STANDARD SHEETS OF HYPERBOLIC PATTERNS FOR SURVEY USE

by J. Th. VERSTELLE

Research Section, Hydrographic Office,
Royal Netherlands Navy

Article presented to the 11th General Assembly of IUGG, Toronto

1. — INTRODUCTION

After the end of World War II several electronic systems came into use for the navigation of ships and aircraft; some of these systems could be developed to such a high degree of accuracy that improved versions could be used for survey purposes, helping the surveyor to solve some of his greatest problems*. Consequently these systems are used nowadays at a rapidly increasing scale.

For instance, it is now possible to measure distances of many hundreds of kilometres directly, thus enabling the rapid establishment of trilateration networks, constituting geodetic frameworks over whole continents. Canada is not the only, but a very illustrative, example of how such a geodetic framework could be established in a few years and it should be realized that this would have been virtually impossible by means of conventional terrestrial triangulations.

Connecting existing terrestrial triangulations by the hitherto impossible bridging of large areas of water is another possibility of which the connection Norway-Scotland-Faroes is but one example.

Although the accuracy of electronic measurements is to a certain extent affected by atmospheric and other conditions, their use is not limited by these factors and they do away completely with the requirement of mutual visibility in terrestrial surveying. Quite important also is that this sort of measurement is not affected by plumb-line deflections. Still another very important advantage is that they do away to a large extent with accumulation of errors, a source of trouble which demands that terrestrial measurements should commence with extreme accuracies in order to achieve acceptable accuracy over a large area; the practical freedom from accumulation of errors therefore compensates to a certain extent for the relatively limited accuracy of electronic measurements.

As to absolute accuracy, it should always be borne in mind that although primary accuracy in the usual geodetic sense of that expression might be desirable from a theoretical point of view, this does not necessarily mean that practical charting and mapping would in fact require that ultimate accuracy. Let us never forget that a very large — many will say an immensely large — part of the land and sea surface of our globe is still not adequately surveyed or not surveyed at all, and that in very many areas mapping and charting with *acceptable* accuracy is a very urgent economic

---

* For details of many of these systems see reference (11).

requirement. In many of these cases it is not a question of whether any electronic system can compete in accuracy with primary terrestrial triangulation, but whether it can help to do the work in a fast and economic way with an accuracy that is acceptable for practical purposes. In this respect the trilateration of Canada again is one of the outstanding examples of a good compromise.

Nowadays there is a choice between several electronic survey systems, working on different principles. Some of them tend to be more suitable than others for solving any specific geodetic problem. For instance, some systems are particularly intended for measurement of long distances and others are more suitable for detailed measurements within a framework of known points, while still others may serve both purposes with a certain degree of accuracy. This paper does not intend to compare the advantages and disadvantages of the different systems for any particular application.

From the point of view of the user, however, they all have one point in common, which is that they may be looked upon as radiating stable patterns of radio-position lines; geodetically speaking, these lines constitute a system of coordinates. A specialized radio receiver now measures with respect to these invisible coordinate lines; in some survey systems these take the form of spheroidal hyperbolae and in other systems circular position lines are radiated by the transmitters. Unfortunately these coordinate systems are not suitable for practical survey work and the hyperbolic or circular coordinates have to be converted first into the more usual coordinates of geographical latitude φ and longitude λ or rectangular coordinates X, Y (or E/asting) and (N/orthing).

The simplest and most widely used method of conversion is to compute a number of points along selected hyperbolae or circles in terms of φ, λ or X, Y, and to plot them on a chart on which the *graticule* (φ, λ) and/or *grid* (X, Y) has already been drawn. The computation is carried out in such a way that these chart patterns are the correct representations of the radiated patterns. The *observed* coordinates are then plotted by interpolation in the chart patterns and the desired geodetic coordinates are scaled off either from the graticule or the grid. This conversion method is therefore in fact a graphical transformation. The scaling off takes very little time; the accuracy is dependent on the scale of the chart. Complete mathematical conversion of observed coordinates is used only in those cases where extreme accuracy is required (for instance in trilateration, using Shoran, Hiran, Decca or other systems).

The graphical conversion itself is very simple and takes little time, but the drawback of the method is that *accurate* computation and plotting of the chart patterns (the chart « lattice ») takes very considerable time, especially in hyperbolic systems. It is this fact that tends to override the advantage of electronic survey systems. An important consideration also is that any electronic survey system is always quite expensive in purchasing cost and it is therefore an economic requirement that the actual survey should start as soon as the chain of transmitters has been put into operation, which means that latticed charts must be available for the conversion of coordinates.

Different existing methods of producing latticed charts will be described in the following sections, but none of them can make the charts available at very short notice. This paper describes a new method of so-called *standard sheets;* they are prepared beforehand and can be used in connection with *any hyperbolic survey system anywhere in the world*. In this system, latticed charts of the whole area of coverage of a chain can be made available at very short notice (if required, about two days) by a simple photographic process which brings the standard sheets to the correct scale and azimuthal orientation.

The idea has been developed having in mind the production of hyperbolic Decca survey sheets; it could, however, just as well be used for other systems, including those that employ circular patterns.

Some general considerations with respect to the computation and plotting of chart patterns now follow.

The universally adopted surface for geodetic computations and for charting is the spheroid (= ellipsoid); the international (Hayford) spheroid is used throughout this paper. The universally adopted coordinate system on the spheroid is a system of parallels determining the latitude φ and of meridians for λ. The *graticule* is the chart representation of this coordinate system φ, λ.

The actual electronic survey measurements either take place on the physical surface of the earth (terrestrial surveys) — which for most practical purposes may be regarded as sufficiently identical with the spheroid — or in a plane parallel to it at some altitude (aerial surveys).

The most obvious method therefore is to compute the hyperbolic chart patterns in terms of φ and λ and to plot the hyperbolae with respect to the chart graticule.

The geometrical form of the graticule is of course dependent on the choice of the *chart projection*. For the usual chart projections tables are available giving intersections of parallels and meridians in terms of coordinates X, Y (or E, N) which constitute the plane rectangular *grid* of the map or chart; see (1).

Clearly the advantage of computing the hyperbolic chart patterns in terms of φ and λ is that they can be plotted in any desired chart projection, thus leaving this open to the choice of the charting agency.

2. — METHODS OF COMPUTING SPHEROIDAL HYPERBOLAE

Hyperbolae computed in terms of φ and λ are by the nature of the method of computation, spheroidal hyperbolae.

There are different methods of computation, the best known and most widely used being * :

(*a*) a method referred to herein as the « British » method (2).

(*b*) the method of Professer Ballarin (3).

(*c*) the method of Professor Hugon (4).

(*d*) the method of Professor Dupuy (5).

Reference (6) describes another British method — now generally used for latticing nautical charts for *navigation* (not survey) purposes. *Flat* hyperbolae are used which are precomputed and tabulated. The rectangular coordinates are later converted into geographical coordinates and then plotted in the desired chart projection. The finally plotted lattices are identical to *spheroidal* hyperbolae, notwithstanding the fact that they are computed and tabulated as flat hyperbolae. Some more details are given in section 6.

---

* See article by the same author in the present issue (page 54).

3. — METHODS OF COMPUTATION AND CONSTRUCTION OF FLAT HYPERBOLAE

Computation of flat hyperbolae can be carried out in various ways with simple formulae. Reference (7) describes the method recently used for the computation of the patterns of the Decca Survey Chain in Netherlands New Guinea.

The following methods of *construction* of flat hyperbolae are mentioned here :

(*a*) A Swedish method, which has been used also by other agencies, described in (8).

(*b*) A method developed by Messrs. Wernink and Rombach of the Netherlands Hydrographic Office at the Hague.

