervals of about 28.5 lunar days. In order to get the best possible repetition certain entries in the tables are duplicated, so that an entry in the marked-off space at the foot of a column is repeated at the top of the next column, also in a space which has been marked off.

(e) For $s=0$, the time of high water is approximately the same for all the days entered at the tops of the columns. Therefore the time for this sequence is noted for the central day and for all the columns, for the days stated, the corresponding high water is circled by a ring and marked $s=0$. Beginning with the first column the values of O—P are entered in order, at intervals of about a solar day, and for each column the work is checked by seeing that the next column begins at the high water which has been marked $s=0$. The duplication of entries must also be observed.

Proceed similarly for the sequence s=2, which is 12 hours later than the sequence s=0, and similarly complete the tables for s=1 and s=3, which are respectively 6 and 18 hours later than for s=0.

It is well to verify the entries for the last day, December 25 or day 177, and great care must be taken to maintain the sequence. Note that when the times pass through 24 hr., there may be a gap in the table of observations but there should be no gap in the sequences on this account.

Also note that the column headings for the days are given in all cases for s=0 only, though one or more of the sequences will have entries a day later than stated, but it is inadvisable to alter the dates for the different sequences, and it is not necessary to do so if sequence s=0 has been correctly marked for each column.

The example should be carefully studied.

(f) If observations are missing then there will be gaps in the sequences but enough spaces should be left for the number of days elapsed. When each table has been filled up then the gaps may be filled by interpolation in the columns or in the rows or both. Also if there are obvious errors then these may be rectified by judicious smoothing, but excessive smoothing should be avoided. In the example no smoothing has been attempted though in some instances it might have been advisable.

(g) Table III gives sets of multipliers for the first process of analysis, which is the combination of the data O—P. The multipliers are applied to each row and the products are summed so as to obtain from each row of each table functions with suffixes as follows:

0.0, 0.1, 0.2, 0.3, 0.a, 0.b, 0.c, from s = 0
1.0, 1.1, 1.2, 1.3, 1.a, 1.b, 1.c, from s = 1

and similarly for s = 2 and 3.

The central dot in the notation is used because we are ultimately to obtain quantities denoted by T_{sdm} and H_{sdm} where T and H refer to times and heights, s denotes the sequence, d indicates the daily multiplier, and m denotes a monthly multiplier. It is convenient to use the multiplier first for the rows, because we then get 29 quantities in a column which can be scrutinised for smoothness. See table 4 for an example. The results can be checked by a summation method. The last row of table III contains the sum of the multipliers and if these are applied to the data for O—P the result should agree with the sum of the seven quantities which have been entered for the row which is under test.

(h) The second process of analysis uses the daily multipliers of Table IV. These are placed alongside the columns of $T_{0.0}$, etc., and for each column the products are summed and divided by 1000. The results are indicated by suffixes in which the middle figure or letter indicates the daily multiplier used. The results are arranged in groups as follows:

000; 001, 00a; 002, 00b; 010, 0a0; 011, 01a, 0a1, 0aa;
020, 0b0; 022, 02b, 0b2, 0bb; 031, 03a, 0c1, 0ca;
033, 03c, 0c3, 0cc; 040, 0d0; 042, 04b, 0d2, 0db.

Similar results are obtained for other sequences. The results are placed in order as in table 5. This process is most conveniently checked by repetition.

(i) The third process of analysis combines the sequences s=0, 2 and s=1, 3 in order to separate, as far as possible by a simple process, the terms which change sign every 12 hours from those which retain the same sign; that is, the terms which have diurnal characteristics are separated approximately from the rest. The combinations given in tables V and VI give first approximations to R cos δ and R sin δ. Thus, for C(02) the multipliers

2.67, 2.67, —0.01, —0.01

applied to

T_{002} T_{202} T_{00b} T_{20b}

give a first approximation to R cos δ for C(02) in T.

The results are placed in the order given in table 6. Checks are possible by applying the multipliers in the last column of tables V and VI, and the result gives the sum of values of R cos δ, and R sin δ, for the constituents mentioned. Thus the check applied to s11, s1a, sa1, saa, gives the sum of R cos δ and R sin δ for both C(11) and C(13).

(j) The first approximations do not sufficiently eliminate the effects of one constituent upon another, and this can only be remedied by applying the multipliers of tables VII to X. The process consists of summing the products of the multipliers with the first approximations. Thus the column of multipliers given under C(11) in table VII, when placed alongside the values under HT in table 6, give a total sum of products which, after dividing by 1000, is R cos δ for C(11) for HT. It should be noted that the entries of table 6 should be carefully set out so that they can be aligned with the tables of multipliers, and that the whole column under HT, whether derived from first approximations to R cos δ or R sin δ, should be used with the whole column of multipliers. The results are placed in the appropriate spaces in tables 7 and 8. Checks can be made by applying the multipliers in table XI in the same manner to give sums for groups of constituents. The effects of this process are small, so that there should not be much difference between the first and second approximations.

(k) The computation of R and δ is effected in tables 7 and 8 in the same way as in normal tidal analyses.

(l) As parts of tables 7 and 8 were filled in at the commencement of the analysis it only remains to compute W and $\chi = W + \delta$. The expressions for W in terms of s, h, p are given in the tables and the combinations are made with the values at the time origin on the central day. There are some simple checks on W which are sufficient, and these are indicated in the tables.

11. *Prediction by using normal tide-predicting machines.*

It is, of course, obvious that the synthesis of these special tidal constituents could best be effected by means of a special machine, but the expense of construction could not be justified at the time when the method was first designed, as it was felt that the method should be used for a long period until experience had been gained. At first the method was only used for places where diurnal tides were small, so that the constituents C'( ) were not needed. As experience was gained the method was extended to more general types of problems, and though it has been in use now for 30 years it has not been found absolutely necessary to make a special machine, though one would be useful.

It has been indicated that these shallow-water constituents have periods of a month and sub-multiples thereof, so that a constituent with a monthly period could be set on a diurnal component of a machine, and a constituent with a fortnightly period could be set on a semidiurnal component, and so on, provided that the speeds of those components were suitable. A special time-scale would be required since a diurnal component has a speed of about $360^\circ/24$ per hour while a monthly constituent has a speed of about $360^\circ/28$ per lunar day, so that a time scale of about 24/28 solar hour per lunar day would be required. By trial it was found that a time scale which was most appropriate for the larger constituents was 48/55 solar hour per lunar day.

It was found that it was possible to choose machine components which are found normally on the larger machines and that these components needed only small changes of angle at infrequent intervals of time for a satisfactory process of prediction. It is convenient to introduce the notation:

$$MT = \text{machine time.}$$

A special dial is necessary to indicate intervals of half a lunar day, the average interval from one high water to the next one. For the Kelvin and Légé machines first used by the Tidal Institute a paper scale round the drum for the record was used, this paper scale being divided into 55 equal parts, alternately by lines in black and red ink. For all the Doodson-Légé machines which have been constructed in recent years a small dial has been included with the time dials, and this small dial rotates 55 times in 48 hours of solar or machine time, and it has two graduations, one in white and one in red, at 180° apart. The white line is used for sequences $s=0$ and $s=1$, and the red line for sequences $s=2$ and $s=3$. This device facilitates prediction when there are constituents of diurnal origin.

Table XII shows the components used for the constituents, with fundamental data as follows:

μ = speed of constituent in degrees per lunar day,

$(48/55)\ \sigma$ = increment of phase of component in 2/55 of one day of MT,

c = correction to phase of component after each day of MT,

= 27.5 times the correction per lunar day.

It will be noted, by comparing with table II, that the Doodson-Légé machines have not enough third-diurnal components to be able to represent C(36), C'(36), C'(40) in addition to C(38) and C'(38). The three components omitted are much smaller than those included, and it should be remembered that there are other small constituents which have not been considered worthy of representation.

If constituents derived from diurnal species are not required then an alternative choice may be made for the rest, by representing C(36), C(38) on MO_3 and MK_3, with $c = -14^\circ.15$ and $9^\circ.27$ respectively.

Similarly if the constituents derived from the semidiurnal tide were of little importance better choices could be made for the other species.

Some of the Doodson-Légé machines have only 30 components, and for one of these machines (Thailand) the constituents $2SM_2$ and S_4 are not available. The constituents S_2 and MS_4 can be chosen instead, with $c = 30^\circ.1$ and $28^\circ.5$ respectively.

It is not possible to include instructions for all existing machines, but the principles are sufficiently explained to make it possible to adapt the method to the available machines.

The corrections c may appear to be large in some cases, but it will be realized that when the correction is made the error changes from $-0.5c$ to $0.5c$;

also that the larger angle corrections apply to the smaller constituents, so that actually the corrections do not give any visible evidence of error in the indications of the machine by reason of any obvious discontinuities.

It will be noted that all species are run together, but this can only be for intervals of a lunar day, approximately. Thus the machine is first set to run off the values of HSWC for the sequence s = 0. Then the machine is reset with the angles for C'( ) changed by 180°, so as to give the sequence s = 2. Similarly the low-water corrections are run off. Though this means 8 runnings of the machine they are only for 13 days each, and the whole is less than the running of the normal high and low water times and heights.

One important point needs to be mentioned. No nodal quantities f and u can be used for these constituents since each is an aggregate of normal harmonic constituents of many species, each with different laws for f and u, and there is no possibility of obtaining theoretical guidance for the composite values. But if the basic predictions include all major constituents of the tide then the nodal variations of the remaining constituents are quite negligible.

Certain precautions need to be taken. Thus it is advisable to mark on the basic predictions the high water for the sequence s = 0, which is the one following the commencement of each day of MT by a certain interval appropriate to the place, being the time of high water of M_2 for the basic prediction, after the lower transit of the mean moon. This is facilitated by the use of the first column of table XIII, which gives the mean solar time corresponding to the zero hour of each day of MT. This part of the table is also given on the panels of the DL machine, and a table for the interpretation of the number of the day, as in table XV, is also given on the panels.

As there may be times when doubt has arisen as to the synchronization of the corrections with the basic predictions, table XIII has been prepared to provide a check. This table can readily be interpolated or it can be extended very easily for more frequent intervals in n, the number of the division on the drum (55 in all), or for the corresponding times on the dials. T + 6s has to be added according to the sequence.

There is also the possibility of making errors when adding the correction c at the end of the day of MT, and so it is advisable to check the results against calculations of the angular settings for each day of MT, or at least for every third or fourth day. This requires the use of table XIV.

