AUTOMATIC COMPUTATION OF HYPERBOLIC LATTICES

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When erecting a transmitting chain for hyperbolic navigation, the problem arises as to how to construct the hyperbolic constant-phase-difference lines on charts with sufficient accuracy for existing requirements. The demand for exactitude increases on routes, in harbours and in the vicinity of the coast. Especially in an archipelago, like the one outside Stockholm in the northern Baltic, where the narrowest channels wind between rocks and islands and where a divergence of 50 metres could mean grounding, is high accuracy essential.

This article discusses the possibility of improving the accuracy with the aid of electronic computing equipment, when drawing up the hyperbolic lines. It also gives some information on computations with the Swedish computing machine BESK. Some of the circumstances mentioned are peculiar to Sweden, but on the whole the problem would be much the same here as elsewhere.

Sweden is, because of its longish form and its North-South orientation, well adapted for representation on the Gauss projection with the tangential meridian in the neighbourhood of the town Orebro (about 15° E. of Greenwich). In this projection, a right-angled co-ordinate system has been introduced with the equator as Y-axis. At right-angles to the equator a line is placed as X-axis 1,500,000 m. west of the tangential meridian. The latter is situated 2.5° west of Stockholm Observatory (X- and Y-axes are here reversed compared with normal use in analytic geometry). In such a system the co-ordinates of the transmitting stations are not whole numbers. The hyperbolas also lie obliquely and therefore the work of computation would be very laborious if carried out by hand. A computing machine is, however, very suitable for such work and there are many possible methods of computation.

a) With given boundary conditions, we write the equations of the hyperbolas as a differential equation, which thereafter is integrated in accordance with, for example, the method of Runge-Kutta. The advantage with this method is that we only deal with one hyperbola at a time and we do not risk mixing together points from different hyperbolas when plotting on a chart. Further, we can space the points more closely near the stations where the hyperbolas bend the most, and more widely but equidistantly further away. The equation is, however, unattractive (especially because of the correction of the projection). With the accuracy required the distances between the points must be made so short that the computation takes rather a long time even with a rapid machine.

b) We determine the points of intersection of the hyperbolas with whole numbers with a certain X-co-ordinate, for example, by solving the equation

$$Y = \alpha X + \beta \pm \sqrt{\gamma X^2 + \delta X + \varepsilon}$$

Then we must, however, test for false roots; moreover, the constants $\alpha \ldots \varepsilon$ are complicated functions of the constants involved.

By drawing a locally square pattern orientated as the axis of the hyperbolas and with the origin in the centre of the hyperbolas, these become simplified, but in turn the correction of the chart becomes a function of both X and Y. When the square roots have been obtained, these must be transformed back to the old system of co-ordinates.

Moreover, the co-ordinate with which the intersections have been taken becomes oblique, so that the result becomes ungainly to deal with. A transformation of the co-ordinates would probably not be simpler.

c) The method shown to be most suitable is the following, which closely resembles the calculations formerly carried out by hand.

The machine proceeds forward by steps along a certain X- (or Y-) co-ordinate. In each step it will calculate the lane numbers of the hyperbolas of the point in question, and will also detect the passage of a whole-number hyperbola. For the correction of the chart the following formula has been used :

$$S = s - \frac{s\ y^2}{2\ R^2} - \frac{s(y_a - y_b)^2}{24\ R^2}$$

where S = the distance on the earth's surface.

s = the distance on the projection.

R = curvature radius.

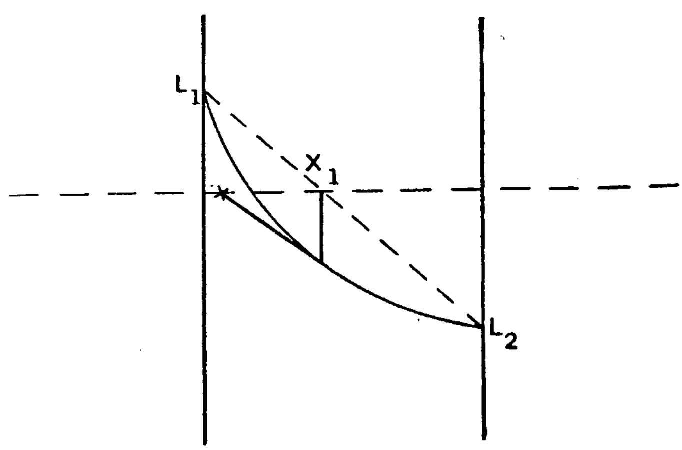
y_a and y_b = the Y-co-ordinates of points A and B.

y = the distance of the mid point of AB to the tangential meridian,

$$\text{i.e. } \frac{y_a + y_b}{2}$$

It is considered that the formula has sufficient accuracy for the requirement.