(c) An apparatus for the drawing of flat hyperbolae, devised by Mr Kuipers of the Royal Shell at the Hague.

Methods (*a*) and (*b*) are based on drawing the hyperbolae as smooth curves through the intersections of circles drawn round the positions of the transmitters (radii equal to whole numbers of wavelengths at the « comparison » frequency). Method (*a*) makes use of long compass arms, while in method (*b*) a modified coordinatograph is used for drawing the circles by turning round the sheet of dimensionally stable material on which the patterns are to be drawn. The accuracy of both methods is critical at small intersection angles of the circles, because in this case a small error in radius displaces the intersection of the circles to a large amount in the direction perpendicular to the hyperbola; in this respect, however, method (*b*) is somewhat less critical than (*a*), because it uses more accurate and rigid equipment. Only in method (*a*) are the patterns drawn to the desired scale.

In Kuipers' method the flat hyperbolae are directly drawn without the intermediate circles and as such it is more elegant. The hyperbolae are drawn or engraved on dimensionally stable material at a small scale; they are brought to the desired scale by photographic enlargement. In all three methods the patterns have to be drawn separately.

4. — COMPARISON OF EXISTING METHODS

An obvious requirement is that the chart patterns should be the correct representation of the radiated patterns in terms of which the receiver is measuring. The numerous factors that have to be taken into account — among them the propagation speed of the radio waves — are discussed in (7). As far as mathematical computation is concerned, the agreement between computed and radiated patterns can be achieved* by *computation of spheroidal hyperbolae*. Among the different methods of computation, Ballarin's is in my opinion the most elegant because it gives the intersections of the wanted hyperbolae in terms of latitude and longitude.

The drawback of this and any other methods of computation of spheroidal or spherical hyperbolae is that the computations are complicated and time-consuming and it is for this reason that the much simpler and speedier *computation of flat hyperbolae* is often used as a substitute. Excepting applications in small areas where the effect of the earth's curvature can be neglected, these flat hyperbolae are

---

* « Pattern-corrections » due to the effect of different ground-conductivities are still necessary; see (7).

naturally not the correct representation of the radiated (spheroidal) patterns. There are two ways out of this difficulty. One is to compute the difference — expressed in fractions of lanes — between the spheroidal and flat patterns for a number of points in the coverage area of the survey chain and apply these differences with negative sign as corrections to the observed readings (in Decca, « Decometer » readings) before plotting in the flat patterns; these corrections can be given in the form of a correction chartlet. The other way is to compute and plot the *chart's graticule* in such a way that it compensates for the difference between flat and spheroidal hyperbolae. This latter method has been successfully used in New Guinea and is described in detail in (7). In both these solutions the plotted observer's position, as scaled off against the chart's graticule, is correct*.

Computations could be speeded up considerably by the use of electronic computers. For flat hyperbolae a relatively simple computer could do the job, but for spheroidal hyperbolae a more complicated instrument would be required and the formulae are not particularly suitable for programming the computations.

Construction of flat hyperbolae is undoubtedly a faster method than any form of computation. The accuracy, however, is limited and cannot be improved beyond a certain point. The practical problem is that higher accuracy can generally be obtained only from construction at a larger scale. In method (*a*) — *see section* 3 — this would necessitate still longer compass arms, reducing the rigidity and increasing the linear errors due to the effect of temperature expansion. In method (*b*) the equipment is more rigid and perhaps better compensated for temperature effects, but the dimensions of a coordinatograph — even a large one — set the limit to the scale of construction. Temperature effects also limit the accuracy of Kuipers' instrument and any remaining errors are enlarged by the photographic process of bringing the original drawing or engraving to the desired scale. As far as is known, this instrument has not yet been used in actual practice. It is, however, expected to have a number of advantages over the two other methods, because the mechanical stability will be considerably better as the instrument is constructed for this special purpose; a big advantage also must be that the hyperbolae are drawn directly and continuously, avoiding the intermediate process of drawing circles which is necessary in the other methods.

The following example is intended to give an idea of the accuracy requirements for the three construction methods:

Latticed charts for the New Guinea Survey Chain (7) *at a scale of* 1/50 000.

(*a*) Construction of circles at a scale of 1/50 000 ; maximum length of compass-arms 7.5 metres (25 feet).

(*b*) Construction of circles by means of a large coordinatograph; scale of construction 1/400 000; photographic enlargement by a factor of 8.

(*c*) Continuous tracing of hyperbolae at a scale of approximately 1/750 000; photographic enlargement by a factor of 15.

To achieve an accuracy of 0.2 mm (about 0.01 inch) in the final latticed chart would therefore require extremely high accuracies in the original construction; 0.2 mm is not an excessive requirement, because it represents 10 terrestrial metres at a scale of 1/50 000, which near a Decca baseline is equivalent to 0.02 lane, while the inherent accuracy of the system is 0.01 lane or better (7).

---

* « Pattern-corrections » due to the effect of different ground-conductivities are still necessary; see (7).

In surveying undeveloped areas one is not always interested in the ultimate accuracy of position fixing of all the objects in the terrain — especially not when an electronic survey system is used, because propagation of errors plays no part in the usual geodetic sense. In these cases the limited accuracy of constructed patterns may be acceptable.

Whenever it is required to make full use of the inherent accuracy of the system, computation must be considered necessary.

5. — THE BASIC PROBLEM OF HYPERBOLIC CHART PRODUCTION FOR SURVEY WORK

In section 4 it was concluded that *computation* of hyperbolic patterns — either spheroidal or flat — is a necessity when it is required to make full use of the inherent accuracy of the electronic survey system.

In addition to the work involved in computation, there is, however, the problem of *plotting* the computed coordinates and *drawing* the hyperbolae as smooth curves through these points. Practical experience has shown that this plotting and drawing is by no means a small problem and takes very considerable time. (Drawing the hyperbolae takes just as much time in the construction methods, based on circles; only in Kuipers' method are the hyperbolae drawn directly.)

Electronic survey systems offer great possibilities, but are expensive in purchasing cost. It is therefore highly desirable that the fullest possible use is made of them and that as little time as possible is lost in the process of setting up, putting into operation and making the latticed charts available, without which the actual survey cannot start. The time required for building the transmitting stations and arriving at the constants which determine the radiated patterns, is to a high degree dependent on the nature of the terrain and the available transport and labour facilities. For example, under the extremely difficult circumstances in Netherlands New Guinea, this took 12 weeks for a Decca Survey Chain covering an area of 420 × 250 kilometres; under more normal conditions it might well have taken less than one week.

From the point of view of economy, many survey agencies consider it a requirement that no additional time — or at least the minimum — should be lost in computation, or in construction and drawing of the chart patterns, after the chain has been put into operation and the pattern constants have been determined. It is in this respect that all the known methods fall short. Even electronic computation only solves part of the whole problem, because it leaves the plotting and drawing still to be done.

This problem is especially urgent when a survey system is to be used in the most economical way, namely when it is to be employed by several survey parties simultaneously, spread out over the whole or a large part of the coverage, implying that the *complete* chart-patterns are needed at short notice.

Note: The opinion is often expressed that this sort of problem is of minor importance when a survey system with circular position lines is used instead of a hyperbolic system. The argument then is that the radiated circular position lines can be represented by circles on the chart, providing that a suitable chart projection is used and appropriate corrections are applied to reduce spheroidal distances to chart distances before setting the radii of the chart circles.

Whether this method is acceptable or not, depends entirely on the final accuracy required. In the first place it is a construction method which as such has its limitations, because long compass-arms have to be used. In the second place it should be borne in mind that — for instance with the Decca 2-range system and also with some other survey systems — distances can be measured with an accuracy of 5 metres and in many cases even considerably better. Except at small distances, there is no chart projection in which the chart radii can be corrected in a simple way with that sort of accuracy and moreover these corrections would vary considerably in different azimuthal directions; this of course complicates the construction.