12. *The use of special dials.*

The need for ensuring that there is no error in making the angular correction each lunar day necessitates some initial calculation of angles which can be rather tedious. On the Doodson-Légé machine in use at the Tidal Institute, this has been overcome by adding supplementary dials to the component angle-dials. Figs. 1 and 2 illustrate two of these attachments, which were made, engraved, and silvered by the staff of the Institute. The dials rotate with the angle-dials but are movable with respect to them against frictional clamps. An accessory pointer is used for these supplementary dials; see, for example, fig. 1, where the accessory pointer is at 106° on the angular scale. When the corrections have been set in amplitude and angle for zero day of the machine the inner dials are rotated until zero day is shown against the accessory pointer. Then the corrections are read off from the machine for about a month until the machine time-dial is at day 1. The accessory pointer will then be nearly at day 1, on the inner dial, and the necessary correction in angle is made by unclamping the component and revolving it slightly until the day 1 appears against the pointer. In fig. 1 the component is C(25) and the correction of angle required per day of machine time is —2°.

For those corrections which are diurnal in character the inner dials are graduated in red.

It has been found that by the use of these supplementary dials and by the use of the improved methods now available the corrections for a year may be run off in 8 hours.

13. *Instructions for predicting HSWC.*

(a) A specimen calculation of $W + \frac{1}{2}c$ for the year 1952 is given in table XVI. The values of *s, h, p* for zero hour on January 1, 1952 and for 1953 are obtained from the standard tables, and from these we compute:

$$t_0 = (s-h)/14^\circ.492$$

and then increments for this time are added to *s, h, p* to give values for the origin of time, which is the time of lower transit of the mean moon at Greenwich.

The values of W are obtained from the formulae; thus for C(11) the formula gives $W = s - 2h + p$. Independent calculations are made for 1952 and 1953 and then the differences are compared with the check values appropriate to the value of the increment in t_0 which depends upon the number of days elapsed. It may be noted that as t_0 may be positive or negative we have to allow for all cases of increment in t_0 both for common years (CY) and for Leap Years (LY). The constituents are arranged in the example to suit the Doodson-Légé machine so as to require minimum movement of the sliding doors. Labour would be saved by a standard table of $W+\frac{1}{2}c$.

(b) A setting card for the amplitudes and phases of the components is then prepared. It is helpful to enter all values for C( ) in black ink and those for C'( ) in red ink. For the times HT and LT enter the values of R/10 so as to use a scale of 10 minutes per 2 cms of the dial. For the heights HH and LH enter the values of R so as to use a scale of 2 cms per foot. Enter χ for all constituents and compute:

$$\alpha = W + \tfrac{1}{2}c - \chi$$

for the time origin on January 1, 1952 and 1953. A check on the computation of α is made in the same way as for $W + \frac{1}{2}c$, according to the increment in t_0 and the character of the year. Enter (g of M)/28°.984 from the lists of harmonic constants and add t_0 which gives

T = approximate time of high water on January 1, 1952.

Enter the value of MT at the end of the year from the card for $W+\frac{1}{2}c$ according to the increment in t_0 and the character of the year.

(c) Table XIII, n=0, gives a list of dates and approximate times of HW shortly after the commencement of MT=0, 1, ... 13, using the value of T on the setting card. For even values of MT a ring in black ink should be placed round the times in the basic predictions which correspond to those given in the table, and a ring in red ink should be placed round the times corresponding to odd values of MT.

In the following instructions (M) will refer to any machine which uses a special scale graduated with 55 divisions per day of the drum, while (DL) will refer to the Doodson-Légé machine which is fitted with a small dial alongside the time-dials, and which has a white line and a red line engraved upon it.

(d) Adjust the zero readings in the usual manner, and set C(00) on the scale used, which will usually be the scale with graduations of 2 cms per ft. Verify for the DL machine that the white line is showing when the time dials read Od, Oh, Om. If the red line is showing, run the machine for one day and reset the day dial. For (M) the special scale on the drum should read zero.

(e) Set the amplitudes and angles in the usual manner from the setting card for HT. For the DL machine turn the inner dials so that the special pointer is opposite 0.

(f) Commence to take readings. For (DL) the machine is turned by hand, after removing the driving belt from the motor, or it may be otherwise controlled. The first reading is entered against the HW marked with a ring in black ink on the first day, and subsequent readings are taken when the white line shows on the dial at approximate intervals of a lunar day.

For (M) the readings are taken at intervals of 2 graduations on the special scale.

(g) Table XIII is useful for checking the relation of MT with the day and approximate hour of high water. This is useful, if at any time there is doubt or if computers are changing duties, or at the beginning of a day's work. As an example, if T is 14 and the machine shows 000d, 08h, 44m, then the time of high water is approximately 11d, 22h, for M_2 in the basic tide. This check only indicates the sequence, for the actual time of HW may differ appreciably from that of M_2. Table XV interprets this time as Jan. 11, 22h. If the machine shows 003d, 08h, 44m then the high water of M occurs at 96d, 18h plus *T*, or 97d, 8h, and this day is April 7 for CY.

(h) If all is correct at the beginning of day MT = 1, then the angles of the components should be amended. This may be done by unfastening the clutch, and for the DL machine by rotating the angular dial until the inner scale reads 1 against the special pointer, after which the clutch is tightened again. For other machines the correction may be made by adding, according to sign, the amount c which will be shown by a small tab attached to the component. If a table of angle settings for MT = 1, 2, ... 13 has been prepared as in table XIV then this may be used with less risk than depending on the tab, as was indicated in § 11.

(i) Continue running in the same way, reading only against the white line, and continuing in the same sequence. This second day will terminate with a white line at the beginning of day MT = 2, and the entry for that line should be against the basic reading which has been marked with a black ring. The angular corrections are again made, and the running is continued, with the days terminating alternately with white lines and red lines on the small dial for the DL machine.

(j) Towards the end of MT = 12, the end of the year will be reached. Without entering further readings run the machine to zero hour of MT = 13, and make the corrections, verifying that the changes are approximately as indicated also by the tab. Then run back a few days to the end of the year for which the MT will be as indicated on the card for W + $\frac{1}{2}$ c for the appropriate increment in t_0 or on the setting card, while the approximate time of HW will be T hours later. This verifies that no error has been made in the sequences. Then proceed forward until day MT = 13 is reached, but enter the readings on the « comparisons sheet » which is used as a check when doing the predictions for the next year. Verify that the readings for zero hour on MT = 13 agrees with the entry on the basic predictions which has been enclosed in a red ring.

(k) HT has now been completed for sequence $s=0$, and the procedure for sequence $s=2$ is similar, but the amplitudes remain as already set, so all that is necessary for setting is to run the machine forward to zero hour, with a white line showing, then change the time dials to zero for the day, and alter all angles to read as are given on the setting card for MT=0, but with a change of 180° for all components C'( ); i.e. for those indicated in red ink on the setting card. Do not forget C'(00). Rotate all inner scales to read 0.

(l) If a prediction is required on Jan. 1 about 12 hours before the one already obtained run the machine back until the red line appears; otherwise turn forward to the red line and enter the reading 12 hours later than the one marked with a black ring.

(m) Proceed for this sequence $s=2$, but reading always on the red line.

(n) For any checks or comparisons remember that all angles for components C'( ), those given in red ink, will differ by 180° from those on the card.

(o) When comparisons are completed, reduce all amplitudes to zero, verify that C(00) is correctly shown on the dial, and then check that all amplitudes are zero, paying special attention to the long-period components.

HT is now fully completed, and LT may be done in the same way, but noting that for all checks the time $T+6$ is used instead of T. In a similar manner HH and LH are completed.

14. *Predictions when diurnal corrections are not required.*

When the diurnal corrections are negligible then it is not necessary to have two runnings to give readings for $s=0$ and $s=2$, for the latter can be obtained by simple interpolation in those for $s=0$. Similarly readings for $s=1$ can be interpolated to give readings for $s=3$. This reduces the labour very considerably.

15. *Analysis when the basic prediction includes some HSWC.*

It is a simple deduction that when a prediction has been obtained for a year, and that prediction includes some HSWC, then we may take that prediction as basic for a subsequent analysis. There are no changes in procedure for the analysis, but the results are corrections to the Corrections used in the prediction. The two should be vectorially averaged. If in the predictions the corrections used R_0 and χ_0 and if the small corrections give R' and χ', then if the former are derived from N years of analysis the new constants should be vectorially obtained from:

$$R_0 \cos \chi_0 + \frac{1}{N+1} R' \cos \chi', \; R_0 \sin \chi_0 + \frac{1}{N+1} R' \sin \chi'$$

16. *Alternative method of prediction.*

An alternative method of prediction is possible and was used for many years after the year 1936 for the prediction of corrections which included those with diurnal characteristics. Consider the constituent C(25) which has a phase increment per lunar day which is identical with that of the normal constituent S_2. If therefore the values of R and α were set on this component of the machine, while the angle of M_2 was set at zero at zero hour, then at every two revolutions of the component M_2 the angle shown on S would agree with that for C(25). No corrections c are required and there is no difficulty with regard to the synchronization of basic prediction and correction. But since the component S_2 makes two revolutions in the day there are two oscillations superposed upon the required fortnightly variation; that is, in all there are 28 oscillations in the fortnight and yet the variation sought has only one. It is essential therefore to have very exact readings of the zero angle in M_2 when the indications of the machine are taken.

The following table gives the components used for the shallow-water constituents:

C(01)	C(02)	C(11)	C(13)	C(25)	C(27)	C(36)	C(38)	C(50)	C(52)
Sa	Ssa	SN_4	L_2	S_2	K_2	$2SN_6$	MNS_2	S_4	MSK_6

C'(00)	C'(11)	C'(12)	C'(13)	C'(27)	C'(36)	C'(38)	C'(40)
M_3	P_1	S_1	K_1	J_1		SK_3	

It will be noted that some constituents are not represented by components for the old Légé machine which was used with this method. It may well be that other machines may not be so well adapted to employ this method.

It is necessary to remark that the component MNS_2 needs special treatment for its increment in a lunar day is negative. Therefore, instead of setting α as for the other constituents it is necessary to set $(360° - \alpha)$.

The constituent M_3 takes $\alpha = 0°$ when C'(00) is positive and $\alpha = 180°$ when C'(00) is negative. The constituent M_1 may be used in place of M_3.

