HYPERBOLIC CURVES APPLIED TO ECHO SOUNDING

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ABSTRACT. — The mechanically accurate hyperbolae recorded by the Precision Depth Recorder (PDR) can be used in the solution of many problems of ocean bottom surveying that could previously be solved only by more complex procedures, or not be solved at all. An adaptation of the equations for the hyperbola in the construction of various types of templates allows the observer to determine directly the beam width of the echo sounding transmission, the speed of the survey ship, the phasing of the PDR, or the slope of the bottom.

A knowledge of the effect of bottom topography in producing hyperbolae on the PDR trace can be very useful in interpreting the bottom configuration in areas of great complexity, and may prove to be a major factor in the solution of some Deep Scattering Layer problems.

DISCUSSION

Distinct hyperbolae will be recorded by the PDR (Luskin, Heezen, Ewing, and Landisman, 1954) when it is used to record soundings in conjunction with a wide beam transducer, except when the sounding areas have a flat bottom or are of constant gradient. How these hyperbolae develop on the echogram of a submarine feature is exemplified by a simple seapeak in Figure 1.

Assume that the ship's track passes directly over the feature. As the ship approaches the peak, the trace will indicate the minimum depth which is the distance of the ship from the peak rather than the actual depth of the sea bottom. When the ship is directly over the apex, the minimum depth is true and is recorded as the vertex of the hyperbola. As the ship moves away from this position, the echoes from the seapeak rather than from the bottom will continue to record as the minimum depth. The distance that the ship must move before the seapeak ceases to be recorded is dependent upon the depth and shape of the seapeak, and the width of the sounding cone.

That the reflections thus engendered delineate a hyperbola on the echogram can readily be shown. In Figure 1, the horizontal distance of the ship from the apex of the seapeak is x; y is the difference between the minimum depth d, as recorded at the vertex, and the apparent depth at some other point along the same hyperbolic trace. The relationship of these quantities can be expressed by:

$$(d + y)^2 = x^2 + d^2 \quad \textit{or}$$

$$\frac{(d + y)^2}{d^2} - \frac{x^2}{d^2} = 1 \qquad (1)$$

which is one form of the general equation of the hyperbola.

N. B. — Non metric units, such as fathoms and feet, have been used in this paper since the equipment described (PDR) is graduated in fathoms. The fathom is equal to 1.8288 meters and the foot equal to 0.3048 meter.

The angle θ in Figure 1 is measured from the vertical below the transducer and varies from 0°, when the ship is directly over the seapeak, to its maximum of one-half the effective beam width when the ship ceases to receive reflections from the apex.

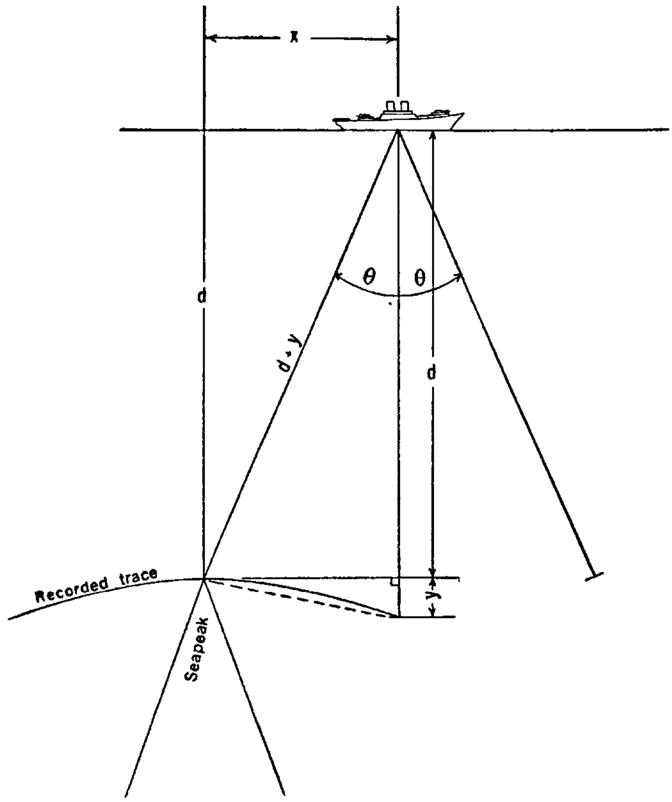

Fig. 1.
Development of a hyperbola.

The relationship of x, y, d, and θ can be expressed by :

$$x = d \tan \theta \qquad (2)$$

$$y = \frac{d}{\cos \theta} - d \qquad (3)$$

which are the parametric equations for the hyperbola described by equation (1).

Because the paper speed of the PDR is constant, the horizontal extent of any specific hyperbola varies inversely with the speed of the ship. Similarly, the depth d as recorded is influenced by the velocity of sound in water, resulting in a deeper recorded depth if the actual velocity is less than that for which the PDR is calibrated, and lesser depth if the actual velocity is greater.