I am therefore of the opinion that in all cases where it is required to make full use of the inherent accuracy of the electronic survey system, it is always necessary to draw the circular *position lines* through a fairly great number of *points* computed. However, as the curvature of circles is of a simpler nature, the number of points will be considerably smaller than in a hyperbolic system.

6. — GENERAL OUTLINE OF THE PROPOSED SOLUTION

In the second paragraph of section 4 it has already been pointed out how *flat chart hyperbolae* can be used for *survey work* as a substitute for the spheroidal hyperbolae that would be the correct representation of the radiated patterns, provided that the difference between the two is applied as corrections to the observations (in the case of Decca : decometer-readings). There can in fact be no sound objection against this much simpler method of computation. As already shown, this leaves the choice of chart projection entirely open to the surveyor.

Based on the ideas of Mr. Atherton of the British Hydrographic Office and of Mr. Sadler of H.M. Nautical Almanac Office, the British Hydrographic Department has prepared tables of flat hyperbolae and these tables are used for latticing British Admiralty nautical charts for *navigation* in the manner described in (6). For this navigation requirement these flat hyperbolae are plotted on the nautical chart in such a way that they become the correct representation of the radiated spheroidal hyperbolae. The method is very handy for that particular purpose because the charts are always at a small scale, but it cannot be considered to be accurate enough for the much larger scales used in survey work, especially when it is required to make full use of the inherent accuracy of the electronic survey system.

In the British tables X (or Y) values are given at certain intervals of Y (or X) for points along a homofocal system of 400.0 flat hyperbolae on an arbitrary baseline comprising 400 *units;* the hyperbolae are thus separated by one unit and the baseline therefore comprises 400.0 *unit-lanes.* An actual baseline however comprises a certain number of « normal » lanes (360° difference in phase) dependent on the actual spheroidal length of the baseline and on the radio frequency; that physically existing (radiated) number of lanes is determined by the process of « lane-counting », described in (7). If, for instance, the actual number of lanes turns out to be 225.63, the rectangular coordinates along the *full* hyperbolae (consecutive hyperbolae of 0° phase difference) can be obtained from these tables by interpolation; to this end the table-interval has been chosen such that it is sufficiently small to allow for linear interpolation. The full hyperbolae can — and could also for survey work — now be plotted in a rectangular system which has its origin in the Master and its X-axis in the direction of the baseline; after a simple transformation these coordinates could of course also be directly plotted in the system of the chart's grid.

The advantage of the tables — also for survey work — evidently is that they do away with any later computations; the only objection for survey applications is that the very time-consuming plotting and drawing still has to be done.

It should be noted that the computation of the corrections (difference between flat and spheroidal) is comparatively simple and needs only to be carried out for a limited number of points in the coverage; it does not take much time, as explained in (7).

As to the use of such tables for *survey work*, my proposal goes one step further by introducing these unit-lanes into practical use by assuming that on any baseline of any length there is radiated an invariable number of 400 *unit-lanes*. This needs some further explanation.

The usual definition of a (Decca) lane is the distance between two consecutive hyperbolae of 0° phase difference, and these hyperbolae are computed and plotted in the chart. In Decca or other phase-comparison systems it is however by no means essential that hyperbolae of 0° phase difference should be used as reference lines. *The essential property of these systems is that they radiate patterns which have a stable position in space; any system of subdivision of the number of actually radiated lanes can just as well be used for a pattern of reference lines to be computed and plotted on the chart.* In this system of 400 unit hyperbolae, the coordinates can be taken directly from the tables, which saves the work of interpolation.

This choice of 400 unit-lanes, however, is not in agreement with the display on normal Decometers, because their unit is $\frac{360^\circ}{100} = 3.6^\circ$ phase difference. To overcome this discrepancy between chart and Decometer units, it is therefore necessary to change the latter. As actual baselines vary widely in length (and consequently the number of normal lanes), the reading-scale of the Decometers must be made continuously variable within a wide range and then set for any particular baseline. In principle, this could be achieved by an electronic modification, although some sort of mechanical solution appears to be more simple. The ratio to be set to existing Decometers is of course 400 divided by the physically existing number of lanes as determined by a lane-count with normally divided Decometers.

As in this proposal there will always be 400 unit-lanes, *any hyperbolic pattern on any baseline anywhere on the globe will be the same* and this makes it possible to prepare chart patterns in advance in the form of *standard sheets* as will be described in more detail in the following sections. These standard sheets can be kept in the office at the home base, and for any particular use can be brought to the required scale and correct azimuthal orientation by a process of photographic enlargement and simultaneous fitting into the known chart coordinates of the transmitters.

This enables all the required chart patterns to be photographed or printed directly on the working sheets to be used by the surveyor.

Standard sheets on a dimensionally stable plastic material can be prepared in advance once and for all, and the speed of production of latticed survey sheets is only a matter of availability of suitable photographic enlarging equipment of the