The principal disadvantage of this method is that the four runnings of the machine to give the corrections take about 18 hours.

17. *Mean tide level.*

The time of mean tide level can be dealt with in exactly the same way as the times of high and low water. The value of a prediction of mean tide level would be that it would then be possible to employ the so-called « reduction curves » for much shorter intervals than at present. Reduction curves give the average shapes of the tide curves from low water through high water to low water, usually related to the time of high water, but often related to high or low water from mean tide level to mean tide level. If the time of mean tide level is accurately predicted then it would only be necessary to deal with a « quadrant » from high water or low water to the adjacent mean tide level. This would be more accurate than the use of longer spans of time.

18. *Hourly heights of tide.*

If heights of tide at regular hourly intervals are required then the preceding methods may be applied if the heights are at intervals in lunar time. It is not possible to employ the methods to give heights at intervals of solar hours. The use of reduction curves could also give heights at times related to high water.

If heights at intervals of hours in solar time are required then some diminution in accuracy must be accepted. A possible method of correcting the machine predictions is to assume that the eighth, tenth, and twelfth diurnal tides can be functionally related to the semidiurnal tide as was indicated in § 4. It is possible that a correction to the semidiurnal tide might also be found to be necessary as arising from the shallow water variations which are generated along with the sixth diurnal tide.

The Doodson-Légé machines are fitted with components M_8, M_{10}, and M_{12} for this method to be used, along with M_2 if semidiurnal corrections are necessary. This method is still under investigation and no general rules can as yet be given. The components named would have to be altered in amplitude and phase every day according to the variation in the semidiurnal tide.

TABLE I: VALUES OF μ FOR PRINCIPAL CONSTITUENTS.

μ = increment of phase in degrees per mean lunar day

μ	Constituents	μ	Constituents	
-40°·6841	$2Q_1$	...		
−27·1612	Q_1	-27°·0458		$2N_2$
...		−25·2361		μ_2
−13·6382	O_1, MO_3	−13·5229		N_2, MN_4, $2MN_6$
...		−11·7132		ν_2
0·0000	M_1, M_3	0·0000	M_0,	M_2, M_4, M_6
...		1·0202	Sa	
...		2·0404	Ssa	
11·5978	P_1	11·7132		λ_2, SN_4, MSN_6
12·6180	S_1	...		
13·6382	K_1, MK_3	13·5229		L_2
...		24·2159		T_2
...		25·2361		S_2, MS_4, $2MS_6$
27·1612	J_1	27·2765		K_2, MK_4, $2MK_6$
40·9147	OO_1	...		
		50·4721		$2SM_2$, S_4, $2SM_6$

TABLE II: LIST OF PRINCIPAL HARMONIC SHALLOW-WATER CONSTITUENTS.

Symbol	μ	Constituents represented	
C(00)	00°000	M_4, M_6, M_8, M_{10}, M_{12}, ...	
C(01)	01·020	Sa, MST_2, MTS_2, ...	
C(02)	02·040	Ssa, MKS_2, MSK_2, $2MKS_4$, $2MSK_4$, OP_2, ...	
C(11)	11·713	SN_4, MSN_6, $2MSN_8$, SNM_2, ...	
C(13)	13·523	MN_4, ML_4, $2MN_6$, $3MN_8$, ...	
C(25)	25·236	MS_4, $2MS_6$, $3MS_8$, $4MS_{10}$, ...	
C(27)	27·276	MK_4, $2MK_6$, $3MK_8$, ...	
C(36)	36·949	$2SN_6$, $2SMN_8$, ...	
C(38)	38·759	MSN_2, MNS_2, $2MSN_4$, $2MNS_4$, ...	
C(50)	50·472	$2SM_2$, S_4, $2SM_6$, $2(MS)_8$, ...	
C(52)	52·513	SK_4, MSK_6, $2MSK_8$, ...	
C'(11)	11·598	P_1, MP_1, SO_3	also 5th diurnal and 7th diurnal
C'(12)	12·618	S_1, MS_1	
C'(13)	13·638	K_1, O_1, MK_3, MO_3	
C'(27)	27·161	J_1, Q_1, MJ_3, MQ_3	
C'(36)	36·834	SP_3	
C'(38)	38·874	SO_1, SK_3	
C'(40)	40·915	OO_1, KO_3	

TABLE III: MONTHLY MULTIPLIERS.

Suffix	Monthly Multipliers											
0	1	1	1	1	1	1	1	1	1	1	1	1
1	-2	-1	-1	1	1	2	2	1	1	-1	-1	-2
2	1	0	-1	-1	0	1	1	0	-1	-1	0	1
3	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
a	-1	-2	-2	-2	-2	-1	1	2	2	2	2	1
b	1	2	1	-1	-2	-1	1	2	1	-1	-2	-1
c	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
Check	-2	0	0	-2	-4	2	8	6	2	0	2	0

TABLE IV: DAILY MULTIPLIERS.

Day	d_0	d_1	d_2	d_3	d_4	d_a	d_b	d_c	d_d
-14	1	-2	2	-2	2	0	-1	1	-1
-13	1	-2	2	-1	1	1	-1	2	-1
-12	1	-2	1	0	-2	1	-2	2	-2
-11	1	-1	0	1	-2	1	-2	1	0
-10	1	-1	-1	2	-1	2	-2	0	1
-9	1	-1	-2	2	0	2	-1	-1	2
-8	1	0	-2	1	2	2	-1	-2	1
-7	1	0	-2	-1	2	2	1	-2	0
-6	1	1	-2	-1	1	2	1	-1	-2
-5	1	1	-1	-2	-1	2	2	-1	-2
-4	1	1	0	-2	-2	1	2	1	-1
-3	1	1	1	-1	-2	1	2	2	1
-2	1	2	1	1	0	1	1	2	2
-1	1	2	2	2	1	1	1	1	2
0	1	2	2	2	2	0	0	0	0
1	1	2	2	2	1	-1	-1	-1	-2
2	1	2	1	1	0	-1	-1	-2	-2
3	1	1	1	-1	-2	-1	-2	-2	-1
4	1	1	0	-2	-2	-1	-2	-1	1
5	1	1	-1	-2	-1	-2	-2	1	2
6	1	1	-2	-1	1	-2	-1	1	2
7	1	0	-2	-1	2	-2	-1	2	0
8	1	0	-2	1	2	-2	1	2	-1
9	1	-1	-2	2	0	-2	1	1	-2
10	1	-1	-1	2	-1	-2	2	0	-1
11	1	-1	0	1	-2	-1	2	-1	0
12	1	-2	1	0	-2	-1	2	-2	2
13	1	-2	2	-1	1	-1	1	-2	1
14	1	-2	2	-2	2	0	1	-1	1

TABLE V: COMBINATIONS OF SEQUENCES 0 and 2, OR 1 and 3,
TO GIVE FIRST APPROXIMATIONS TO $R\cos\delta$, $R\sin\delta$.

	$R\cos\delta$		$R\sin\delta$		$R\cos\delta$		$R\sin\delta$		Check	
	C(00)								C(00)+C'(00)	
S00	1·44	1·44	···	···					2·88	···
	C(01)		C(01)						C(01)	
S01	1·53	1·53	···	···					1·53	1·53
0a	···	···	1·19	1·19					1·19	1·19
	C(02)		C(02)						C(02)	
S02	2·67	2·67	0·02	0·02					2·69	2·69
0b	-0·01	-0·01	1·48	1·48					1·47	1·47
S10										
a0										
	C(11)		C(11)		C(13)		C(13)		C(11)+C(13)	
S11	-0·82	-0·81	···	-0·08	-0·88	-0·88	···	-0·10	-1·70	-1·87
1a	···	-0·06	0·56	0·55	···	0·07	-0·59	-0·59	-0·03	-0·03
a1	···	-0·09	0·86	0·85	···	-0·10	0·81	0·80	1·67	1·46
aa	0·58	0·58	···	0·06	-0·54	-0·54	···	-0·08	0·04	0·04
	C(25)		C(25)						C(25)	
S20	1·33	1·33	0·15	0·15					1·48	1·48
b0	0·16	0·16	-1·44	-1·44					-1·28	-1·28
	C(27)		C(27)						C(27)	
S22	1·24	1·20	-0·01	0·30					1·23	1·50
2b	···	-0·16	0·68	0·67					0·68	0·51
b2	-0·01	0·36	-1·48	-1·44					-1·49	-1·08
bb	0·82	0·80	0·01	0·19					0·83	0·99
	C(36)		C(36)		C(38)		C(38)		C(36)+C(38)	
S31	-0·78	-0·74	···	-0·24	-0·84	-0·79	···	-0·28	-1·62	-2·05
3a	···	-0·17	0·54	0·51	···	0·19	-0·56	-0·53	-0·02	···
c1	···	-0·28	0·87	0·82	···	-0·28	0·86	0·81	1·73	1·07
ca	0·60	0·57	···	0·19	-0·57	-0·54	···	-0·19	0·03	0·03
S33										
3c										
c3										
cc										
	C(50)		C(50)						C(50)	
S40	1·38	1·38	0·31	0·31					1·69	1·69
d0	0·34	0·34	-1·51	-1·51					-1·17	-1·17
	C(52)		C(52)						C(52)	
S42	1·01	1·01	0·24	0·24					1·25	1·25
4b	-0·23	-0·23	0·99	0·99					0·76	0·76
d2	0·23	0·23	-0·99	-0·99					-0·76	-0·76
db	1·01	1·01	0·24	0·24					1·25	1·25

TABLE VI: COMBINATIONS OF SEQUENCES 0 and 2, OR 1 and 3, TO GIVE FIRST APPROXIMATIONS TO R cos δ, R sin δ.