When the computer has passed a whole-number hyperbola a procedure of interpolation begins to find the Y-(orX-) co-ordinate. The method is in principle as follows :



Curvature is exaggerated Whole number value

Fig. 1.

L_1 is the passed lane value and L_2 the next number. First a linear interpolation is made. The lane-value of x_1 is calculated as the first approximate value and the correction is made at the same time as the linear interpolation (the derivative is approximated to the chord).

This very home-made and unmathematical method has been chosen because of the difficulty of determining the derivative, because the function is almost linear, because the accuracy required is not so high (0.001 Lane is more than sufficient), and finally because the method suits computing machines. (Naturally it does not converge, but with small modifications we can increase the accuracy where necessary).

Thus the machine moves along a co-ordinate to the end of the chart, where it changes co-ordinate and starts moving again. This will be repeated until the whole area is calculated. Then the machine stops and is immediately ready to calculate a new area.

Each Decca chain can be handled. We can determine arbitrary horizontal, vertical and 45° oblique chart boundaries, location of stations, comparison frequencies, propagation speed and bending radii, and can calculate multiples and fractions of a lane. Then we only give BESK the information mentioned in appendix 1.

The example is for Gauss' projection.

Line 1 gives the setting-in order.

Line 2 gives the number of figures in the information. (Noughts in the beginning of numbers are not written out).

8 figures mean that every constant is given in decimetres (one exception). The calculations are, however, carried out as if they had been given with 12 correct figures.

Lines 3-10 contain information about the boundary line of the areas.

Line 11 gives the distance between the co-ordinates along which the stepping occurs. Line 12 is the length of the steps. Line 13 is half of the curvature radius. Line 14 gives wavelength (in 1/10th of mm.). Lines 16-19 are the co-ordinates of the stations. The last line is the sum (Σ) of all information and A indicates the end of the information.

The machine calculates in the hexadecimal system. At first the machine calculates every item of information into this system and it must also convert every result from the hexadecimal into the decimal system.

The result is given in appendix 2. On the first line is the X- or Y-co-ordinate along which the machine steps. On the next line follows the lane number of the first intersection with the co-ordinate of the associated point, followed by a further five points of intersection without any lane numbers.

(To the X-co-ordinates, 4 is, however, to be added to the first figure; consequently 2505 010 becomes 6505 010).

A special problem is the checking of mistakes. If the machine makes any mistakes, they are mostly of the kind that are immediately apparent, i.e. it places hyperbolas north of the North Pole, stops, etc., but there can also be mistakes that are serious.

1. When the machine reads the constants and translates them into the hexadecimal system, it adds them at the same time. If the addition does not correspond to the last given, the machine stops.

2. When the calculations are made, BESK checks the result. If the interpolation has been wrong, amounting to more than a certain allowed (arbitrary) tolerance, which may happen in the immediate vicinity of a transmitting station if the length of the steps is too great (in the nearest area to the station, a shorter step length should be used), or if the machine calculates wrongly, the result is given as it is, but the machine writes an asterisk in front of the result.

3. The writing is most difficult to check. The result which BESK gives is correct, but the automatic typewriter can make mistakes and write 6 instead of 7. Also here it can write the sum.

This must, however, be checked either by hand or by checking in the machine; but both methods are too complicated, bearing in mind that such faults are very rare. Besides, we can see on the chart if a point is wrongly placed and it can then be checked. Such fault finding has therefore not been catered for. We can also do the computations twice and compare the results. It is very improbable that the same fault will be repeated. But on the other hand the cost then will be twice as high.

This method of computing has proved to be the fastest and safest. When the value is to be plotted with a co-ordinatograph, one of the arms may be fastened and the other alone varied, thus making the plotting twice as fast.