STANDARD SHEET INDEX

Standard sheets 40.000 by 40.000 units

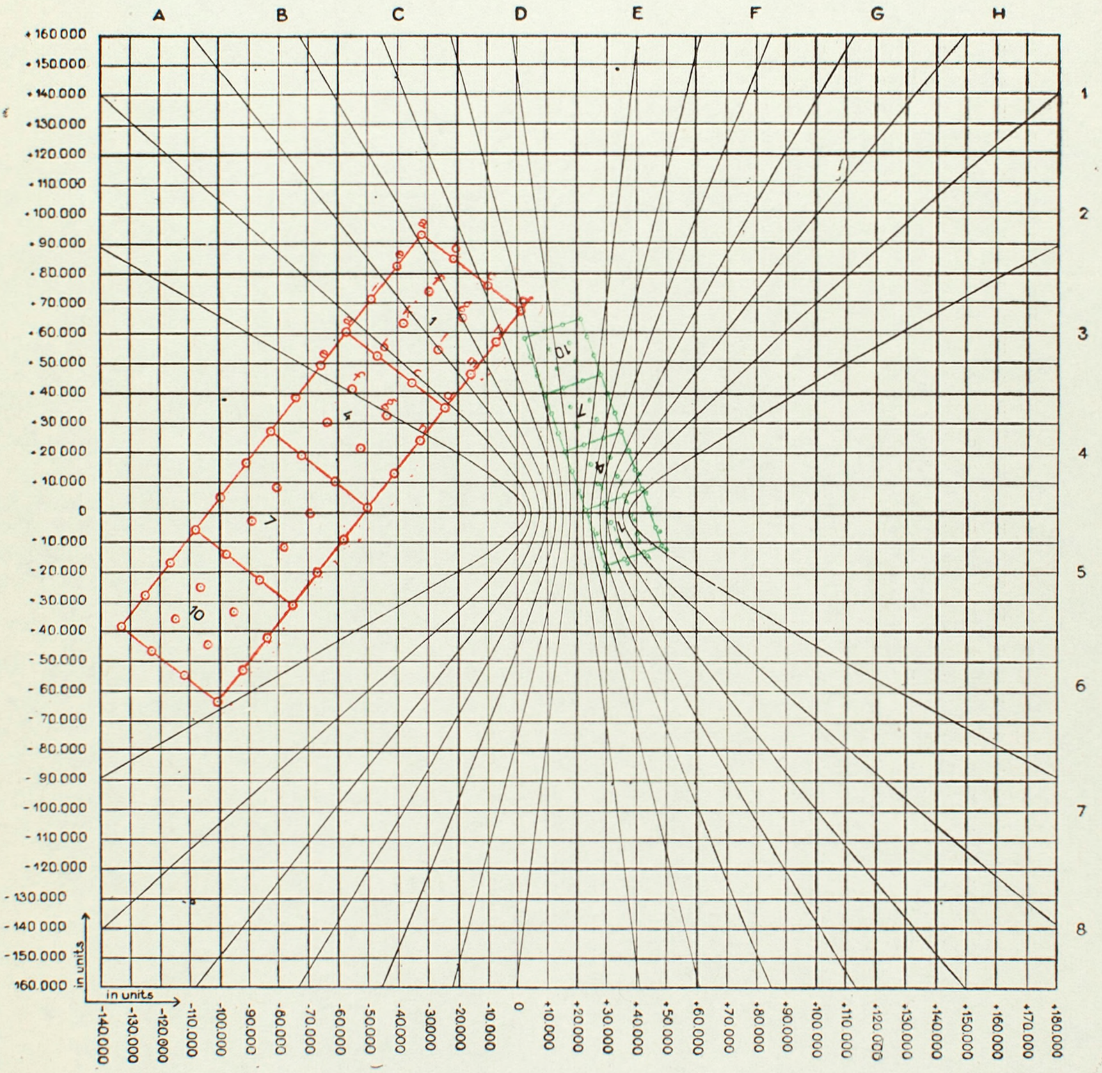


Fig. 1.

type generally used by chart reproduction agencies. For survey in remote countries this can be done in the home office of the chart agency and the charts can be mailed to the surveyor in the field.

Summarizing, this universal method requires:

(*a*) Preparation in advance of standard sheets.

(*b*) Bringing to the correct scale and azimuth by means of a photographic process, using generally available equipment.

(*c*) Modification of Decometers or other display meters.

Evidently the idea in principle is applicable also to other systems than Decca, including circular systems.

7. — STANDARD SHEETS

Coordinates of 400 flat hyperbolae can be plotted with the aid of the British (Sadler/Atherton) or similar tables and the standard hyperbolae can then be drawn through these plotted points. As the standard sheets are made once and for all, every precaution can be taken to draw them as accurately as possible on one of the commercially-available stable plastic materials; from these originals any desired number of photographic copies can be made for use by other charting agencies. (It should be noted that no separate standard sheets are needed for Red, Green and Purple baselines.)

Notwithstanding the utmost care taken in their plotting and drawing, these standard sheets will be within certain limits of accuracy; as they must be photographically enlarged to bring them to the scale of the working sheets, only a moderate enlargement factor is acceptable. This means that the standard sheets should not be constructed on too small a scale.

For all imaginable practical applications — that is, working sheet scales up to 1/25 000 — the enlargement factor is always smaller than 8 when the total area of coverage is subdivided into standard sheets of 1 × 1 metre as indicated on the *standard sheet index* of which fig. 1 is a reproduction.

As flat homofocal hyperbolae are used, the standard sheets are symmetrical with respect to the X-axis and to its perpendicular halfway along the baseline. Consequently it is necessary only to prepare a quarter of the total number of standard sheets of fig. 1, that is to say:

16 *standard sheets cover all needs anywhere in the world.*

Fig. 2 is an example of a standard sheet ($C_3 = C_6 = F_3 = F_6$). Because of reduced scale of this reproduction, only every fifth hyperbola is shown.

8. — WORKING SHEETS

The scale of the working sheets is dependent on the requirements of the survey. Fig. 3 is an example of a *working-sheet index* for actual sheets at a scale of 1/100 000 for part of the area of coverage of the Decca Survey Chain in Netherlands New Guinea.

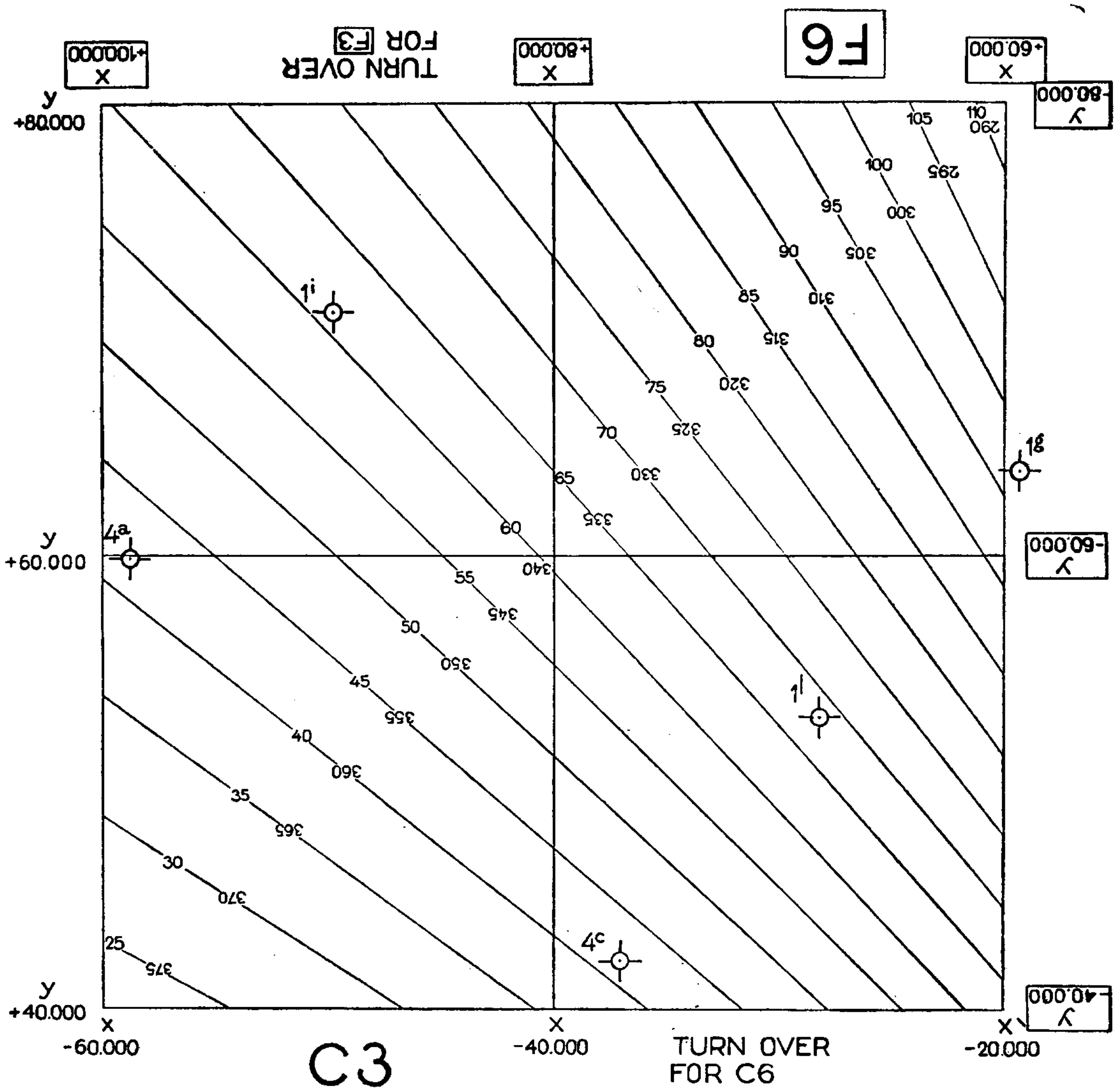


fig 2
SAMPLE STANDARD SHEET
AT REDUCED SCALE
(only every 5th lane is drawn here)

Fig. 2.