	R cos δ	R sin δ	R cos δ	R sin δ	Check
	C'(00)				See other sheet
S00	1·44 −1·44				
S01					
0a					
S02					
0b					
	C'(12)	C'(12)			C'(12)
S10	−1·46 1·46	−0·08 0·08			−1·54 1·54
a0	−0·07 0·07	1·43 −1·43			1·36 −1·36
	C'(11)	C'(11)	C'(13)	C'(13)	C'(11)+C'(13)
S11	−0·75 0·74	··· 0·08	−0·81 0·80	··· 0·10	−1·56 1·72
1a	··· 0·06	0·58 −0·58	··· −0·07	−0·62 0·62	−0·04 0·03
a1	··· 0·08	0·79 −0·79	··· 0·09	0·73 −0·72	1·52 −1·34
aa	0·61 −0·61	··· −0·06	−0·56 0·56	··· 0·07	0·05 −0·04
S20					
b0					
	C'(27)	C'(27)			C'(27)
S22	1·33 −1·29	0·01 −0·30			1·34 −1·59
2b	0·01 0·17	0·70 −0·68			0·71 −0·51
b2	0·01 −0·36	−1·58 1·54			−1·57 1·18
bb	0·83 −0·80	−0·01 −0·20			0·82 −1·00
	C'(36)	C'(36)	C'(38)	C'(38)	C'(36)+C'(38)
S31	−0·71 0·68	··· 0·23	−0·76 0·72	··· 0·25	−1·47 1·88
3a	··· 0·18	0·56 −0·53	··· −0·19	−0·59 0·56	−0·03 0·02
c1	··· 0·25	0·81 −0·77	··· 0·26	0·77 −0·73	1·58 −0·99
ca	0·63 −0·60	··· −0·20	−0·60 0·57	··· 0·20	0·03 −0·03
	C'(40)	C'(40)			C'(40)
S33	−1·21 1·21	−0·22 0·22			−1·43 1·43
3c	0·23 −0·23	−1·25 1·25			−1·02 1·02
c3	−0·23 0·23	1·25 −1·25			1·02 −1·02
cc	−1·21 1·21	−0·22 0·22			−1·43 1·43
S40					
d0					
S42					
4b					
d2					
db					

TABLE VII: COMPUTATION OF 1000 R cos δ
(Multiples of First Approximations)

	First Approximation	C(00)	C(01)	C(02)	C(11)	C(13)	C(25)	C(27)	C(36)	C(38)	C(50)	C(52)
$R \cos \delta$	C(00)	998	...	...	...	...	...	...	...	...	...	...
	C(01)	-31	987	20	-75	82	-1	-1	-31	32	1	-1
	C(02)	34	-87	987	4	-2	4	98	...	1	2	70
	C(11)	7	56	-2	1000	10	1	-5	-6	-47	-8	-4
	C(13)	-12	-76	7	1	989	13	-21	-92	64	-2	-6
	C(25)	-16	...	1	-1	1	998	-15	2	-2	37	-2
	C(27)	2	-4	-93	-14	-23	42	981	10	22	-3	131
	C(36)	...	8	-1	-1	-17	-7	-3	997	1	...	1
	C(38)	-7	-32	2	-49	-84	-43	34	6	995	27	-29
	C(50)	-15	...	2	2	-2	-22	3	-4	4	999	-39
	C(52)	1	-3	-43	-4	-2	3	-78	-10	-20	49	988
	C'(00)	...	...	...	...	...	...	...	...	...	...	...
	C'(11)	...	...	...	...	...	...	...	...	-2	...	...
	C'(12)	...	...	...	...	...	...	...	...	...	...	...
	C'(13)	...	...	...	-3	4	...	...	-5	...	...	...
	C'(27)	...	...	-1	-1	...	...	...	-1	...	...	-1
	C'(36)	...	...	...	...	-1	...	...	1	-3	...	...
	C'(38)	...	-1	...	-3	-1	...	...	-1	...	...	...
	C'(40)	...	...	...	...	1	...	...	-1	2	...	-1
$R \sin \delta$	C(01)	...	-4	...	-4	-5	...	...	-3	-5	...	...
	C(02)	...	-1	-1	-1	-1	1	-11	-1	-1	1	-9
	C(11)	...	3	...	2	3	1	...	...	-10	1	1
	C(13)	-1	-4	...	...	1	-3	1	-21	-7	-1	-1
	C(25)	-2	...	...	...	...	...	...	...	...	-6	...
	C(27)	...	...	-10	-1	-1	1	-1	-2	-2	...	-12
	C(36)	...	1	...	...	-4	...	...	...	-3	...	...
	C(38)	-1	-6	1	-12	-10	-4	1	...	...	-3	1
	C(50)	-3	...	...	...	...	-7	...	-1	...	1	1
	C(52)	...	-1	-11	-1	-1	1	-9	-1	-1	...	-2
	C'(11)	...	-3	...	-3	-4	...	...	-1	10	...	...
	C'(12)	...	...	...	...	...	-2	...	...	...	2	...
	C'(13)	...	5	...	1	...	...	...	23	10	...	...
	C'(27)	-1	2	10	3	4	-11	-3	5	4	...	-15
	C'(36)	...	-1	...	-1	3	...	...	-2	2	...	...
	C'(38)	...	7	...	11	12	3	-1	1	1	-2	2
	C'(40)	...	-1	2	-2	-2	2	3	-1	-3	-3	-10

TABLE VIII: COMPUTATION OF 1000 R cos δ
(Multiples of First Approximations)

	First Approximation	C'(00)	C'(11)	C'(12)	C'(13)	C'(27)	C'(36)	C'(38)	C'(40)
R cos δ	C(00)	...	...	...	...	...	...	...	...
	C(01)	...	...	...	...	...	-1	1	...
	C(02)	...	...	...	...	2	...	...	...
	C(11)	...	...	...	-3	...	...	-2	...
	C(13)	...	-3	...	...	...	-4	...	...
	C(25)	...	...	-1	...	...	...	...	...
	C(27)	...	...	...	...	-2	...	...	...
	C(36)	...	...	...	-1	...	3	...	...
	C(38)	...	-3	...	-1	...	1	2	...
	C(50)	...	...	...	...	...	...	...	...
	C(52)	...	...	...	...	-1	...	...	...
	C'(00)	998	...	...	...	...	...	...	...
	C'(11)	1	1002	-18	1	-1	-10	-45	2
	C'(12)	-16	-14	999	14	...	...	1	...
	C'(13)	-4	-6	-46	996	-8	-96	72	...
	C'(27)	8	-29	26	-49	1003	11	50	-56
	C'(36)	...	5	...	-14	...	1000	-2	-4
	C'(38)	-3	-45	-6	-94	16	4	994	-45
	C'(40)	-3	14	-10	10	-45	91	-10	1001
R sin δ	C(01)	...	4	...	5	...	4	4	...
	C(02)	...	1	...	1	11	1	1	...
	C(11)	...	...	7	-2	...	...	9	...
	C(13)	1	...	10	...	-1	20	6	-1
	C(25)	2	...	10	...	...	...	...	1
	C(27)	...	1	-1	1	-2	2	2	3
	C(36)	...	...	...	4	...	2	-2	-7
	C(38)	1	11	4	8	-1	2	2	18
	C(50)	3	...	-3	...	...	1	-1	...
	C(52)	-1	1	-1	1	8	1	1	-4
	C'(11)	...	1	...	3	...	1	-10	...
	C'(12)	-1	...	-6	...	...	...	...	...
	C'(13)	...	-1	-1	-1	...	-21	-8	...
	C'(27)	1	-3	...	-3	-1	-4	-4	3
	C'(36)	...	1	...	-3	...	-1	3	...
	C'(38)	...	-10	-1	-11	1	-3	-3	1
	C'(40)	-1	2	-1	2	-3	1	3	3

TABLE IX: COMPUTATION OF 1000 R sin δ
(Multiples of First Approximations)

	First Approximation	C(01)	C(02)	C(11)	C(13)	C(25)	C(27)	C(36)	C(38)	C(50)	C(52)
R cos δ	C(00)	...	...	...	...	...	...	...	...	...	...
	C(01)	4	...	-3	6	...	...	-3	4	...	...
	C(02)	...	...	...	...	...	11	...	...	-1	15
	C(11)	4	...	...	...	...	...	...	-11	-1	...
	C(13)	5	-1	2	3	...	-2	-21	8	3	...
	C(25)	...	...	...	...	-4	...	...	...	...	...
	C(27)	...	9	...	1	...	-1	...	1	...	18
	C(36)	1	...	...	-4	...	...	...	-1	...	...
	C(38)	7	...	-10	9	...	-2	2	...	...	-3
	C(50)	...	...	...	...	-6	...	...	...	...	-1
	C(52)	1	9	...	...	1	9	...	1	...	-2
	C'(00)	...	...	...	...	...	...	...	...	...	...
	C'(11)	-4	...	-1	-1	...	...	2	10	...	...
	C'(12)	...	...	...	...	4	...	...	...	...	...
	C'(13)	-4	...	-2	-3	...	1	21	-6	...	...
	C'(27)	-1	-9	...	-1	9	...	2	-2	-1	12
	C'(36)	-2	...	...	3	...	...	-2	1	...	...
	C'(38)	-7	...	9	-11	-2	...	-1	1	2	-2
	C'(40)	-2	-2	2	-3	-1	-3	2	-3	3	11
R sin δ	C(01)	981	42	89	91	5	5	34	37	2	1
	C(02)	-28	991	15	13	-2	101	6	8	...	45
	C(11)	-65	8	997	-11	-6	3	-8	47	-6	-3
	C(13)	-89	13	-1	983	30	-15	90	66	6	2
	C(25)	1	1	-2	2	1000	-14	2	-2	47	-3
	C(27)	-2	-92	-3	-16	37	983	17	27	1	118
	C(36)	-9	1	-1	16	-8	-2	1000	4	1	...
	C(38)	-37	5	48	-83	-32	36	-4	993	34	-26
	C(50)	1	1	1	-1	17	2	-1	2	999	-41
	C(52)	-2	-43	3	1	-2	-73	-6	-17	48	984
	C'(11)	1	...	-5	-3	...	...	...	2	...	...
	C'(12)	...	...	...	...	-1	...	...	...	...	...
	C'(13)	...	...	1	...	...	...	5	1	...	...
	C'(27)	...	-1	...	...	...	4	1	1	...	-2
	C'(36)	...	...	...	5	...	...	-4	-1	...	...
	C'(38)	-1	...	3	-1	...	...	...	1	...	...
	C'(40)	...	...	...	-1	...	-1	...	-1	...	...