The relationship of x and y to the hyperbola as it appears on the PDR trace is shown by Figure 2. The hyperbola depicted coincides with the trace that would be produced by a seapeak over which d is 500 fathoms, the ship's speed 5 knots, and the beam width is 50°; x is 466 yards, y is 51.7 fathoms, and at the speed of 5 knots the vertical exaggeration is 10/1.

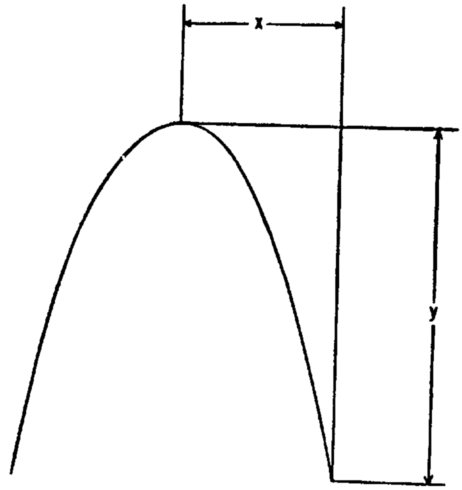

Fig. 2.

Relationship of x and y on trace.

From the preceding discussion of the development of hyperbolae, it is apparent that each distinct hyperbola appearing on the PDR trace is the result of successive reflections from a single point.

As the PDR has a constant paper speed of 24 inches per hour and records at a calibrated velocity of sound in water of 4,800 feet per second, it is possible to assign values to certain of the variables and construct transparent templates which, when laid over the recorded hyperbolae, can be used to determine the effective width of the sound cone, the speed of the ship, the phasing of the PDR, and the slope of the bottom.

Table I provides the basic parameters for the construction of such templates. For any specific depth d and constant ship's speed v, a template can be made to have a family of hyperbolae whose form is a function of half the effective sounding beam width; for a specific θ and v, a family of hyperbolae is formed whose shape is a function of d; and for a specific θ and d, the curves are a function of v. The construction and application of these templates in the analysis of recorded hyperbolae is described in the following sections.

USE OF TEMPLATES

Determining the Speed of the Ship.

The speed of the ship can be determined by measuring the vertical exaggeration of a recorded hyperbola. This requires that the depth of the feature and effective width of the sound beam be known. Holding the correct d and θ constant, a template is made on which the horizontal extent of the hyperbolae is a function of the speed of the ship. (See Figure 3). The hyperbola most nearly matching the recorded curve indicates the ship's speed.

When the vessel is moving slowly the speed can be accurately determined, but as the speed increases the accuracy obtainable lessens. This effect could be

TABLE 1

A

x in fathoms (1)

$x_{fm} = d_{fm} \tan \theta$

θ	tan θ d=	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
5°	.08749	8.7	17.5	26.2	35.0	43.7	52.5	61.2	70.0	78.7	87.4	96.2	105.0	113.7	122.5	131.2
10°	.17633	17.6	35.3	52.9	70.5	88.2	105.8	123.4	141.1	158.7	176.3	194.0	211.6	229.2	246.9	264.5
15°	.26795	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2	268.0	294.7	321.5	348.3	375.1	401.9
20°	.36397	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6	364.0	400.4	436.8	473.2	509.6	546.0
25°	.46631	46.6	93.3	139.9	186.5	233.2	279.8	326.4	373.0	419.7	466.3	512.9	559.6	606.2	652.8	699.5
30°	.57735	57.7	115.5	173.2	230.1	288.7	346.4	404.1	461.9	519.6	577.4	635.1	692.8	750.6	808.3	866.0

B

y in fathoms (2)

$$y = \frac{d_{fm}}{\cos \theta} - d_{fm}$$

θ	cos θ d=	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
5°	.99619	.4	.8	1.1	1.5	1.9	2.3	2.7	3.1	3.4	3.8	4.2	4.6	5.0	5.3	5.7
10°	.98481	1.5	3.1	4.6	6.2	7.7	9.3	10.8	12.4	13.9	15.4	17.0	18.5	20.0	21.6	23.1
15°	.96593	3.5	7.1	10.6	14.1	17.6	21.2	24.7	28.2	31.7	35.3	38.8	42.3	45.9	49.4	52.9
20°	.93969	6.4	12.8	19.3	25.7	32.1	38.5	44.9	51.3	57.8	64.2	70.6	77.0	83.4	89.9	96.3
25°	.90631	10.3	20.7	31.0	41.1	51.7	62.0	72.4	82.7	93.0	103.4	113.7	124.1	134.4	144.7	155.1
30°	.86603	15.5	30.9	46.4	61.9	77.3	92.8	108.3	123.8	139.2	154.7	170.2	185.6	201.1	216.6	232.0

(1) The value of x is in terms of distance on the surface of the sea and must be converted to echogram horizontal distance for template construction; e.g., 1 inch horizontally on the PDR trace = 42.2 fathoms on the surface when the ship's speed is 1 knot.