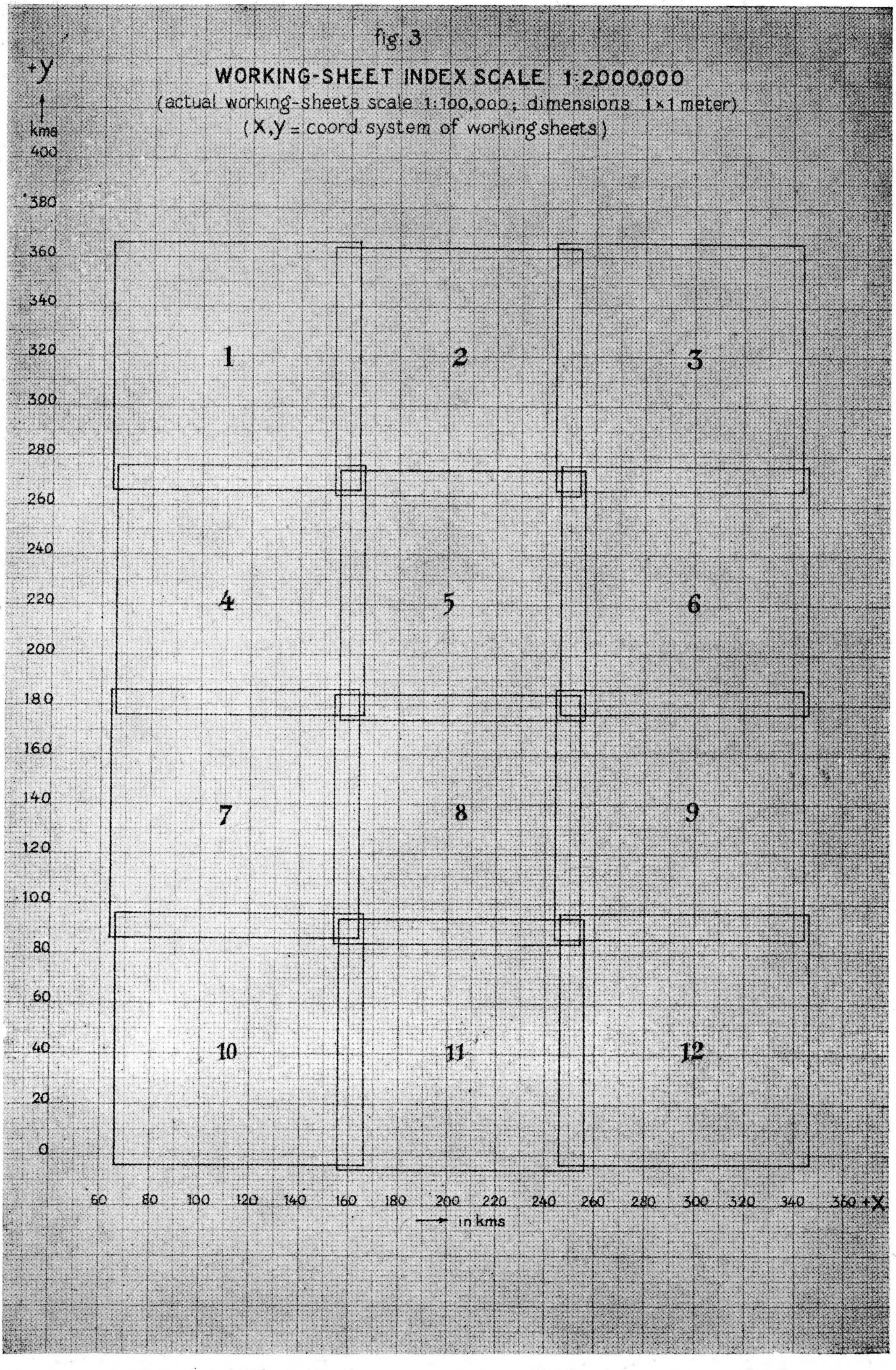


fig. 3
WORKING-SHEET INDEX SCALE 1:2.000.000
(actual working-sheets scale 1:100,000; dimensions 1×1 meter)
(X,Y = coord system of working sheets)
+Y
kms
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
11
12
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360 +X
in kms

Fig. 3.

The origin of a rectangular coordinate system X, Y — to be used as the plane grid on *all* the working sheets — can be arbitrarily adopted somewhere beyond the left hand lower corner of the area of coverage of the chain. In New Guinea the origin has been chosen such that the Master coordinates become:

$$X_M = +250\,000 \text{ m}$$

$$Y_M = +250\,000 \text{ m}$$

The X, Y coordinates of the working-sheet corners, as planned on the index-sheet, can be seen from fig. 3. At the scale of 1/100 000 the dimensions of these working sheets are 1 × 1 metre.

These working sheets are prepared and gridded either by the local survey party or at the home base *before the coordinates of the transmitters have been determined**. Fig. 4 is an example of working sheet No. 1 (at a reduced scale); nine reference points — to be used as will be explained later — (intersections of grid lines) are indicated on this sheet.

9. — TRANSFER OF STANDARD SHEET HYPERBOLIC PATTERNS TO WORKING SHEETS

In order to be able to transfer the standard sheet hyperbolic patterns to the working sheets, it is necessary to know the mathematical relationship between the two plane coordinate systems X, Y (working sheets) and x, y (standard sheets). To this end it is necessary that the coordinates X, Y of the transmitters are known. The relationship is computed by means of a conformal coordinate transformation as will be explained later.

In those cases where the transmitters can be tied in to an existing terrestrial triangulation, the relationship can be determined and hyperbolic patterns can then be transferred (photographically) to the working sheets before even the transmitting equipment has been installed; the latticed charts can therefore be made available in advance in order that the actual survey can start immediately after the chain is put into full operation. The only thing necessary is to « count » the number of radiated lanes ** and use this number for setting the multiplication factor in the Decometers (see section 6).

In undeveloped countries such as New Guinea it often happens that no reliable terrestrial triangulation, or none at all, is available; in these cases the transfer has to wait until the chain is in full operation and the lanecount has been made, because the coordinates of the transmitters then have to be determined from measurements in the radiated patterns as explained in (7). Here also the counted number of lanes would be used for setting the multiplication factor in the Decometers. The economic advantage of the proposed method of standard sheets is that the latticed working sheets can be made available to the surveyor at such short notice that only a couple of days need be lost before the actual survey can start.

---

* For the method of determination of the coordinates of the New Guinea transmitters see (7).

** This need not take more than one day when the lanecounts are carried out by an aircraft.

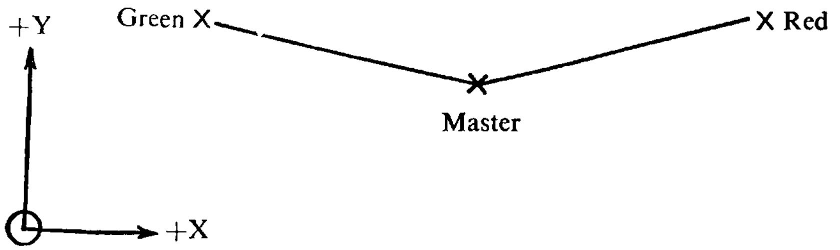
In both cases the latticed working sheets can be prepared either by the local survey party or at the home office. In many cases the latter will be preferred because suitable photographic enlarging equipment will be more readily available.

Once the X, Y coordinates of the transmitters are known, it is only a matter of half an hour's computation to derive the transformation formulae as is shown in the following example:

Example

Rectangular plane coordinates of transmitters of the New Guinea Decca Survey Chain, as adopted in the geodetic framework:

(+Y-axis through Master corresponds with true North).



	X	Y
Master	+250 000	+250 000
Red	+316 884	+304 781
Green	+ 81 284	+306 771

coordinates in metres.