TABLE X: COMPUTATION OF 1000 R sin δ
(Multiples of First Approximations)

	First Approximation	C'(11)	C'(12)	C'(13)	C'(27)	C'(36)	C'(38)	C'(40)
R cos δ	C(00)	...	...	...	...	...	...	...
	C(01)	4	...	-5	...	4	-4	...
	C(02)	...	...	...	-12	...	...	...
	C(11)	...	-7	-1	...	...	10	...
	C(13)	2	-10	4	2	21	-9	...
	C(25)	...	-5	...	...	...	...	...
	C(27)	...	...	...	-2	...	-1	...
	C(36)	...	...	4	...	2	...	9
	C(38)	12	...	-9	3	...	-2	-20
	C(50)	...	7	...	...	...	...	...
	C(52)	1	...	...	-9	...	-1	...
	C'(00)	...	...	...	...	...	...	...
	C'(11)	...	...	...	...	-1	-10	...
	C'(12)	...	...	...	...	...	...	...
	C'(13)	-2	...	-4	-1	-22	8	1
	C'(27)	-1	...	1	2	-2	2	-5
	C'(36)	-1	...	-3	...	-1	...	...
	C'(38)	-10	...	10	-1	...	2	...
	C'(40)	-2	...	3	2	-1	5	-1
R sin δ	C(01)	...	...	1	...	...	...	...
	C(02)	...	...	...	1	...	...	...
	C(11)	3	...	-3	...	...	1	...
	C(13)	...	...	1	...	4	...	...
	C(25)	...	...	...	...	...	...	...
	C(27)	...	...	...	...	...	...	...
	C(36)	...	...	...	...	...	...	...
	C(38)	2	...	-1	...	...	2	...
	C(50)	...	1	...	...	...	...	...
	C(52)	...	...	...	-1	...	...	...
	C'(11)	998	-19	-5	...	-10	45	-1
	C'(12)	-13	1000	15	...	...	...	...
	C'(13)	3	-43	997	-7	96	71	-2
	C'(27)	-1	13	-38	999	28	59	-54
	C'(36)	5	-1	14	...	997	5	-4
	C'(38)	45	1	-93	17	-12	997	-48
	C'(40)	15	-2	-8	-42	90	-28	1002

TABLE XI: CHECKS ON 1000 R cos δ and 1000 R sin δ .
(Multiples of First Approximations)

First Approximation		R cos δ C(00) to C(25)	R cos δ C(27) to C(52)	R sin δ C(01) to C(25)	R sin δ C(27) to C(52)	R cos δ C'(00) to C'(13)	R cos δ C'(27) to C'(40)	R sin δ C'(11) to C'(13)	R sin δ C'(27) to C'(40)
R cos δ	C(00)	998	...	...	...	...	...	...	...
	C(01)	982	...	7	1	...	...	-1	...
	C(02)	940	171	...	25	...	2	...	-12
	C(11)	1072	-70	4	-12	-3	-2	-8	10
	C(13)	922	-57	9	-12	-3	-4	-4	14
	C(25)	983	20	-4	...	-1	...	-5	...
	C(27)	-90	1141	10	18	...	-2	...	-3
	C(36)	-18	996	-3	-1	-1	3	4	11
	C(38)	-213	1033	6	-3	-4	3	3	-19
	C(50)	-35	963	-6	-1	...	...	7	...
	C(52)	-48	929	11	8	...	-1	1	-10
	C'(00)	...	...	...	...	998	...	...	...
	C'(11)	...	-2	-6	12	986	-54	...	-11
	C'(12)	...	...	4	...	983	1	...	...
	C'(13)	1	-5	-9	16	940	-32	-6	-14
	C'(27)	-2	-2	-2	11	-44	1008	...	-1
	C'(36)	-1	-2	1	-1	-9	994	-4	-1
	C'(38)	-5	-1	-11	...	-148	969	...	1
	C'(40)	1	...	-6	10	11	1037	1	3
R sin δ	C(01)	-13	-8	1208	79	9	8	1	...
	C(02)	-3	-21	989	160	2	13	...	1
	C(11)	9	-8	923	33	5	9	...	1
	C(13)	-7	-29	941	149	11	24	1	4
	C(25)	-2	-6	1002	30	12	1	...	...
	C(27)	-11	-17	-76	1146	1	5	...	...
	C(36)	-3	-3	-1	1003	4	-7	...	...
	C(38)	-32	-1	-99	1033	24	21	1	2
	C(50)	-10	1	19	961	...	...	1	...
	C(52)	-13	-13	-43	931	...	6	...	-1
	C'(11)	-10	9	-7	2	4	-9	974	34
	C'(12)	-2	2	-1	...	-7	...	1002	...
	C'(13)	6	33	1	6	-3	-29	957	158
	C'(27)	7	-9	-1	4	-5	-6	-26	1032
	C'(36)	1	...	5	-5	-2	2	18	998
	C'(38)	33	1	1	1	-22	-4	-47	954
	C'(40)	-1	-14	-1	-2	2	4	5	1022

TABLE XII: MACHINE COMPONENTS USED FOR HSWC

	μ	Machine Component	$(48/55)\sigma$	Correction per lunar day	c
C(01)	01°0202	MSf	0°8866	0°1336	3°67
C(02)	02·0404	Mf	0·9583	1·0821	29·76
C(11)	11·7132	Q_1	11·6934	0·0198	0·54
C(13)	13·5229	J_1	13·6018	-0·0789	-2·17
C(25)	25·2361	M_2	25·2952	-0·0591	-1·63
C(27)	27·2765	$2SM_2$	27·0684	0·2081	5·72
C(38)	38·7590	MO_3	37·4637	1·2953	35·62
C(50)	50·4721	M_4	50·5904	-0·1183	-3·25
C(52)	52·5125	S_4	52·3636	0·1489	4·09
C'(00)	0·0000	Sa	0·0358	-0·0358	-0·98
C'(11)	11·5978	O_1	12·1685	-0·5707	-15·69
C'(12)	12·6180	M_1	12·6476	-0·0296	-0·81
C'(13)	13·6382	K_1	13·1268	0·5114	14·06
C'(27)	27·1612	K_2	26·2535	0·9077	24·96
C'(38)	38·8743	MK_3	38·4220	0·4523	12·44

TABLE XIII: MT IN DAYS AND HOURS OF SOLAR TIME

MT	0000 n=0	0422 10	0844 20	1305 30	1727 40	2149 50
0	1 00	6 04	11 08	16 13	21 17	26 21
*1	29 11	34 15	39 20	45 00	50 04	55 08
2	57 22	63 02	68 07	73 11	78 15	83 19
*3	86 09	91 14	96 18	101 22	107 02	112 06
4	114 20	120 01	125 05	130 09	135 13	140 17
*5	143 08	148 12	153 16	158 20	164 01	169 05
6	171 19	176 23	182 03	187 07	192 12	197 16
*7	200 06	205 10	210 14	215 19	220 23	226 03
8	228 17	233 21	239 02	244 06	249 10	254 14
*9	257 04	262 08	267 13	272 17	277 21	283 01
10	285 15	290 20	296 00	301 04	306 08	311 12
*11	314 02	319 07	324 11	329 15	334 19	339 23
12	342 14	347 18	352 22	358 02	363 06	368 11
*13	371 01					

* Red line on special dial on Doodson-Légé machines

n = 55 corresponds to 1 day in MT

Add T to give approximate time of HW, sequence s = 0
Add T+6 to give approximate time of LW, sequence s = 1
Add T+12 to give approximate time of HW, sequence s = 2
Add T+18 to give approximate time of LW, sequence s = 3

TABLE XIV: INCREMENTS OF ANGLE FOR DAYS OF MT

MT	C(01)	C(02)	C(11)	C(13)	C(25)	C(27)	C(38)	C(50)	C(52)	C'(11)	C'(12)	C'(13)	C'(27)	C'(38)
0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0	0°·0
1	28·1	56·1	322·1	11·9	334·0	30·1	345·9	308·0	4·1	318·9	347·0	15·0	26·9	349·0
2	56·1	112·2	284·2	23·8	308·0	60·2	331·7	256·0	8·2	277·9	334·0	30·1	53·9	338·1
3	84·2	168·3	246·3	35·6	282·0	90·3	317·6	204·0	12·3	236·8	321·0	45·2	80·8	327·1
4	112·2	224·4	208·4	47·5	256·0	120·4	303·5	151·9	16·4	195·8	308·0	60·2	107·7	316·2
5	140·3	280·6	170·6	59·4	230·0	150·5	289·4	99·9	20·5	154·7	295·0	75·3	134·7	305·2
6	168·3	336·7	132·7	71·3	204·0	180·6	275·2	47·9	24·6	113·6	282·0	90·3	161·6	294·3
7	196·4	32·8	94·8	83·2	177·9	210·7	261·1	355·9	28·7	72·6	269·0	105·4	188·5	283·3
8	224·4	88·9	56·9	95·0	151·9	240·8	247·0	303·9	32·8	31·5	256·0	120·4	215·5	272·3
9	252·5	145·0	19·0	106·9	125·9	270·9	232·8	251·9	36·9	350·5	243·0	135·5	242·4	261·4
10	280·6	201·1	341·1	118·8	99·9	301·0	218·7	199·8	40·9	309·4	230·0	150·5	269·3	250·4
11	308·6	257·2	303·2	130·7	73·9	331·1	204·6	147·8	45·0	268·3	217·0	165·6	296·2	239·5
12	336·7	313·3	265·3	142·6	47·9	1·2	190·5	95·8	49·1	227·3	204·0	180·6	323·2	228·5
13	4·7	9·4	227·5	154·4	21·9	31·3	176·3	43·8	53·2	186·2	191·0	195·7	350·1	217·6

TABLE XV: DAY NUMBERS

Common Year			Leap Year
0	JANUARY	1	0
31	FEBRUARY	1	31
59	MARCH	1	60
90	APRIL	1	91
120	MAY	1	121
151	JUNE	1	152
181	JULY	1	182
212	AUGUST	1	213
243	SEPTEMBER	1	244
273	OCTOBER	1	274
304	NOVEMBER	1	305
334	DECEMBER	1	335
365	JANUARY	1	366

e.g. June 16; add 16
to day number for June 1.