(2) This value of y is in terms of water depth and must be converted to echogram vertical distance for template construction; e.g., 18.85 inches vertically on the PDR trace = 400 fathoms of water depth.

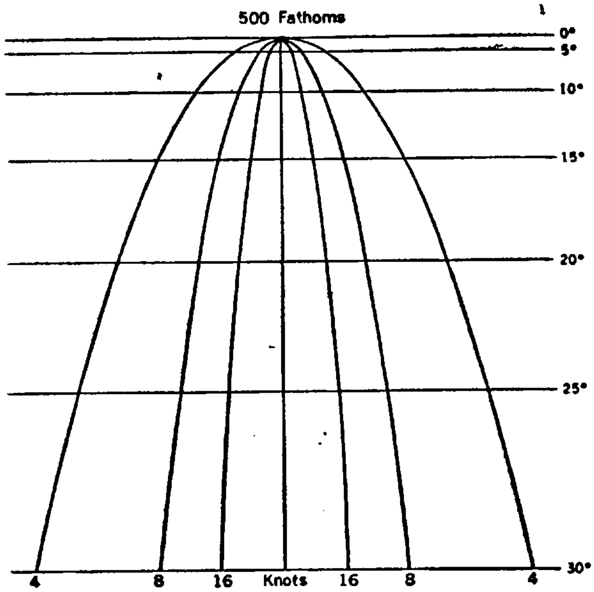

Fig. 3.

Hyperbolae varying in shape with the speed of the ship (constructed for one depth and showing cut-off depths for various beam widths).

remedied if the paper speed of the PDR could be altered, but on the present model this cannot be done.

Hyperbolae caused by marine life may be used to determine the speed, but those recorded from bottom features are preferred since there is an excellent possibility that the suspended sound reflectors may be moving targets.

Determining the Phasing on the PDR.

The PDR records on phasings from 0-400, 400-800, 800-1200 fathoms, etc., and will record from two or more phasings simultaneously.

Figure 10 shows the deep scattering layer and bottom topography recorded on the same part of the echogram, giving the appearance of a seamount with many pinnacles. Actually, the deep scattering layer is at a depth of 140 fathoms while the bottom topography is 940 fathoms.

Knowing the ship's speed and the effective width of the sound beam enables the observer to construct a template on which the forms of the hyperbolae vary with depth. (See Figure 4). The template curve which most closely matches the recorded hyperbola will indicate the approximate depth of the responsible feature. Thus the phase that is recording can be determined and the annotated scale on the recordings checked.

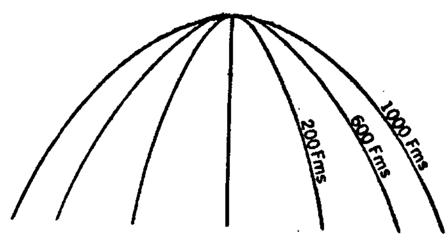

Fig. 4.
Hyperbolae varying in shape with depth of feature.

Determining the Width of the Sound Beam.

The maximum angle θ obtainable with any specific echo sounder varies with the gain setting on the receiver, depth, and acoustical characteristics of the bottom and the water. It also varies not only among different types of echo sounders, but among individual equipments of the same type, and in a given set as transmitted power and receiver performance vary.

To determine θ, the observer must know the speed of the ship and the depth of the feature causing the formation of the hyperbola. A template constructed for these constants is then compared to the trace. The point at which the trace discontinues from coincidence with the template indicates the value of θ, which doubled gives the effective beam width of the transmitted sound. (See Figure 3).

Although the effective width of the sound cone can be determined in areas with steep slopes, the full cone width is indicated and can be determined only when the ship passes directly over a seapeak. Generally, a short hyperbola with weak returns is coming from side features while an extended curve with strong returns indicates a feature very near to or directly below the ship's track.

Determining the Slope of the Bottom.

The procedure for determining the gradient of bottom slopes is allied to the preceding problem of ascertaining the width of the effective sound cone, and the same template is used.

In Figure 5, the hyperbolic effect extends along the curve CAB, but beyond points C and B the theoretical curve is steeper than the recorded trace. Therefore, all the soundings along the curve CAB are coming from the peak of the seamount at point A, while the soundings along CD and BE are reflections from the side slopes of the seamount. Since the recorded trace departs from the template at 20°, the true slope of the sides of this seamount is 20°.