Note: The origin of the system is chosen such that the coordinates of the Master become +250 000 for X as well as for Y.

The *unit-coordinates* of the transmitters are (invariably):

	x	y
Master	0	0
Any Slave	+ 40 000	0

coordinates in units.

Derivation of transformation formula: Red.

$$x_R = +40\,000 \qquad y_R = 0$$
$$x_M = 0 \qquad y_M = 0$$
$$\Delta x = +40\,000 \qquad \Delta y = 0$$
$$\text{tg}\,\alpha = \frac{+40\,000}{0} = +\infty$$
$$\alpha = 90^\circ$$
$$s = 40\,000 \text{ units}$$

$$X_R = +316\,884 \qquad Y_R = +304\,781$$
$$X_M = +250\,000 \qquad Y_M = +250\,000$$
$$\Delta X = +\ 66\,884 \qquad \Delta Y = +\ 54\,781$$
$$\text{tg}\,A = \frac{+\ 66\,884}{+\ 54\,781} = +1.220934$$
$$A = 50^\circ\ 40'\ 51''$$
$$S = 86\,454 \text{ metres.}$$

$$\Delta\alpha = \alpha - A = +39^\circ\ 19'\ 09''$$
$$\lambda = \frac{s}{S} = 0.462674$$

$$\sin\Delta\alpha = +0.633638$$
$$\cos\Delta\alpha = +0.773630$$

$$a = \lambda \cos \Delta\alpha = +0.357938$$
$$b = \lambda \sin \Delta\alpha = +0.293168$$

$$\boxed{\begin{aligned} x &= +\ 0.357938\,X + 0.293168\,Y - 162\,777 \\ y &= -0.293168\,X + 0.357938\,Y - \ \ 16\,192 \end{aligned}}$$

Red patterns; transformation formula to convert rectangular chart coordinates X, Y into unit-coordinates x, y.

Derivation of transformation formula: Green.

The formula is derived in a way similar to that for Red.

$$\boxed{\begin{aligned} x &= -0.212970\,X + 0.071662\,Y + 35\,327 \\ y &= -0.071662\,X - 0.212970\,Y + 71\,158 \end{aligned}}$$

Green patterns; transformation formula to convert rectangular chart coordinates X, Y into unit-coordinates x, y.

These two transformation formulae are now used to convert the X, Y coordinates of nine grid intersections on each sheet (sheet No. 1 : 1^a to 1^i and 4^a to 4^c, see fig. 4) into x, y coordinates. The result of this transformation for sheet 1 is given hereunder; it takes about half an hour on a conventional table-model computing machine. For the 12 working sheets of fig. 3 the transformation would therefore require 6 man-hours, but the work can be speeded up by assigning it to several computers.

These are all the computations to be carried out for the two patterns over the whole area of coverage of the chain and they need not take more than a few hours.

The next — immediate — step is to plot the coordinates x, y from the above list (separately for Red and Green) on the standard sheet index as shown in fig. 1. The only purpose of this plotting is to find out the numbers of the standard sheets that have to be used for the further photographic process, consequently no high accuracy in plotting is required.

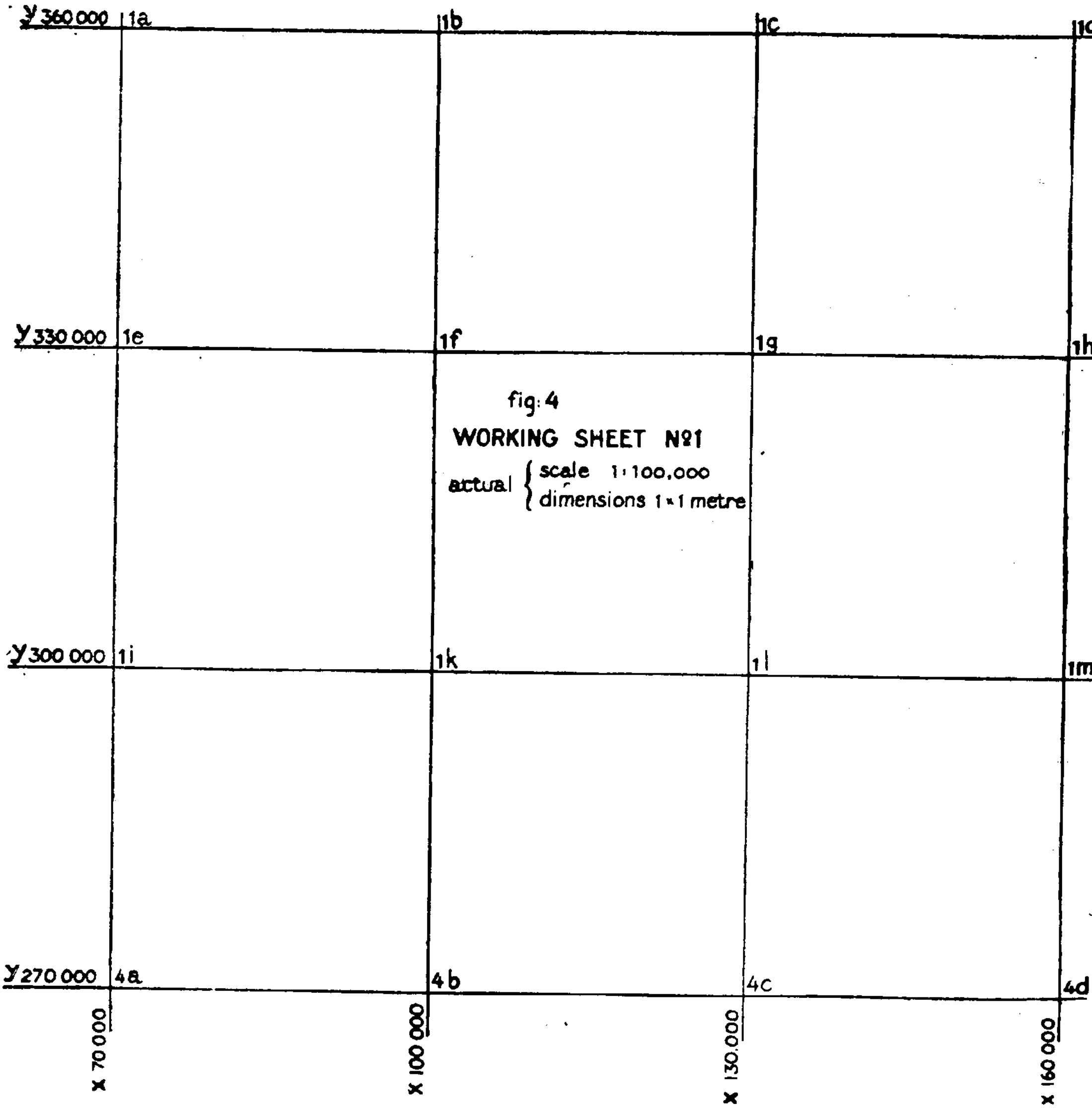

y 360 000
1a
1b
1c
y 330 000
1e
1f
1g
1h
fig: 4
WORKING SHEET Nº1
actual { scale 1:100,000
dimensions 1×1 metre
y 300 000
1i
1k
1l
1m
y 270 000
4a
4b
4c
4d
x 70 000
x 100 000
x 130 000
x 160 000

Fig. 4.

The following step is to plot the x, y coordinates of the grid intersections (1^a to 1^i and 4^a to 4^c for sheet No. 1) accurately on the standard sheets as shown in fig. 2; this plotting is done by means of a coordinatograph.