TABLE XVI: EXAMPLE OF CALCULATION OF $W+\frac{1}{2}c$

		s	h	p	t_o =
	Increment per hour	0°549	0°041	0°005	(s-h)/14·49
1952	Value at 0h. Jan. 1	323·15	279·60	290·16	3·01h.
	Increment for t_o	1·65	0·12	0·02	
	Value at t_o	324·80	279·72	290·18	
1953	Value at 0h. Jan. 1	105·72	280·35	330·94	-12·05h.
	Increment for t_o	-6·62	-0·49	-0·06	
	Value at t_o	99·10	279·86	330·88	

		CHECKS FOR $W+\frac{1}{2}c$ Increment in t_o is				FORMULAE FOR W multiples of				$W+\frac{1}{2}c$	
	Set on	8·94 CY	-15·90 CY	9·79 LY	-15·06 LY	s	h	p	$\frac{1}{2}c$	1952	1953
C(00)	-	-	-	-	-	-	-	-	-	-	-
C'(00)	Sa	0°	0°	0°	0°	0	0	0	-0°5	0	0
C(01)	Msf	0	-1	1	0	0	1	0	1·8	282	282
C(02)	Mf	0	-2	2	0	0	2	0	14·9	214	215
C(50)	M_4	177	124	227	177	4	-4	0	-1·6	179	355
C(25)	M_2	268	243	294	268	2	-2	0	-0·8	89	358
C'(27)	K_2	228	201	255	228	2	0	-1	12·5	12	240
C'(13)	K_1	134	121	148	134	1	0	0	7·0	332	106
C'(11)	O_1	134	122	146	134	1	-2	0	-7·8	118	252
C(11)	Q_1	175	163	186	175	1	-2	1	0·3	56	231
C'(12)	M_1	134	122	147	134	1	-1	0	-0·4	45	179
C(13)	J_1	94	80	107	94	1	0	-1	-1·1	34	127
C(52)	S_4	177	124	229	177	4	-2	0	2·0	22	199
C(27)	$2SM_2$	269	241	296	269	2	0	0	2·9	292	201
C'(38)	MK_3	43	4	82	43	3	-2	0	6·2	61	104
C(38)	MO_3	2	323	41	2	3	-2	-1	17·8	143	144

	Increment in MT			
	h. m.	h. m.	h. m.	h. m.
12 days +	20 04	19 12	20 57	20 04

TABLE 1 : OBSERVATIONS AND BASIC PREDICTIONS

(HW and LW at Rosyth, 1945)

		HWT		HWH			HWT		HWH			LWT		LWH			LWT		LWH	
		O	P	O	P		O	P	O	P		O	P	O	P		O	P	O	P
*JAN.5		0728	0703	12•5	13•9	(0)	1921	1904	12•5	14•4		0014	0025	2•4	4•7		1155	1229	3•1	6•1
6	(2)	0803	0749	13•1	13•5		2030	1958	12•6	13•9	(1)	0020	0116	3•0	5•4	(3)	1243	1325	4•5	6•6
7		0905	0840	13•0	13•2		2110	2053	13•0	13•6		0112	0213	4•6	5•9		1340	1429	3•4	6•9
8		0945	0937	12•6	13•2		2215	2154	12•4	13•4		0317	0318	4•3	6•1		1558	1541	5•5	6•9
...																				
...																				
*JUL.1		0636	0637	15•4	15•5	(0)	1914	1904	15•3	14•9		-	-	-	-		1212	1221	1•7	3•5
2	(2)	0730	0728	15•3	15•3		2009	2001	14•9	14•5	(1)	0033	0037	3•2	5•2	(3)	1313	1319	2•3	4•0
3		0823	0826	15•1	15•0		2109	2101	14•6	14•3		0137	0137	3•9	5•7		1428	1428	2•7	4•3
...																				
...																				
*NOV.26		0742	0743	15•0	14•5	(0)	2002	1957	13•9	14•5		0032	0101	4•7	4•7		1302	1323	6•2	6•9
27	(2)	0852	0850	13•4	14•0		2132	2104	14•1	14•2	(1)	0301	0220	3•8	5•3	(3)	1432	1445	6•2	7•3
28		1010	0959	14•1	13•9		2230	2212	13•6	14•2		0330	0343	5•3	5•3		1640	1606	6•3	7•1
...																				
...																				
†DEC.25		0655	0711	14•5	14•7	(0)	1919	1921	14•4	15•0		0021	0033	3•5	4•0		1302	1243	5•2	6•1
26	(2)	0759	0805	13•7	14•0		2025	2015	13•6	14•4	(1)	0119	0132	3•5	4•8	(3)	1359	1343	5•6	6•7

* Day at head of column , for all values of s

† Last day , for all values of s

s = 1, 2, 3 , in this example, are 1 day later than * and †

TABLE 2 : SEQUENCES OF O-P IN HWT AND LWT

(Rosyth, 1945)

T_0

DAY	JAN 5	FEB 4	MAR 5	APR 4	MAY 3	JUN 2	JUL 1	JUL 31	AUG 29	SEP 28	OCT 27	NOV 26
-14	17	22	45	2	6	3	10	17	26	14	-2	5
-13	32	13	28	-13	-21	4	8	15	17	8	4	28
-12	17	-18	23	-15	-5	-1	8	19	8	9	16	18
...												
...												
13	28	37	5	22	18	14	12	20	9	-3	24	17
14	6	45	13	6	8	10	7	26	15	-2	35	-2

T_2

DAY	JAN 5	FEB 4	MAR 5	APR 4	MAY 3	JUN 2	JUL 1	JUL 31	AUG 29	SEP 28	OCT 27	NOV 26
-14	14	-11	26	-1	9	24	20	22	24	15	-2	2
-13	25	14	25	-22	12	17	20	26	29	0	4	11
-12	8	21	12	-11	-13	11	18	21	10	4	-1	-12
...												
...												
13	-5	31	-3	26	32	23	16	2	25	-19	17	-16
14	27	26	3	9	22	20	22	24	20	-2	-2	-6

(Similar tables for T_1 and T_3)

TABLE 3 : SEQUENCES OF O-P IN HWH AND LWH

(Rosyth, 1945)

H_0

DAY	JAN 5	FEB 4	MAR 5	APR 4	MAY 3	JUN 2	JUL 1	JUL 31	AUG 29	SEP 28	OCT 27	NOV 26
-14	-1·9	0·1	-1·9	-1·0	0·0	0·4	0·4	-0·2	-0·2	-0·7	-0·2	-0·6
-13	-1·3	-0·3	-1·6	-0·4	0·1	0·4	0·4	-0·3	-0·2	-0·5	-0·2	-0·1
-12	-0·6	-0·9	-1·5	-1·5	0·4	0·3	0·3	-0·5	-0·1	-0·6	-1·0	-0·6
...												
...												
13	0·2	-1·5	1·4	-0·3	0·7	0·3	0·2	-0·3	-0·2	0·7	-0·6	-0·3
14	-1·0	-1·9	1·0	0·0	0·6	0·4	0·0	-0·2	-0·4	-0·2	-1·2	-0·6

H_2

DAY	JAN 5	FEB 4	MAR 5	APR 4	MAY 3	JUN 2	JUL 1	JUL 31	AUG 29	SEP 28	OCT 27	NOV 26
-14	-0·4	-0·3	-1·2	-0·8	0·1	0·0	0·0	-0·6	-0·2	-0·5	0·4	-0·6
-13	-0·2	0·0	-1·6	-1·1	0·1	0·4	0·1	-0·7	-0·2	-0·7	-0·3	0·2
-12	-0·6	-0·4	-1·2	-1·9	0·3	0·2	0·1	-0·5	0·0	-0·6	-0·5	0·3
...												
...												
13	0·4	-1·6	1·5	0·0	0·0	-0·1	-0·3	-0·9	-0·7	-0·3	-0·3	-0·2
14	-1·4	-1·2	1·0	0·1	0·0	0·0	-0·5	-0·2	-0·6	0·4	0·5	-0·3

(Similar tables for H_1 and H_3)

TABLE 4 : RESULTS OF APPLICATION OF MONTHLY MULTIPLIERS

(Rosyth, 1945)

DAY	$T_{0.0}$	$T_{0.1}$	$T_{0.2}$	$T_{0.3}$	$T_{0.a}$	$T_{0.b}$	$T_{0.c}$	$H_{0.0}$	$H_{0.1}$	$H_{0.2}$	$H_{0.3}$	$H_{0.a}$	$H_{0.b}$	$H_{0.c}$
-14	165	-46	-52	19	-45	144	-11	-5•8	7•9	2•1	2•0	4•3	-1•5	-1•2
-13	123	-151	32	7	74	148	17	-4•0	6•2	2•1	0•4	3•2	-3•1	-0•4
-12	79	-79	17	-5	144	25	59	-6•3	5•9	3•1	-0•5	2•6	-1•1	-3•3
-11	74	-73	39	-8	143	47	20	-4•7	2•0	3•2	1•3	7•3	0•0	-1•5
-10	194	-91	-4	28	113	10	-18	-3•1	3•1	3•9	0•5	10•6	-2•7	-0•3
-9	190	44	20	-56	100	-90	52	-5•9	0•2	0•5	-3•5	5•1	0•5	3•3
-8	212	-6	-22	-24	38	0	28	-2•6	1•1	1•3	-0•8	2•5	-1•9	0•4
-7	191	-16	10	13	45	14	23	-2•6	3•6	-0•4	0•6	1•0	-4•0	0•4
-6	221	-32	-2	-1	93	-6	5	-4•7	3•5	-0•9	0•7	2•8	-4•7	0•5
-5	178	96	-51	42	124	-35	-16	-2•1	2•4	0•8	0•9	2•1	-4•6	1•1
-4	220	2	20	-8	120	-76	16	-3•6	0•8	-1•1	1•6	0•6	0•5	0•6
-3	205	87	-18	57	164	-56	-5	-0•8	-0•9	-0•4	1•2	-2•9	-1•6	-4•4
-2	238	39	-31	30	77	-31	-22	-4•2	-1•8	-1•6	-0•6	0•4	-3•0	-1•4
-1	201	144	-63	27	69	29	35	-2•9	-2•2	0•9	-1•7	-4•7	-2•7	-1•3
0	214	88	-42	38	-64	90	-6	-5•6	-3•6	-0•4	-0•6	0•6	-1•6	2•2
1	122	160	42	44	-50	78	-6	-4•1	1•6	-2•3	-0•7	1•5	-2•1	0•5
2	18	188	46	16	-196	38	-20	-3•5	2•2	-3•3	-1•3	1•3	-2•5	1•3
3	62	132	34	24	-164	12	-4	-2•9	3•6	-1•0	-0•3	-1•1	3•4	-0•3
4	155	76	17	53	-291	-35	-27	-2•4	0•9	0•7	-1•8	1•7	-2•5	2•0
5	216	85	-25	26	-281	-35	-84	-1•6	0•4	-0•8	-3•4	7•0	-1•4	2•4
6	215	105	-63	63	-160	-37	-35	0•9	-0•6	0•7	-2•1	4•7	0•9	2•3
7	226	34	-54	20	-214	-18	-14	0•1	-0•4	-0•4	-3•5	8•5	-0•8	-0•1
8	230	50	-28	10	-130	18	8	-0•3	-0•1	-1•4	-1•7	7•0	-4•8	0•5
9	257	-3	-21	-15	-194	1	-23	-1•7	-1•0	0•3	0•7	8•1	-5•7	4•9
10	212	43	-23	-46	-51	-135	86	-0•1	0•2	-1•8	0•7	4•3	-1•8	0•1
11	228	-36	8	24	-114	-12	28	1•9	1•3	-1•8	-0•1	5•2	-3•8	-0•1
12	247	10	13	5	-99	33	1	-0•5	1•2	-0•7	0•9	-2•1	-1•5	0•7
13	203	-32	38	-25	-77	34	-3	0•3	1•1	-1•2	0•7	-2•0	-2•6	-0•1
14	167	-10	-11	49	-7	85	3	-3•5	6•3	-1•6	-0•3	-3•4	-3•0	1•5