An alternate method of determining the slope of the bottom is to measure an angle BED, which will give the apparent slope of the side of the seamount in terms of the angle m, where BED = m. This value can be used to find the true

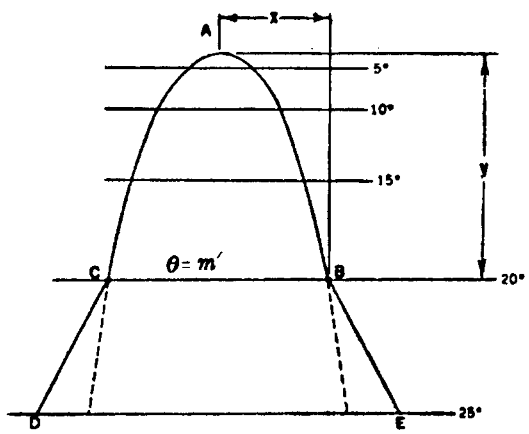

Fig. 5.

Hyperbola recorded when half the cone width is a greater angle than the slope of the seapeak.

slope of the bottom (angle m') by the equation (De Vanssay De Blavous 1930) :

$$\sin m' = \tan m \qquad (4)$$

The angle m' is equal to θ and can be used as a check on the first method described.

Thus, by using hyperbolic templates, it is possible to determine the true gradient of the sides of a seamount without first plotting the ship's track.

MISCELLANEOUS USES OF THE HYPERBOLIC EFFECT

Interpretation of Multiple Echoes.

If, in the course of a survey, a ship's track passes from a sloped to a flat area, or *vice versa,* multiple echoes may be recorded. (See Figures 6 and 10). To illustrate their development consider a ship sounding the bottom profile ADF in Figure 6.

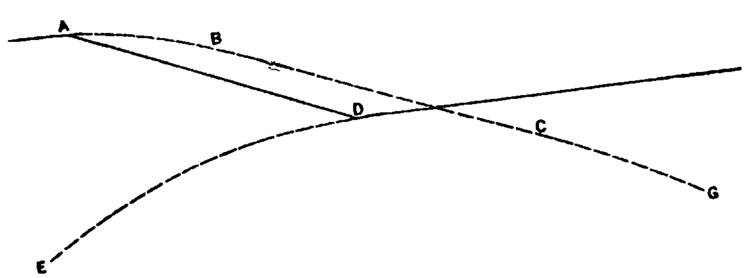

Fig. 6.

Multiple echoes resulting from a change in bottom gradient.

One trace will record a hyperbola that extends from A to B, as θ increases from 0° at point A to an angle equal to the slope AD, which occurs at point B. From B to C the trace will be a straight line whose slope is identical with that of AD, since the echoes recording are perpendicular reflections from AD. From C to G another hyperbola will develop as a result of scattered returns from the point source D, as θ increases from an angle equal to the slope AD at point C to its maximum which occurs at point G.

While this trace is recording, a second trace is also being developed by the same features. The ship, in moving from A to D, receives some reflections from the point source D which record as the hyperbolic section ED. From D to F some returns that are perpendicular reflections from the flat bottom will record as the straight line DF.

It should be noted that because of the hyperbolic effect the profile AD will not appear on the trace; it is superimposed on the two described traces in Figure 6 merely to show the relative position of points on the traces to points on the bottom.

Determining true bottom topography

Wherever there is a change in bottom gradient, a hyperbolic curve will develop on the echogram. In Figure 7, the slope of EF is angle a and the slope of FG is angle b. This feature will produce a trace of the form represented by