The (transparent) gridded working sheet is now mounted on the glass screen and the appropriate standard sheet(s) are fitted to the copy board of a conventional rectifying camera (fig. 5). For the Red pattern of working sheet No. 1, that would be the standard sheets C2, C3, C4 and D3. The copy board is now turned and moved until the points on the standard sheet are seen to be in coincidence with the corresponding points on the working sheet. By now placing a photosensitive sheet of material in contact with the working sheet, a photographic exposure will result in a new gridded working sheet with one hyperbolic pattern superimposed on it at the correct scale and correct azimuthal orientation.

The process has to be repeated to « overprint » the second hyperbolic pattern.

Evidently the speed of production of latticed working sheets is mainly a matter of availability of rectifying cameras. They are, however, of the type that will be in the possession of every chart production agency and it will therefore generally be possible to do the work simultaneously by using several of them. It would not be difficult to organize the work in such a way that all the latticed working sheets became available within say 2 days.

The idea of the use of standard sheets for a speedy production of accurate latticed charts has not yet been put to an actual test. It is therefore possible and even likely that the procedure as described above will have to be altered in some of the details; the most suitable method of photographic rectifying and reproduction will no doubt have to be learned from practical experience. It is, for instance, likely that it will prove impractical to mount more than one or two standard sheets on the copy board. In the example of the Red pattern on working sheet No. 1 — covered by four standard sheets — that would then necessitate more than one photographic exposure which, however, would not impose great difficulties (the Green pattern on sheet 1 is covered by the two standard sheets E4 and E5).

Summarizing, it can be concluded that the method of speedy production of latticed charts for accurate survey purposes by means of the use of standard sheets lends itself ideally to a fine subdivision of the work and can be carried out by chart production agencies with generally available equipment. The cost of production will be small compared with other methods, the more so because the actual survey operations can start very shortly after the chain has been put into operation.

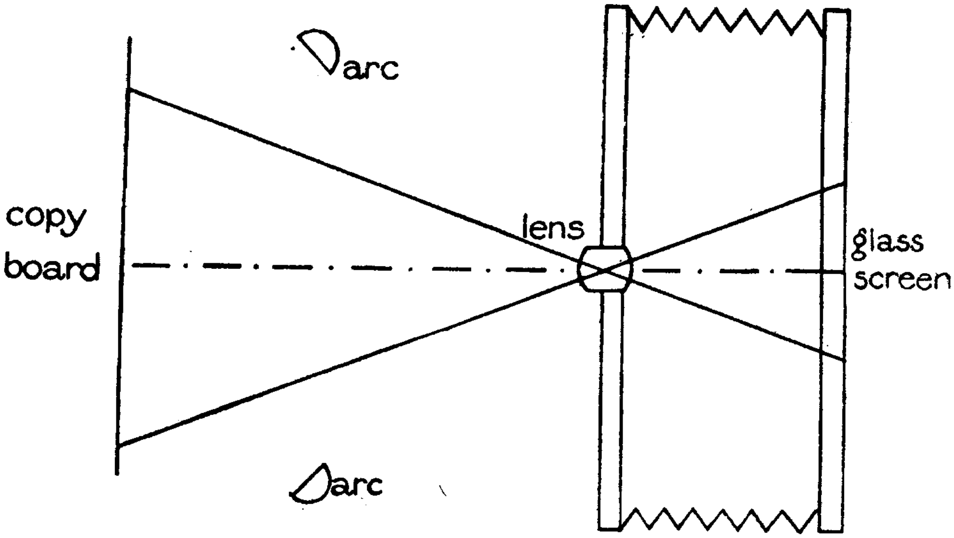


Fig. 5.

TRANSFORMATION OF METRES INTO UNITS

Working sheets and working sheet index (METRES)			Standard sheets and standard sheet index (UNITS)					
			RED			GREEN		
No.	+X	+Y	x	y	No.	x	y	No.
1	70 000	360 000	—32 131	+92 144	C2	+46 217	—10 528	E5
1		330 000	—40 976	+81 406	C2	+44 068	— 4 138	E5
1		300 000	—49 771	+70 668	C3	+41 918	+ 2 251	E4
1 & 4		270 000	—58 566	+59 930	C3	+39 768	+ 8 640	E4
↓	↓	↓	↓	↓	↓	↓	↓	↓
1	100 000	360 000	—21 443	+83 349	C2	+39 828	—12 677	E5
1		330 000	—30 238	+72 611	C3	+37 678	— 6 288	E5
1		300 000	—39 033	+61 873	C3	+35 529	+ 101	E5 E4
1 & 4		270 000	—47 828	+51 134	C3	+33 379	+ 6 490	E4
↓	↓	↓	↓	↓	↓	↓	↓	↓
1	130 000	360 000	—10 705	+74 552	D3	+33 439	—14 827	E5
1		330 000	—19 500	+63 816	D3 C3	+31 289	— 8 438	E5
1		300 000	—28 295	+53 078	C3	+29 140	— 2 049	E5
1 & 4		270 000	—37 090	+42 339	C3	+26 990	+ 4 340	E4
↓	↓	↓	↓	↓	↓	↓	↓	↓
1 & 2	160 000	360 000	+ 34	+65 759	D3	+27 050	—16 977	E5
1 & 2		330 000	— 8 761	+55 021	D3	+24 900	—10 588	E5
1 & 2		300 000	—17 557	+44 283	D3	+22 750	— 4 199	E5
1 & 2 4 & 5		270 000	—26 352	+33 544	C4	+20 601	+ 2 190	E4
↓	↓	↓	↓	↓	↓	↓	↓	↓

10. — CORRECTION FOR DIFFERENCE BETWEEN FLAT AND SPHEROIDAL HYPERBOLAE

(*Example for U.T.M. chart projection*)

In section 4 it has already been explained that — for survey work — there are no objections against the use of chart patterns of flat hyperbolae, provided that corrections are applied that make good the difference between flat and spheroidal hyperbolae. Another provision to be made with respect to practical operational applicability is that these corrections should not become too large at any point within the area of coverage of the chain.

The flat chart hyperbolae need not themselves be corrected for these differences; they can be taken into account much more easily by applying the differences as corrections to the Decometer-readings before plotting them in the (flat) chart-patterns.

As these corrections change slowly and gradually from place to place, they need only be computed for a limited number of points; usually some 40 points (intersections of parallels and meridians or of grid lines) distributed over the whole area of coverage will be sufficient to draw smooth correction curves on a *correction chartlet* of the type of fig. 6; see also (7). This computation of corrections involves the computation (for each of the chosen intersections) of the lane numbers in the spheroidal hyperbolic patterns, using one of the methods mentioned in section 2 and taking the corresponding flat lane numbers from the table; in both cases unit lanes have to be used. Such a computation of corrections for two patterns will take about three man-days, but can of course be split up between different computers; it can be carried out during the photographic enlargement operations mentioned in section 9 and consequently no extra time need be lost. The computations can be carried out either at the home office or by the local survey party.

An example of how such a correction chartlet would look for a hypothetical Decca survey chain and flat hyperbolae plotted with respect to the rectangular grid in the U.T.M. chart projection, is given in fig. 6. The magnitude of the corrections of fig. 6 will seldom be exceeded in any practical application anywhere in the world; applying the corrections to the Decometer-readings before plotting would constitute no practical problems or difficulty whatsoever, the more so because Decometer corrections of another nature — to be described in section 11 — have to be applied in any case and the two corrections can be combined into one single correction.

Note: In many cases these corrections will be much smaller than those of fig. 6. For instance, for a particular Decca survey chain envisaged for installation next year, this type of correction will not exceed 0.005 lane (0.001 standard-lane) within the area of coverage of 4000 sq. km (1600 sq. st. mi.).