By comparison with calculation by hand the advantages become still more outstanding. With the machine we can allow ourselves to take the points at much closer intervals. (By hand calculations, the distance was 1.5 km.). The accuracy has been increased many times as compared with earlier procedures, for instance by making linear interpolations over 5 whole-number hyperbolas. Furthermore, wrong calculations become much more rare.

Hitherto there has been a certain reluctance in handling the problem numerically, graphical and other methods having been used instead; but with a machine which gives 10,000 correct values per hour, any hesitation in this regard should disappear.

The problem now consists in discovering how to plot a large number of values with a co-ordinatograph, this being precision work which takes time. It is therefore suggested that the values should be fed direct from the computer into an automatic co-ordinatograph, which could plot them on a chart twenty-four hours a day. (Such a co-ordinatograph — if it does not already exist — should not be difficult to make.)

The problem of land correction is more difficult to handle. For Sweden the propagation velocity of radio waves over water and land differs by about 2 % according to calculations so far made. This means, that if one transmission path travels for 10 kilometres over land, the hyperbolic pattern lies 200 metres in error. The difference is also too large to enable use of a mean value. For hyperbolas on land and at sea it is easy for the machine to calculate the distance over land and then correct its own results. In an archipelago with its thousands of islands it can, however, lead to an interesting problem of coding. In addition, according to experts, the shape of the coast and islands also affects propagation. As the chain calculated by machine covers an area in the archipelago, the land correction has not been included; this will be a subject of future studies. The charts calculated consequently have to be modified with the help of surveying, but it is easier to correct charts than to draw them and also easier to make small corrections than large ones.

However, it should be possible in principle to feed a computer with such information that it delivers final hyperbolas to within reading accuracy, i.e. the thickness of the lines on the chart.

The Royal Swedish Navy Board has ordered a calculation as a trial of a chain in the northern Baltic where the method described above has been used. The strip which has been especially made for that purpose can be used for every Decca Chain everywhere. The procedure is very rapid. The calculation of a point takes about 2/10 of a second. The time required for writing must be added.

Although the problem of calculating a hyperbolic pattern is not solved with finality, great progress has been made : we can produce values faster, more correctly and more cheaply.

This alone is a sufficient reason for relieving people from very tedious work.

Captain F. Melcher of the Swedish Navy first informed me of this interesting problem and originated the work described in this paper. I also thank Doctor S. Hilding of the National Hydrographic Office and Mr. H. Larsson, Chief of the Research Institute of National Defence.

NOTE: There now moreover exists a code for Mercator co-ordinates.

APPENDIX I

1D00C00000	Starting order.
8C	The maximal number of figures.
24145000D	
24530030D	
16700000D	
17280000D	Boundaries.
50000000D	
100D	
50000000D	
100D	
15000D	
12000D	Step-length, the radius of the earth, frequency etc.
31936845D	
7065134D	
10D	
25379159D	
25251311D	The coordinates of the transmitting stations.
16237671D	
15441276D	
303993636B	The negative sum.
A	The end.

APPENDIX 2

2521500

027	1618815	1622088	1624422	1626333	1628000	1629506
033	1630900	1632212	1633462	1634664	1635827	1636960
039	1638068	1639155	1640226	1641283	1642329	1643366
045	1644395	1645419	1646437	1647453	1648465	1649476
051	1650486	1651496	1652507	1653518	1654531	

2523000

025	1619617	1622745	1624965	1626780	1628368	1629807
031	1631144	1632406	1633612	1634773	1635901	1637001
037	1638078	1639138	1640184	1641217	1642241	1643257
043	1644267	1645272	1646274	1647273	1648270	1649266
049	1650263	1651259	1652257	1653256	1654257	

1614500

050	2504281	2504996	2505711	2506425	2507138	2507851
044	2508564	2509276	2509988	2510700	2511412	2512124
038	2512836	2513549	2514262	2514975	2515689	2516404
032	2517120	2517837	2518555	2519276	2519998	2520722
026	2521449	2522180	2522914	2523652	2524395	2525143
020	2525899	2526662	2527434	2528218	2529015	2529829
014	2530662	2531520	2532411	2533343	2534329	2535390
008	2536558	2537892	2539492	2541615	2545078	