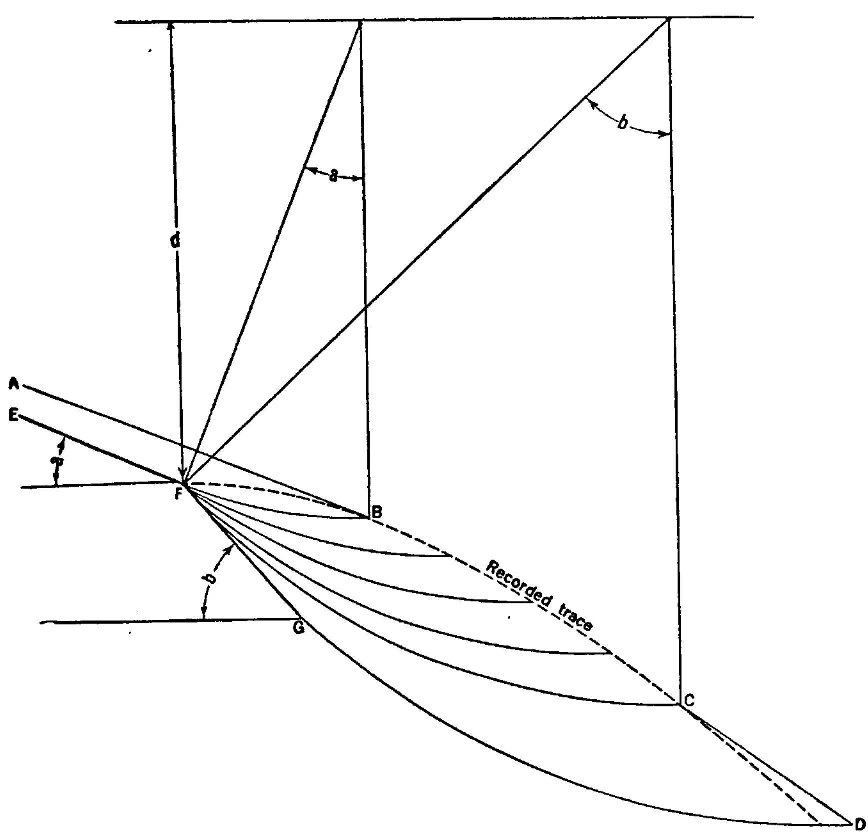

Fig. 7.
Method of swinging arcs to determine true bottom gradient

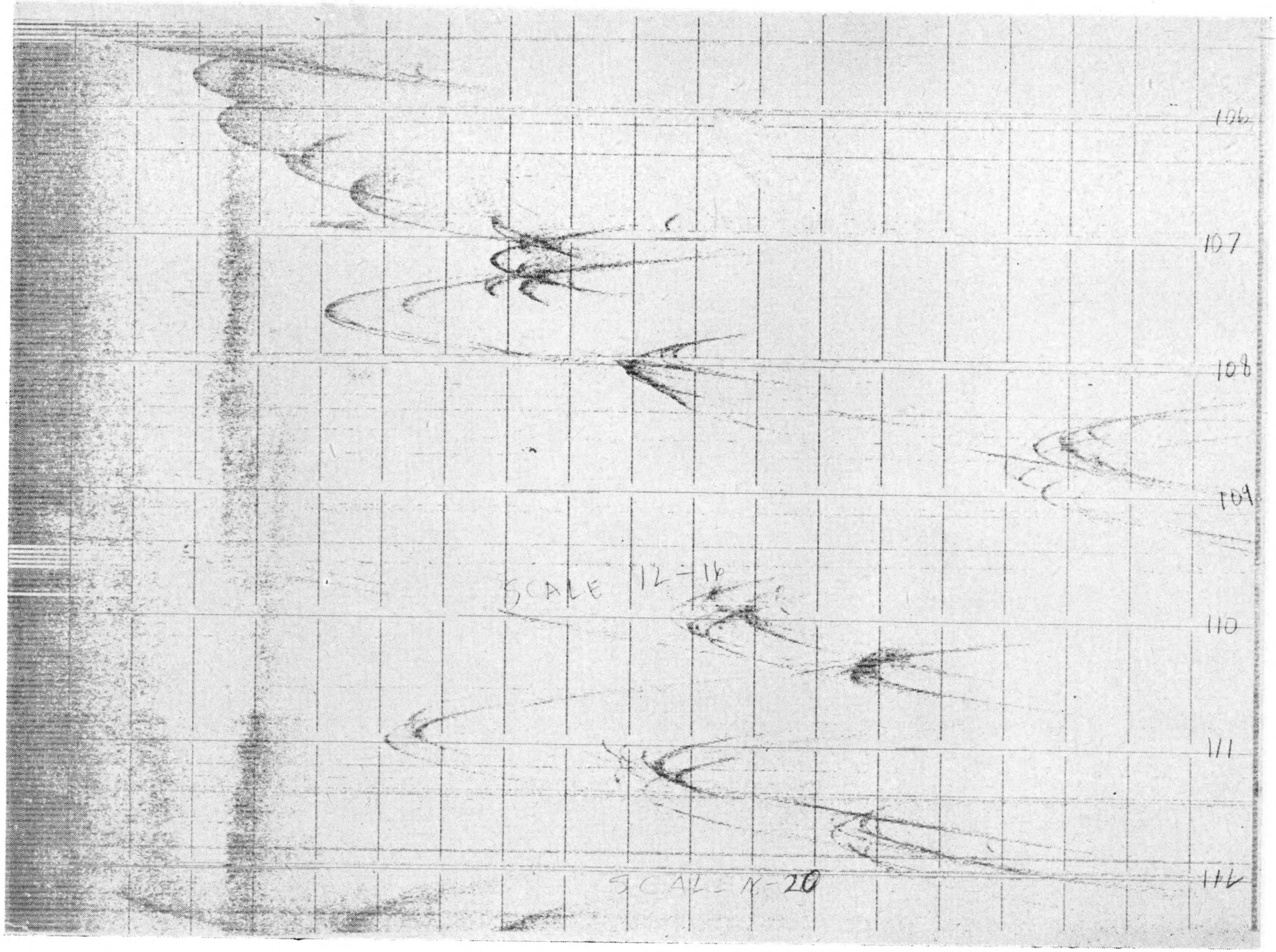

Fig. 8.
An echogram showing multiple echoes received from each emitted signal.

the line ABCD. From A to B the trace indicates a bottom with constant gradient of angle a; from B to C the trace is a portion of a hyperbolic curve; from C to D it is a straight line indicating a constant bottom gradient of angle b. The extent of the hyperbolic segment BC for any given speed is dependent upon the angles a and b. Since the exact location of point B, and consequently the position and depth of F, may be difficult to determine on the trace, the true bottom topography and depth of F can be determined by making a one-to-one profile of this recorded bottom. Then, by swinging a series of arcs from points along the surface with a radius equal to the vertical recorded depth, point F is located at the common intersection of these arcs. The true picture of the bottom topography can then be shown by noting that all the recorded soundings along AB and CD are reflections from the surface EF and FG respectively, while all the soundings along BC are returns from point F.