11. — CORRECTION FOR DIFFERENT PROPAGATION SPEEDS

In order to simplify pattern computation or tabulation — either spheroidal or flat — the usual procedure is to assume a constant speed of propagation of the radio waves (in Decca: constant phase-velocity) throughout the coverage of the chain. Actually, however, the waves travel with a speed that is dependent on the electrical conductivity of the terrain, which usually varies in different parts of the coverage.

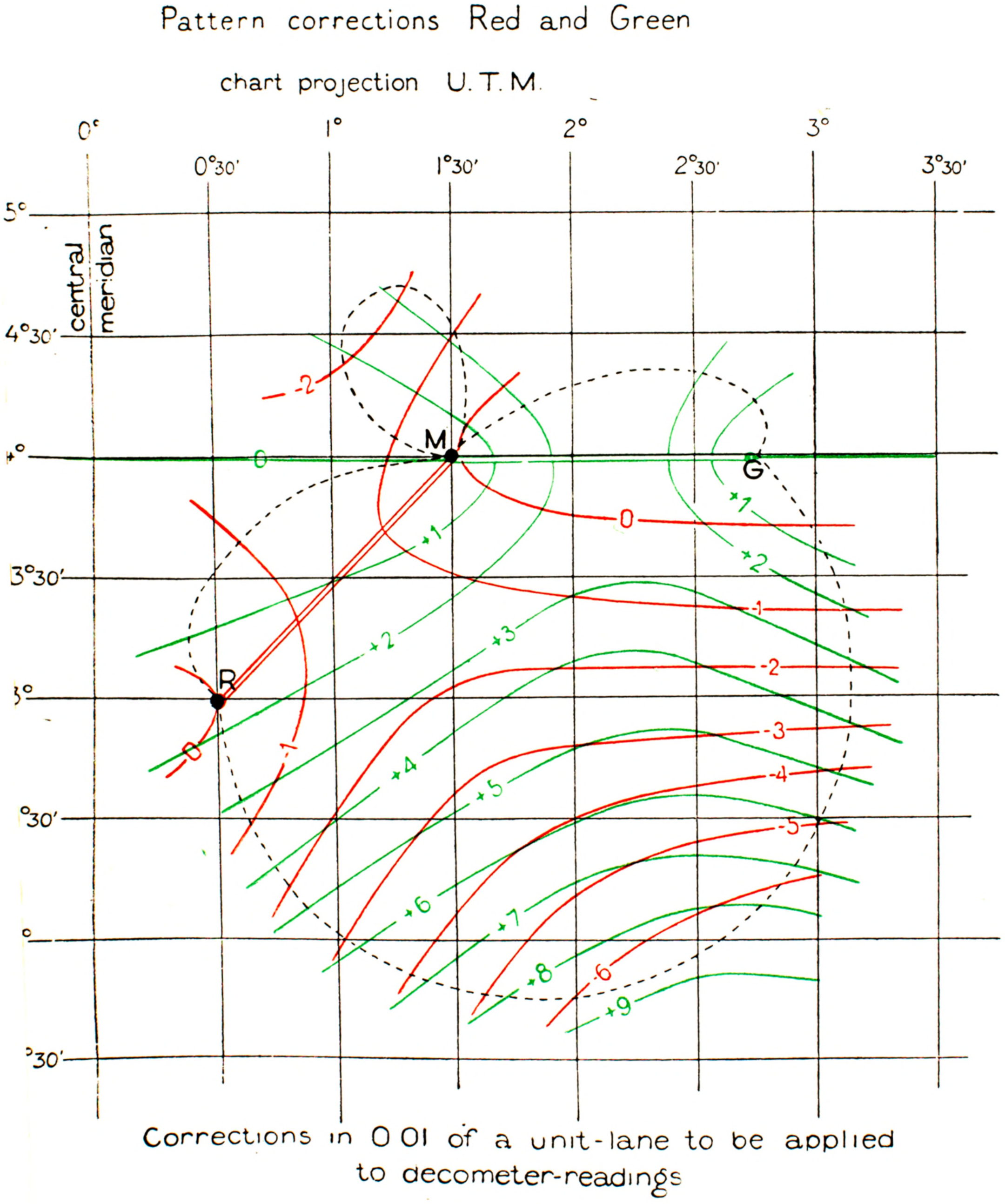

Pattern corrections Red and Green

chart projection U.T.M.

Corrections in 0.01 of a unit-lane to be applied to decometer-readings

Fig. **6.**

Consequently the computed or tabulated hyperbolae are an idealized representation of the patterns actually radiated. Here also it would be possible to correct the chart patterns, but a much simpler way is to compute corrections and apply them to the (Decometer) readings before plotting in the uncorrected chart patterns. References (7) to (10) describe how the effect of different propagation speeds over different parts of the trajectories travelled by the radio waves from transmitter(s) to receiver, can be taken into account in the form of Decometer-corrections; an example is given in (7).

Evidently the most practical way to take into account both the corrections of sections 10 and 11 simultaneously, is to add them algebraically and to plot them in a *combined correction chartlet* (fig. 6).

12. — ACKNOWLEDGMENTS

The author wishes to thank Mr. Wernink, Mr. Raasveld and Mr. Rombach (Netherlands Hydrographic Office) for their help and advice during our many discussions in relation to the idea of the use of standard sheets.

The principles have been discussed also with Mr. Atherton and Mr. Pascoe of the British Hydrographic Office and Mr. Sadler of H.M. Nautical Almanac Office, who not only gave me valuable advice, but also were kind enough to make a copy of their table of flat hyperbolae available to me.

It was a great pleasure to have the opportunity to discuss the idea of standard sheets with Mr. Hawker, Mr. Powell and Mr. Oliver of the Decca Navigator Co. Ltd.

REFERENCES

(1) C. A. WHITTEN. Rapport Général No. 2 sur les projections; liste des tables publiées; Travaux de l'Association Internationale de Géodésie. Vol. 17, Paris 1952.

(2) HYDROGRAPHIC DEPARTMENT PROFESSIONNAL PAPER No. 9. An elementary account of hyperbolic lattice computation; Admiralty, London, 1950.

(3) S. BALLARIN. Geometric properties of position lines in hyperbolic navigation and their lay-out on the reference ellipsoid; the International Hydrographic Review, Vol. XXVII, No. 2, Monaco, Nov. 1950.

(4) P. HUGON. Note sur une représentation rectiligne des hyperboles sphériques; publications scientifiques et techniques du Ministère de l'Air, Paris, 1950.

(5) M. DUPUY. Méthodes nouvelles employées pour le calcul des réseaux d'hyperboles homofocales sur les cartes en projection conforme; Navigation No. 8, 1954.

(6) D. H. SADLER. The construction of hyperbolic lattices; Journal of the Institute of Navigation (British), Vol. IV, No. 2, July 1951.

(7) J. TH. VERSTELLE. Use of the Decca Navigator Survey System in New Guinea for Hydrography and as a Geodetic Framework. Paper presented to the XIth Assembly of IUGG. Toronto, 1957.

(8) H. LARSSON. Investigation of the accuracy obtained with the Decca system for survey in the Southern Baltic, with Appendix II. A new technique for the construction of hyperbolic charts; the International Hydrographic Review, Vol. XXVI, No. 2, Monaco, November 1949.

(9) H. LARSSON. Use of the Decca Navigator by the Royal Swedish Hydrographic Office; the International Hydrographic Review, Vol. XXXIII, No. I, Monaco, May, 1956.

(10) P. HUGON. Note sur un moyen approché permettant de prévoir les déformations des hyperboles équiphases en franchissement des lignes de côte; Annales de Radioélectricité, Vol. XII, No. 47, January 1957.

(11) INTERNATIONAL HYDROGRAPHIC BUREAU. Radio aids to maritime navigation and hydrography; special publication No. 39, Monaco, July 1956.