AUTOMATIC COMPUTATION OF HYPERBOLIC LATTICES
by Lars Nystedt, Sweden

When erecting a transmitting chain for hyperbolic navigation, the problem arises as to how to construct the hyperbolic constant-phase-difference lines on charts with sufficient accuracy for existing requirements. The demand for exactitude increases on routes, in harbours and in the vicinity of the coast. Especially in an archipelago, like the one outside Stockholm in the northern Baltic, where the narrowest channels wind between rocks and islands and where a divergence of 50 metres could mean grounding, is high accuracy essential.

This article discusses the possibility of improving the accuracy with the aid of electronic computing equipment, when drawing up the hyperbolic lines. It also gives some information on computations with the Swedish computing machine BESK. Some of the circumstances mentioned are peculiar to Sweden, but on the whole the problem would be much the same here as elsewhere.

Sweden is, because of its longish form and its North-South orientation, well adapted for representation on the Gauss projection with the tangential meridian in the neighbourhood of the town Orebro (about 15° E. of Greenwich). In this projection, a right-angled co-ordinate system has been introduced with the equator as Y-axis. At right-angles to the equator a line is placed as X-axis 1,500,000 m. west of the tangential meridian. The latter is situated 2.5° west of Stockholm Observatory (X- and Y-axes are here reversed compared with normal use in analytic geometry). In such a system the co-ordinates of the transmitting stations are not whole numbers. The hyperbolas also lie obliquely and therefore the work of computation would be very laborious if carried out by hand. A computing machine is, however, very suitable for such work and there are many possible methods of computation.

a) With given boundary conditions, we write the equations of the hyperbolas as a differential equation, which thereafter is integrated in accordance with, for example, the method of Runge-Kutta. The advantage with this method is that we only deal with one hyperbola at a time and we do not risk mixing together points from different hyperbolas when plotting on a chart. Further, we can space the points more closely near the stations where the hyperbolas bend the most, and more widely but equidistantly further away. The equation is, however, unattractive (especially because of the correction of the projection). With the accuracy required the distances between the points must be made so short that the computation takes rather a long time even with a rapid machine.

b) We determine the points of intersection of the hyperbolas with whole numbers with a certain X-co-ordinate, for example, by solving the equation

\[ Y = aX + \beta \pm \sqrt{\gamma X^2 + \delta X + \varepsilon} \]

Then we must, however, test for false roots; moreover, the constants \(a\ldots\varepsilon\) are complicated functions of the constants involved.

By drawing a locally square pattern orientated as the axis of the hyperbolas and with the origin in the centre of the hyperbolas, these become simplified, but
in turn the correction of the chart becomes a function of both $X$ and $Y$. When the square roots have been obtained, these must be transformed back to the old system of co-ordinates.

Moreover, the co-ordinate with which the intersections have been taken becomes oblique, so that the result becomes ungainly to deal with. A transformation of the co-ordinates would probably not be simpler.

c) The method shown to be most suitable is the following, which closely resembles the calculations formerly carried out by hand.

The machine proceeds forward by steps along a certain $X$- (or $Y$-) co-ordinate. In each step it will calculate the lane numbers of the hyperbolas of the point in question, and will also detect the passage of a whole-number hyperbola. For the correction of the chart the following formula has been used:

$$S = s - \frac{s}{2R^2} \frac{y^2}{s} - \frac{(y_a - y_b)^2}{24R^2}$$

where $S$ = the distance on the earth's surface,
$s$ = the distance on the projection,
$R$ = curvature radius.
$y_a$ and $y_b$ = the $Y$-co-ordinates of points A and B.
$y$ = the distance of the mid point of AB to the tangential meridian,
\[ y_a + y_b \]
\[ \text{i.e.} \frac{y_a + y_b}{2} \]

It is considered that the formula has sufficient accuracy for the requirement.

When the computer has passed a whole-number hyperbola a procedure of interpolation begins to find the $Y$-(or $X$-) co-ordinate. The method is in principle as follows:

L_1 is the passed lane value and L_2 the next number. First a linear interpolation is made. The lane-value of $x_1$ is calculated as the first approximate value and the correction is made at the same time as the linear interpolation (the derivative is approximated to the chord).
This very home-made and unmathematical method has been chosen because of the difficulty of determining the derivative, because the function is almost linear, because the accuracy required is not so high (0.001 Lane is more than sufficient), and finally because the method suits computing machines. (Naturally it does not converge, but with small modifications we can increase the accuracy where necessary).

Thus the machine moves along a co-ordinate to the end of the chart, where it changes co-ordinate and starts moving again. This will be repeated until the whole area is calculated. Then the machine stops and is immediately ready to calculate a new area.

Each Decca chain can be handled. We can determine arbitrary horizontal, vertical and 45° oblique chart boundaries, location of stations, comparison frequencies, propagation speed and bending radii, and can calculate multiples and fractions of a lane. Then we only give BESK the information mentioned in appendix 1.

The example is for Gauss' projection.

Line 1 gives the setting-in order.

Line 2 gives the number of figures in the information. (Noughts in the beginning of numbers are not written out).

8 figures mean that every constant is given in decimetres (one exception). The calculations are, however, carried out as if they had been given with 12 correct figures.

Lines 3-10 contain information about the boundary line of the areas.

Line 11 gives the distance between the co-ordinates along which the stepping occurs. Line 12 is the length of the steps. Line 13 is half of the curvature radius. Line 14 gives wavelength (in 1/10th of mm.). Lines 16-19 are the co-ordinates of the stations. The last line is the sum (Σ) of all information and A indicates the end of the information.

The machine calculates in the hexadecimal system. At first the machine calculates every item of information into this system and it must also convert every result from the hexadecimal into the decimal system.