This method substantiates the existence of a hyperbolic curve along the line BC since all the arcs intersect at a common point.

Locating a Submarine Feature and Determining Its Depth.

Hyperbolae are not necessarily formed by features directly below the ship's track, and as many as four or five may be recorded from each emitted signal (See Figure 8). These returns may come from any direction and only by a very detailed and exact hydrographic survey can the point source responsible for the hyperbola be positioned.

When an individual hyperbola is recorded, the feature can be positioned by two sounding runs. The method of accomplishing this is based on the fact that the vertex of the hyperbola is recorded when the ship is in a position such that the normal to its track passes through the point. Hence, by plotting the two tracks and their normals, the position of the feature is determined at the point of intersection of the normals. (See Figure 9). A third run which also sounds the feature will verify its position.

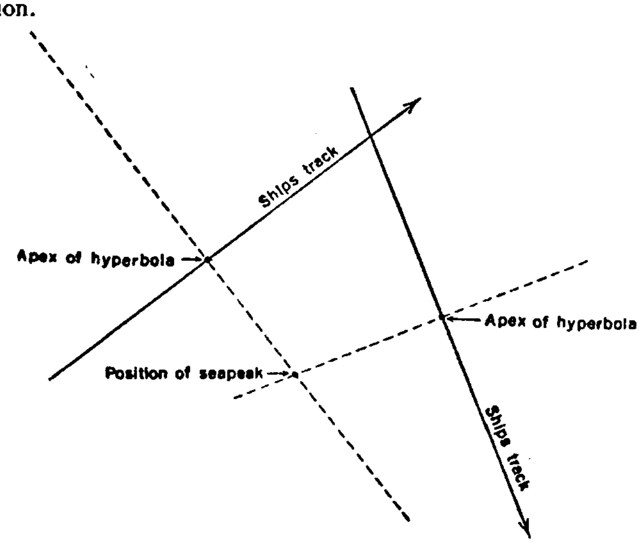

Fig. 9.

Determining the position of a feature that is offset from the ship's track.

The true depth of the feature can be computed from the following equation :

$$d = \sqrt{d_1^2 - h^2} \qquad (5)$$

Where d is the true depth, d_1 is the recorded depth at the vertex of the hyperbola, and h is the horizontal distance between the ship's position when the vertex was recorded and the position of the feature.

Deep Scattering Layer (D.S.L.).

In Figure 10, the deep scattering layer appears as a series of hyperbolae, which indicates the returns are reflections from a few individuals or consolidated masses rather than from an infinite number of points.

In this illustration there is an average of four distinct hyperbolae being recorded from each emitted signal ; thus, assuming the effective beam is 60° and ship's speed is 5 knots and the layer is forty fathoms thick, these four individuals or groups are in a volume of about eighteen million cubic feet. It therefore seems quite unlikely that a trawl towed through this layer could catch any of them, which may explain the consistent lack of success this procedure has met with to-date.

By increasing the gain on the PDR, many small hyperbolae will usually record to depths of eighty fathoms. If the gain is quite intense these small individuals and the D.S.L. will record as a dark massive area similar to that appearing on the trace of less sensitive recorders. This evidence tends to corroborate the contention that the D.S.L. is composed of bathypelagic fish. However, since the PDR, even after gain adjustment, will often record the D.S.L. as a massive area, or a massive area in which distinct hyperbolae are interspersed, it is the author's belief that there are many different kinds of Deep Scattering Layers.

Interpretation of Instrumentation Errors.

Although any submarine feature which produces an extended point return should theoretically cause a hyperbola to develop on the PDR trace, many times this effect is not apparent. It is often necessary to adjust the gain control in order to record them distinctly. A strong gain will cause darkening of hyperbolae that are second or third echoes. When the gain is too low they may not record because the limbs of the hyperbola have generally weaker returns.

Because of the hyperbolic effect, abrupt changes in apparent depth can readily be disproved and can most logically be attributed to instrumentation. Even areas of precipitous submarine topography will be smoothed out on the trace. Figure 11 shows sharp changes in apparent depth resulting from changes in frequency at the power source.

If an abrupt change in depth appears on the trace when a wide-beam transducer is being employed, it can only be the result of an instrumentation error.