TABLE 5 : RESULTS FROM APPLICATION OF DAILY MULTIPLIERS

(Rosyth, 1945)

	HT		LT		HH		LH	
	S=0	S=2	S=1	S=3	S=0	S=2	S=1	S=3
S00	5•26	4•81	-1•92	-2•00	-0•076	-0•080	-0•551	-0•561
S01	0•81	0•16	0•74	0•66	0•045	0•045	0•010	0•001
0a	-0•83	-0•84	1•26	0•94	0•076	0•062	0•059	0•065
S02	-0•17	-0•17	-0•16	0•11	-0•002	0•005	0•005	-0•001
0b	0•24	0•26	0•39	0•50	-0•060	-0•038	-0•097	-0•077
S10	-0•06	0•07	-2•27	-0•24	-0•005	-0•013	0•021	0•013
a0	-0•24	-0•18	-0•44	0•63	-0•052	-0•052	-0•043	-0•053
S11	2•52	-2•60	-2•42	-1•62	-0•060	0•061	0•104	0•073
1a	-0•70	-0•22	-8•50	-5•51	-0•033	0•002	-0•058	-0•038
a1	-1•17	-0•66	4•75	1•52	0•029	0•081	0•032	0•033
aa	4•87	0•70	-3•94	-5•06	-0•029	0•037	-0•011	-0•040
S20	-1•04	-1•35	-3•58	-3•18	-0•029	-0•030	0•027	0•049
b0	1•46	1•32	-1•72	-2•28	0•037	0•020	-0•092	-0•092
S22	0•37	0•18	-0•19	1•41	-0•006	-0•018	-0•025	-0•026
2b	1•67	1•62	-1•66	-1•84	0•012	-0•046	-0•076	-0•021
b2	-0•32	-0•13	0•36	0•10	-0•035	-0•029	-0•008	-0•020
bb	-0•73	-0•54	0•47	1•24	-0•037	-0•035	-0•092	-0•092
S31	0•40	-0•04	0•16	-0•29	-0•052	-0•040	-0•039	-0•007
3a	0•66	-0•18	-2•42	-1•33	0•036	0•025	0•033	-0•045
c1	-0•68	0•18	-0•49	-0•52	-0•017	-0•010	0•007	0•007
ca	0•93	1•01	0•20	1•66	0•051	0•016	0•015	0•009
S33	-0•43	0•09	-0•33	-0•10	-0•008	0•003	0•000	-0•010
3c	0•54	0•42	0•38	0•17	0•007	0•004	0•020	0•014
c3	0•17	0•34	-0•14	-0•56	-0•006	-0•022	-0•004	-0•027
cc	-0•08	0•14	-0•13	-0•32	-0•026	-0•039	-0•027	-0•034
S40	0•56	0•66	-1•28	-1•15	-0•010	-0•006	-0•003	-0•023
d0	0•99	0•81	0•15	-1•11	0•021	0•000	0•019	0•014
S42	-0•57	-0•05	0•71	0•39	-0•008	-0•020	-0•005	-0•016
4b	1•23	0•13	-1•56	-1•39	-0•026	-0•019	-0•010	-0•035
d2	-0•34	-0•57	0•20	0•84	0•006	-0•001	-0•015	0•000
db	-0•54	-0•13	1•56	-0•16	0•020	-0•014	-0•007	-0•011

TABLE 6 : FIRST APPROXIMATIONS TO R cos δ and R sin δ

(Rosyth, 1945)

	CONSTITUENT	HT	LT	HH	LH
R cos δ	C(00)	14·50	-5·64	-0·225	-1·601
	C(01)	1·48	2·14	0·138	0·017
	C(02)	-0·91	-0·14	0·009	0·012
	C(11)	3·34	-1·73	-0·003	-0·175
	C(13)	-2·89	7·88	-0·013	-0·134
	C(25)	-2·73	-9·63	-0·069	0·072
	C(27)	-0·66	3·16	-0·090	-0·215
	C(36)	0·83	1·53	0·108	0·055
	C(38)	-1·46	-1·02	0·045	0·014
	C(50)	2·30	-3·68	-0·015	-0·025
	C(52)	-1·82	3·44	-0·011	-0·032
	C'(00)	0·65	0·12	0·006	0·014
	C'(11)	-1·34	1·09	0·056	-0·006
	C'(12)	0·19	3·04	-0·012	-0·012
	C'(13)	-6·50	0·56	0·142	-0·036
	C'(27)	0·42	-3·04	0·015	0·000
	C'(36)	-0·32	-1·55	0·034	0·021
	C'(38)	-0·23	0·61	-0·018	0·031
	C'(40)	0·96	0·00	-0·005	-0·024
R sin δ	C(01)	-1·99	2·62	0·164	0·148
	C(02)	0·73	1·32	-0·145	-0·257
	C(11)	-1·83	-2·54	0·074	-0·006
	C(13)	-0·71	13·80	0·098	0·104
	C(25)	-4·37	4·75	-0·091	0·276
	C(27)	2·82	-2·37	0·059	-0·051
	C(36)	0·02	-2·45	0·022	0·010
	C(38)	-0·89	0·98	-0·048	0·017
	C(50)	-2·34	0·70	-0·037	-0·058
	C(52)	1·94	-3·35	-0·055	-0·039
	C'(11)	-0·93	0·99	-0·059	-0·004
	C'(12)	-0·08	-1·37	-0·001	0·014
	C'(13)	-0·29	3·71	-0·007	0·016
	C'(27)	0·44	-1·00	0·063	-0·030
	C'(36)	-0·44	-1·05	-0·012	0·039
	C'(38)	-0·95	0·94	-0·020	-0·044
	C'(40)	-0·20	0·27	0·016	0·018

TABLE 7 : CALCULATION OF R AND χ FOR C( )

PLACE: Rosyth YEAR: 1945 CENTRAL DAY: 1st July, 1945

BASIC PREDICTIONS : for Dunbar

Values of W

C(00)=0
(01)=h
(02)=2h
(11)=s-2h+p
(13)=s-p
(25)=2s-2h
(27)=2s
(36)=3s-4h+p
(38)=3s-2h-p
(50)=4s-4h
(52)=4s-2h

Checks on W

(11)+(13)=(25)
(11)+(25)=(36)
(13)+(25)=(38)
(25)x 2 =(50)
(25)+(27)=(52)

Hourly Increments

s = 0•5490°
h = 0•0411
p = 0•0046

$(s-h)_o$ = value at Oh., central day	=	250°•52
$(s-h)_o$/ 14•492 = exact time origin	=	17 •29h
g of M_2 for basic prediction	=	56°•4
(g of M_2) / 28°•984	=	1•94h
Approximate time,s=0, central day	=	19.23h

	s	h	p
Value at Oh. Jan.1st	124•28	280•29	5•42
Increment for month	224•93	178•40	20•16
Increment for day	0•00	0•00	0•00
Value at Oh.central day	349•21	98•69	25•58
Increment to time origin	9•49	0•71	0•08
Value at time origin	358•70	99•40	25•66

		C(00)	C(01)	C(02)	C(11)	C(13)	C(25)	C(27)	C(35)	C(38)	C(50)	C(52)
HT	R cos δ	14•48	2•01	-0•81	3•34	-2•61	-2•77	-0•59	1•06	-1•69	2•08	-2•04
	R sin δ	-	-1•75	0•23	-2•01	-0•82	-4•30	2•69	-0•14	-1•05	-2•48	2•40
	R	14•48	2•66	0•84	3•90	2•73	5•10	2•75	1•07	1•99	3•22	3•15
	δ	-	318•9	164•1	329•1	197•4	237•2	102•4	352•4	211•9	310•0	130•3
	W	-	99•4	198•8	185•6	333•0	158•6	357•4	344•2	131•6	317•2	156•0
	χ	-	58•3	2•9	154•7	170•4	35•8	99•8	336•6	343•5	267•2	286•3
LT	R cos δ	-5•61	1•39	-0•45	-1•91	7•93	-9•30	2•78	0•52	-0•38	-3•95	3•98
	R sin δ	-	1•51	2•03	-2•23	13•84	5•16	-2•15	-1•25	1•95	0•92	-3•53
	R	-5•61	2•06	2•07	2•93	15•96	10•60	3•51	1•35	1•98	4•05	5•32
	δ	-	47•4	102•5	229•5	60•2	151•0	322•3	292•6	101•0	166•9	318•5
	W	-	99•4	198•8	185•6	333•0	158•6	357•4	344•2	131•6	317•2	156•0
	χ	-	146•8	301•3	55•1	33•2	309•6	319•7	276•8	232•6	124•1	114•5
HH	R cos δ	-0•227	0•136	0•020	-0•014	-0•003	-0•076	-0•082	0•101	0•044	-0•016	-0•023
	R sin δ	-	0•153	-0•139	0•082	0•113	-0•084	0•047	0•039	-0•030	-0•045	-0•052
	R	-0•227	0•205	0•141	0•083	0•113	0•113	0•094	0•108	0•053	0•048	0•057
	δ	-	48•4	278•1	99•6	91•5	227•9	150•2	21•1	325•7	250•4	246•2
	W	-	99•4	198•8	185•6	333•0	158•6	357•4	344•2	131•6	317•2	156•0
	χ	-	147•8	116•9	285•2	64•5	26•5	147•6	5•3	97•3	207•6	42•2
LH	R cos δ	-1•599	0•016	0•034	-0•175	-0•131	0•060	-0•201	0•064	0•009	-0•023	-0•054
	R sin δ	-	0•142	-0•242	0•003	0•113	0•277	-0•077	0•025	0•026	-0•046	-0•058
	R	-1•599	0•143	0•244	0•175	0•173	0•283	0•215	0•069	0•027	0•051	0•079
	δ	-	83•6	278•0	179•0	139•2	77•8	201•0	21•3	70•9	243•5	227•1
	W	-	99•4	198•8	185•6	333•0	158•6	357•4	344•2	131•6	317•2	156•0
	χ	-	183•0	116•8	4•6	112•2	236•4	198•4	5•5	202•5	200•7	23•1