The result is given in appendix 2. On the first line is the X- or Y-co-ordinate along which the machine steps. On the next line follows the lane number of the first intersection with the co-ordinate of the associated point, followed by a further five points of intersection without any lane numbers.

(To the X-co-ordinates, 4 is, however, to be added to the first figure; consequently 2505 010 becomes 6505 010).

A special problem is the checking of mistakes. If the machine makes any mistakes, they are mostly of the kind that are immediately apparent, i.e. it places hyperbolas north of the North Pole, stops, etc., but there can also be mistakes that are serious.

1. When the machine reads the constants and translates them into the hexadecimal system, it adds them at the same time. If the addition does not correspond to the last given, the machine stops.
2. When the calculations are made, BESK checks the result. If the interpolation has been wrong, amounting to more than a certain allowed (arbitrary) tolerance, which may happen in the immediate vicinity of a transmitting station if the length of the steps is too great (in the nearest area to the station, a shorter step length should be used), or if the machine calculates wrongly, the result is given as it is, but the machine writes an asterisk in front of the result.

3. The writing is most difficult to check. The result which BESK gives is correct, but the automatic typewriter can make mistakes and write 6 instead of 7. Also here it can write the sum.

This must, however, be checked either by hand or by checking in the machine; but both methods are too complicated, bearing in mind that such faults are very rare. Besides, we can see on the chart if a point is wrongly placed and it can then be checked. Such fault finding has therefore not been catered for. We can also do the computations twice and compare the results. It is very improbable that the same fault will be repeated. But on the other hand the cost then will be twice as high.

This method of computing has proved to be the fastest and safest. When the value is to be plotted with a co-ordinateograph, one of the arms may be fastened and the other alone varied, thus making the plotting twice as fast.

By comparison with calculation by hand the advantages become still more outstanding. With the machine we can allow ourselves to take the points at much closer intervals. (By hand calculations, the distance was 1.5 km.). The accuracy has been increased many times as compared with earlier procedures, for instance by making linear interpolations over 5 whole-number hyperbolas. Furthermore, wrong calculations become much more rare.

Hitherto there has been a certain reluctance in handling the problem numerically, graphical and other methods having been used instead; but with a machine which gives 10,000 correct values per hour, any hesitation in this regard should disappear.

The problem now consists in discovering how to plot a large number of values with a co-ordinateograph, this being precision work which takes time. It is therefore suggested that the values should be fed direct from the computer into an automatic co-ordinateograph, which could plot them on a chart twenty-four hours a day. (Such a co-ordinateograph — if it does not already exist — should not be difficult to make.)

The problem of land correction is more difficult to handle. For Sweden the propagation velocity of radio waves over water and land differs by about 2% according to calculations so far made. This means, that if one transmission path travels for 10 kilometres over land, the hyperbolic pattern lies 200 metres in error. The difference is also too large to enable use of a mean value. For hyperbolas on land and at sea it is easy for the machine to calculate the distance over land and then correct its own results. In an archipelago with its thousands of islands it can, however, lead to an interesting problem of coding. In addition, according to experts, the shape of the coast and islands also affects propagation. As the chain calculated by machine covers an area in the archipelago, the land correction has not been included; this will be a subject of future studies. The charts calculated consequently have to be modified with the help of surveying, but it is easier to correct charts than to draw them and also easier to make small corrections than large ones.
However, it should be possible in principle to feed a computer with such information that it delivers final hyperbolas to within reading accuracy, i.e. the thickness of the lines on the chart.

The Royal Swedish Navy Board has ordered a calculation as a trial of a chain in the northern Baltic where the method described above has been used. The strip which has been especially made for that purpose can be used for every Decca Chain everywhere. The procedure is very rapid. The calculation of a point takes about 2/10 of a second. The time required for writing must be added.

Although the problem of calculating a hyperbolic pattern is not solved with finality, great progress has been made: we can produce values faster, more correctly and more cheaply.

This alone is a sufficient reason for relieving people from very tedious work.

Captain F. Melcher of the Swedish Navy first informed me of this interesting problem and originated the work described in this paper. I also thank Doctor S. Hilding of the National Hydrographic Office and Mr. H. Larsson, Chief of the Research Institute of National Defence.

NOTE: There now moreover exists a code for Mercator co-ordinates.
APPENDIX 1

Starting order.
The maximal number of figures.

Boundaries.
Step-length, the radius of the earth, frequency etc.
The coordinates of the transmitting stations.
The negative sum.
The end.
APPENDIX 2

2521500
027  1618815  1622088  1624422  1626333  1628000  1629506
033  1630900  1632212  1633462  1634664  1635827  1636960
039  1638068  1639155  1640226  1641283  1642329  1643366
045  1644395  1645419  1646437  1647453  1648465  1649476
051  1650486  1651496  1652507  1653518  1654531

2523000
025  1619617  1622745  1624965  1626780  1628368  1629807
031  1631144  1632406  1633612  1634773  1635901  1637001
037  1638078  1639138  1640184  1641217  1642241  1643257
043  1644267  1645272  1646274  1647273  1648270  1649266
049  1650263  1651259  1652257  1653256  1654257

1614500
050  2504281  2504996  2505711  2506425  2507138  2507851
044  2508564  2509276  2509988  2510700  2511412  2512124
038  2512836  2513549  2514262  2514975  2515689  2516404
032  2517120  2517837  2518555  2519276  2519998  2520722
026  2521449  2522180  2522914  2523652  2524395  2525143
020  2525899  2526662  2527434  2528218  2529015  2529829
014  2530662  2531520  2532411  2533343  2534329  2535390
008  2536558  2537892  2539492  2541615  2545078