Conclusion

The hyperbolae described in this paper have only recently become apparent on the echograms. From the methods described, the width of the sounding cone can be determined, the speed of the ship verified, the phasing of the recorder checked, the slope of the bottom topography measured, and another aspect of the D.S.L. presented. The value of the hyperbolic effect is greatly enhanced by the development of the Precision Depth Recorder.

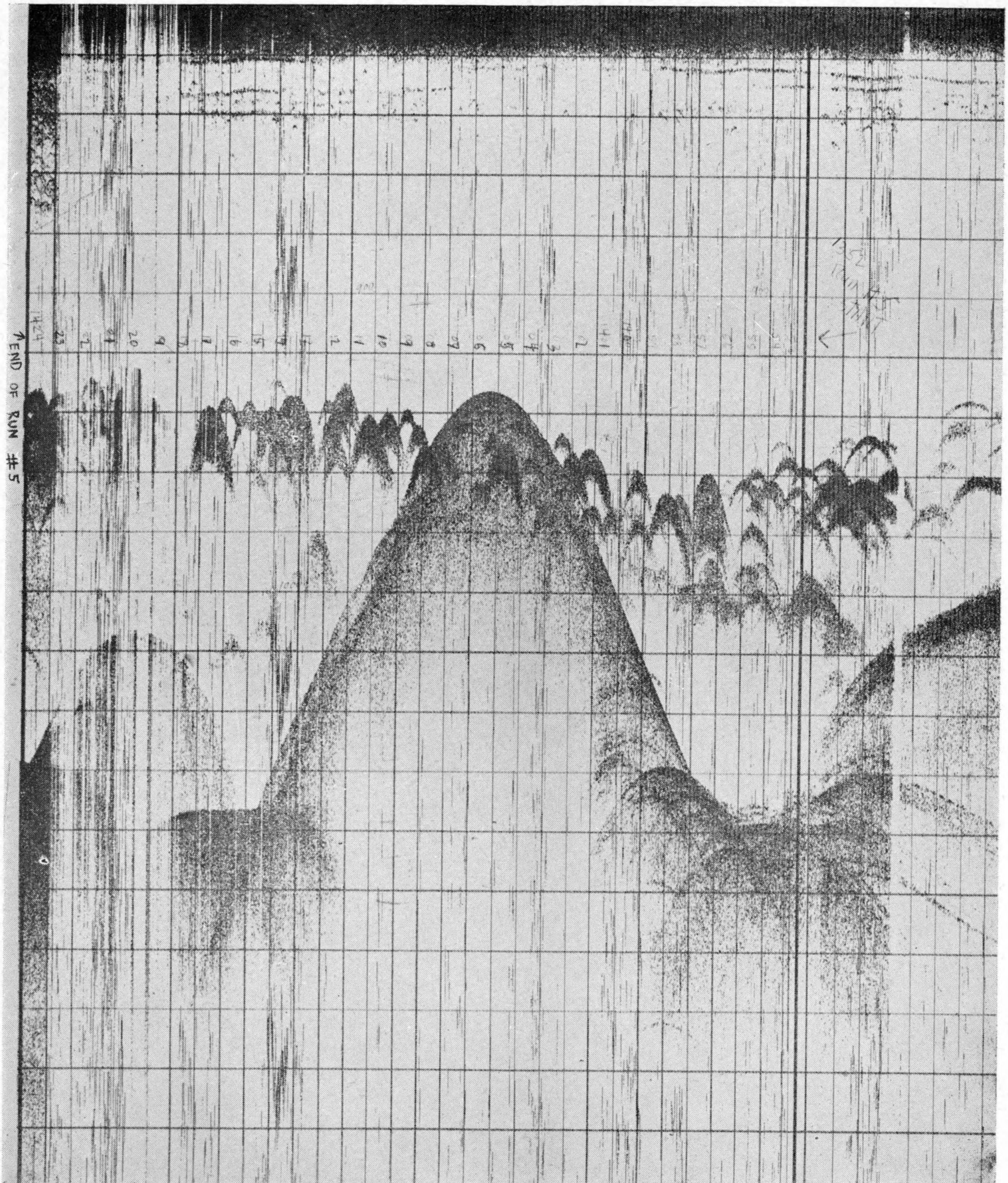

Fig. 10.
An echogram showing the D.S.L. as a series of individual hyperbolae.