TABLE 8 : CALCULATION OF R AND χ FOR O

PLACE: Rosyth YEAR: 1945 CENTRAL DAY: 1st July, 1945

BASIC PREDICTIONS : for Dunbar

Values of W

C'(00)=0
(11)=s-2h
(12)=s-h
(13)=s
(27)=2s-p
(36)=3s-4h
(38)=3s-2h
(40)=3s

Checks on W

C'(11)+C'(27)=C(38)
(11)+ (38)=(50)
(11)+ (40)=(52)
(13)+ (36)=(50)
(12)x 2 =(25)

$(s-h)_o$ = value at 0h., central day = 250°·52
$(s-h)_o$ / 14·492 = exact time origin = 17·29h
g of M_2 for basic prediction = 56°·4
(g of M_2) / 28°·984 = 1·94h
Approximate time, s=0, central day = 19·23h

Hourly Increments

s = 0·5490°
h = 0·0411
p = 0·0046

	s	h	p
Value at 0h. Jan 1st	124·28	280·29	5·42
Increment for month	224·93	178·40	20·16
Increment for day	0·00	0·00	0·00
Value at 0h. central day	349·21	98·69	25·58
Increment to time origin	9·49	0·71	0·08
Value at time origin	358·70	99·40	25·66

		C'(00)	C'(11)	C'(12)	C'(13)	C'(27)	C'(36)	C'(38)	C'(40)			
HT	R cos δ	0·65	-1·29	0·46	-6·48	0·45	0·42	-0·64	0·93			
	R sin δ	-	-0·99	-0·01	-0·19	0·46	-0·36	-0·96	-0·15			
	R	0·65	1·63	0·46	6·48	0·65	0·55	1·15	0·94			
	δ	-	217·5	358·8	181·8	45·6	319·4	236·3	350·8			
	W	-	159·9	259·3	358·7	331·7	318·5	157·3	356·1			
	χ	-	17·4	258·1	180·5	17·3	277·9	33·6	346·9			
LT	R cos δ	0·07	1·09	3·10	0·73	-3·08	-1·47	0·48	0·18			
	R sin δ	-	1·07	-1·60	3·67	-1·05	-0·49	1·06	0·32			
	R	0·07	1·53	3·49	3·71	3·25	1·54	1·16	0·37			
	δ	-	44·5	332·6	78·7	198·9	198·4	65·6	60·6			
	W	-	159·9	259·3	358·7	331·7	318·5	157·3	356·1			
	χ	-	204·4	231·9	77·4	170·6	156·9	222·9	56·7			
HH	R cos δ	0·006	0·056	-0·018	0·142	0·012	0·022	-0·007	-0·006			
	R sin δ	-	-0·059	0·001	-0·009	0·062	-0·011	-0·020	0·014			
	R	0·006	0·081	0·018	0·142	0·063	0·024	0·021	0·015			
	δ	-	313·5	176·9	356·4	79·0	333·4	250·7	113·2			
	W	-	159·9	259·3	358·7	331·7	318·5	157·3	356·1			
	χ	-	113·4	76·2	355·1	50·7	291·9	48·0	109·3			
LH	R cos δ	0·015	-0·006	-0·006	-0·038	-0·001	0·025	0·030	-0·025			
	R sin δ	-	-0·006	0·016	0·022	-0·031	0·040	-0·045	0·022			
	R	0·015	0·008	0·017	0·044	0·031	0·047	0·054	0·033			
	δ	-	225·0	110·6	149·9	268·2	57·9	303·6	138·6			
	W	-	159·9	259·3	358·7	331·7	318·5	157·3	356·1			
	χ	-	24·9	9·9	148·6	239·9	16·4	100·9	134·7			

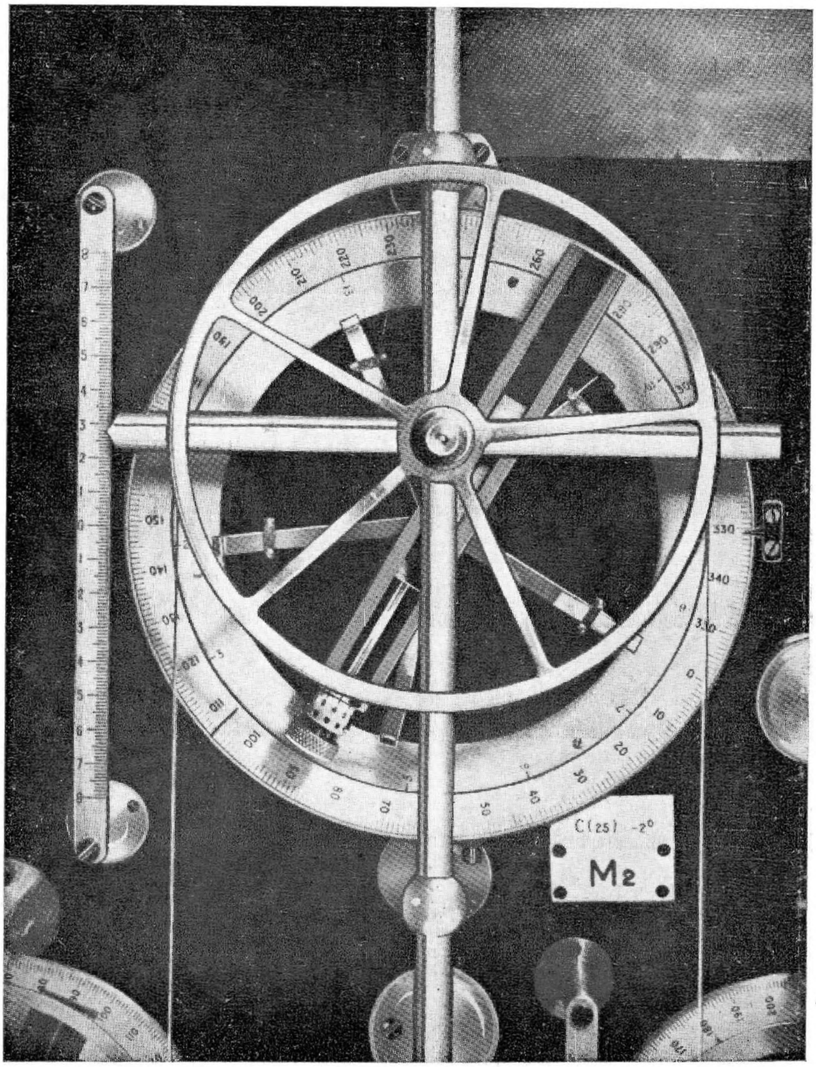

Fig. 1.

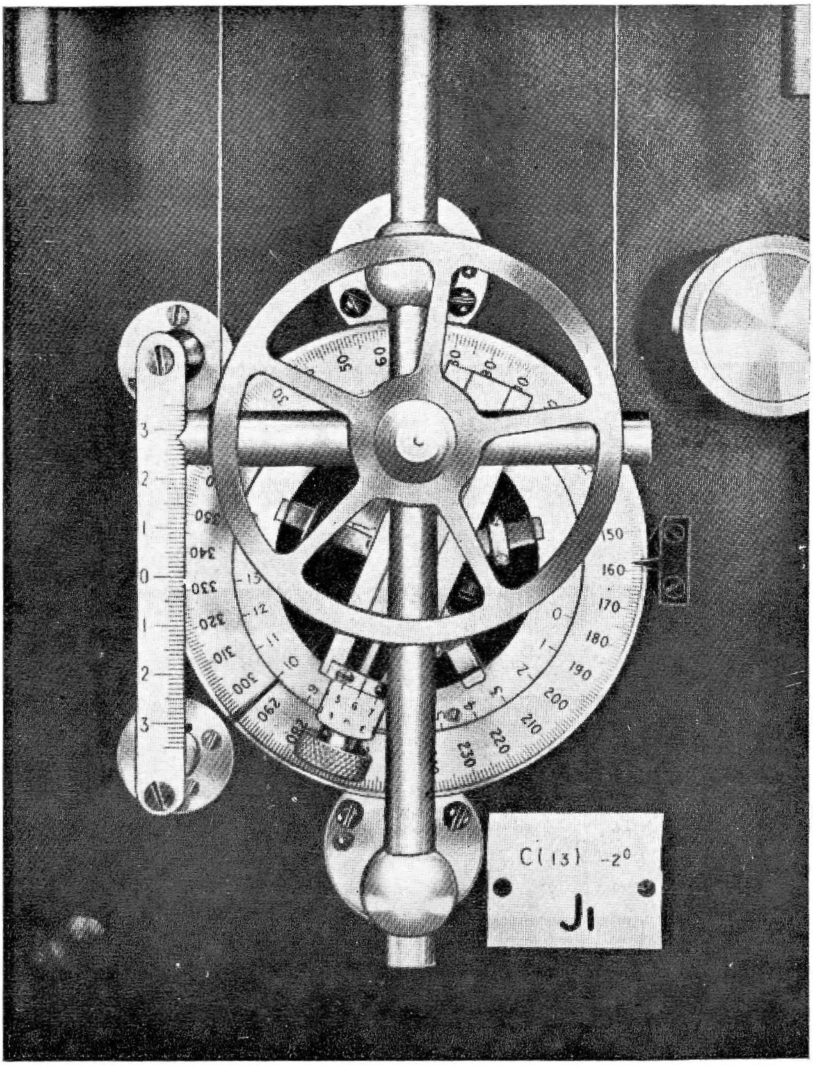

Fig. 2.