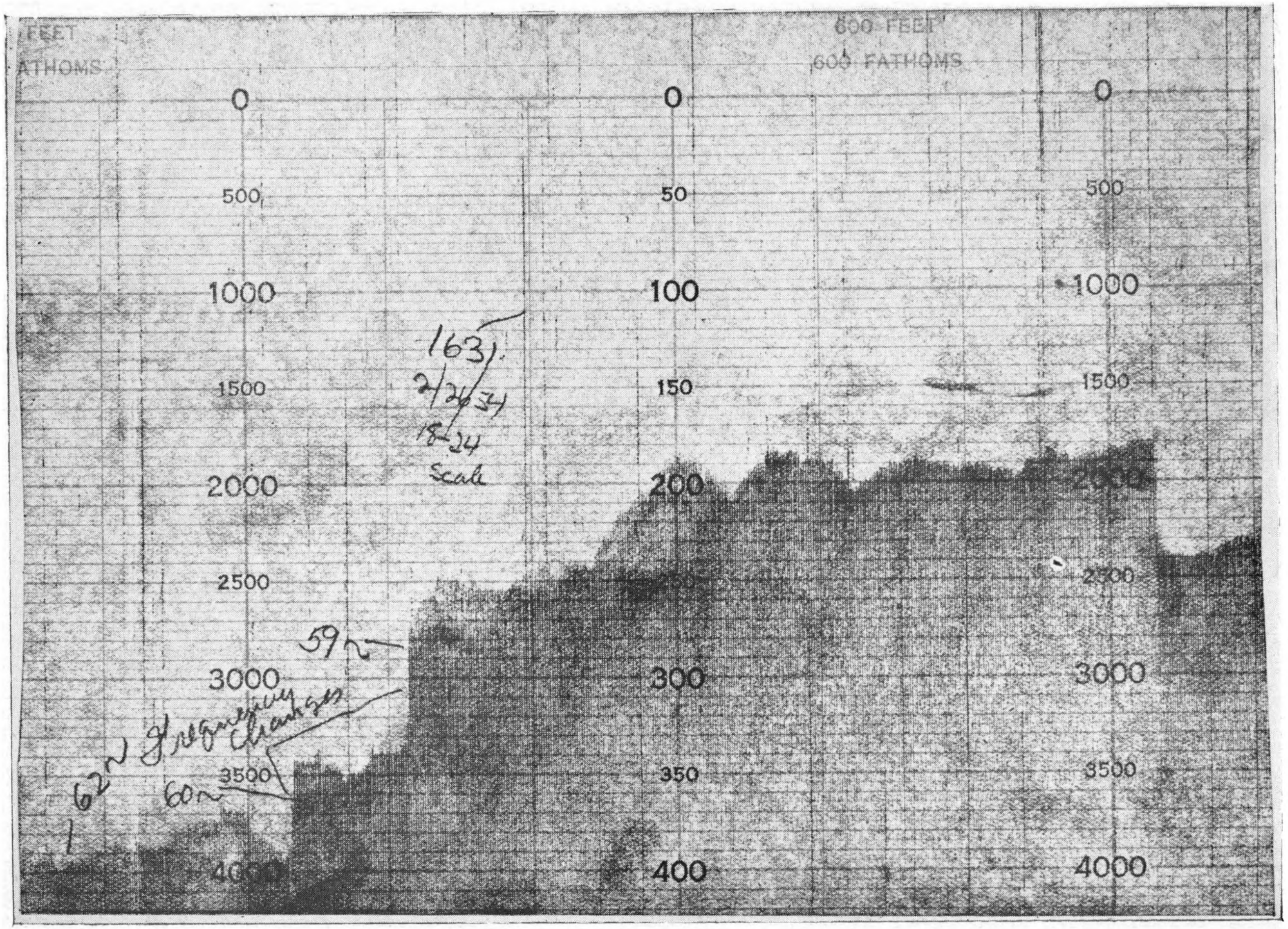

Fig. 11.
An echogram showing abrupt changes in depth as a result of frequency changes.
Annotated frequencies are approximate.

Without the degree of accuracy obtained with the PDR, these curves have little value since they cannot be readily adapted to interpretation through the computed equations of the hyperbola, or by templates constructed from these equations.

REFERENCES

DE VANSSAY DE BLAVOUS (1930), Slope Corrections for Echo Sounding, IHB, Vol. VII, No. 2.

HERDMAN, H.F.D. (July 1955), Directional Echo Sounding, Deep Sea Research, Vol. 2, No. 4.

LUSKIN, B., HEEZEN, B.C., EWING, M., and LANDISMAN, M. (April 1954), Precision Measurement of Ocean Depth, Deep Sea Research, Vol. I, No. 3.

LUSKIN, B., and ISRAEL, H.G. (Jan. 1956), Precision Depth Recorder MK V, Technical Report No. 15 ONRO 27124 Geol., Technical Report No. 12 CU-35-56 No. Bsr 64547, Lamont Geological Observatory, Palisades, New York.

SHALOWITZ, A.L., Slope Corrections for Echo Soundings, USC & GS Special Pub. No. 165.

SMITH, P.A. (Dec. 1939), Some Reflections on Echo Sounding, USC & GS Field Engineers Bulletin No. 13.

STOCKS, T., Die Echolotprofile Wissenshaftliche Ergebnisse der Deutschen Atlantischen Expedition auf den Meteor, 1925-1927, Band II, 1933.