III. — AEROTRIANGULATION ADJUSTMENT

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(Extracts)

INTRODUCTION

The raw product of instrumental aerotriangulation of a strip of photographs is a list of numbers which are the instrument-coordinates of image points, pass points, center points and control points. The coordinates are usually expressed in millimeters, and the reference axes are the *x*-, *y*-, and *z*-axes of the instrument.

The adjustment of aerotriangulation consists of transforming the instrument-coordinates into their proper geodetic ground-coordinates in a systematic way, such that the transformed coordinates of control points are identical respectively to their known geodetic values. The points are then plotted on a grid at an appropriate scale using the adjusted instrument-coordinates, after which the detail mapping is performed by means of the same, or a different, instrument to fit six adjusted points in each model.

In the instrumental process of aerotriangulation, each additional photograph (stereoscopic model) is attached and relatively oriented very carefully to fit the previous one in scale, level and azimuth. Wherever geodetic points are encountered, their instrument-coordinates are observed and recorded just as any other image point, but no effort is made to fit them at this stage.

The chain of models is allowed to deviate without any restraint, as it is realized that the character of the deviation should be systematic.

The method of adjusting this figure to fit control is a practical one which allows the control to be purely horizontal or purely vertical, and which also serves to isolate gross human mistakes which are occasionally committed with regard to such matters as the identification of a control point. The method is divided into two principal steps: (1) corrections to all the image-coordinates for systematic discrepancies indicated at control points, and (2) a uniform coordinate transformation. In the Coast and Geodetic Survey the first step consists of a graphic solution, and the second consists of a numerical computation facilitated by the use of an International Business Machine (which is not necessary for the system as desk calculators might be used instead, without undue difficulty).

Two cases arise depending on the relative arrangement of control in the strip. If sufficient control occurs in the first model for complete absolute orientation, only the graphic step of the adjustment is necessary. On the other hand, if absolute orientation of the first model is not possible, both steps are required. However, a preference exists in this Bureau to employ both steps even where it may not be necessary.

THE GRAPHIC PROCEDURE

The graphic procedure for determining the systematic correction follows a coordinate transformation of the few control points by desk calculator, as explained in the two sections that follow. The horizontal-coordinates are adjusted independently from the vertical-coordinates.

The horizontal adjustment involves two principal types of deviations (1) horizontal bow Δy, and (2) differential scale, Δx.

Each type in turn has an associated logical secondary component: (1a) in a bowed strip, not only does a model required a corrective translation Δy perpendicular to the flight axis, but it also requires a rotation or swing $\Delta x'$, which adjusts the outer points of the model in a direction parallel to the flight line; and (2a) not only does a scale correction Δx adjust a model in the flight direction, but it also moves the outer points in a direction $\Delta y'$ perpendicular to the flight direction, inasmuch as a scale correction applies to both dimensions.

To summarize, these four corrections are: (1) azimuth, Δy, (2) x-scale, Δx, (1a) swing, $\Delta x'$, and (1b) y-scale, $\Delta y'$.

An azimuth or x-scale correction is plotted as an ordinate perpendicular to the flight axis (the x-axis) of the strip at the section where the control point lies. Any convenient scale is used, such as one inch on the graph equaling one millimeter (or ten feet) of correction. The graphs of the two corrections are smooth curves. The azimuth curve is considered as quadratic. The x-scale curve is known to be cubic, but it seldom appears to be different from quadratic in practice.

It can be proved that the swing correction to any point is proportional to (1) the slope of the azimuth curve and (2) the distance the point lies from the flight axis. Hence the swing-curve can be constructed directly from the azimuth-curve by measuring and plotting the slope of the azimuth-curve. It may appear reasonable from the calculus that, inasmuch as the azimuth-curve is quadratic, the swing-curve will be a straight line.

The direction of the correction has understandable significance. The direction of the azimuth correction should be obvious at once upon examination of the y-discrepancies at control points. The proper direction of the swing correction can be ascertained by visualizing how the rectangular model lying on, and aligned with reference to, the azimuth *error* curve must be shifted (Δy) and swung (rotated, $\Delta x'$) in order to move it onto the flight axis. The swing correction will be to the right for points lying on one side of the flight axis, and to the left on the other side. Moreover, the direction of the swing correction is reversed where the swing curve crosses the flight axis.

The y-scale curve is constructed from the x-scale curve in the same manner as the swing-curve is constructed from the azimuth-curve. The directions of the corrections, however, are not so evident. The correction in the x-coordinate is zero at the first control point, and if the corrections to images to the right of the first model are plus, then the model undergoes an enlargement. Thus, the points need to be moved outward also in $\Delta y'$ perpendicularly away from the axis.

The y-scale correction, like swing, is equal to the product of the ordinate of the y-scale curve (which is the slope of the x-scale curve) and the distance the point is from the axis.

Elevation correction for vertical bridging may also be considered as consisting of two components but, ironically, these two curves are not related to each other. In this Bureau, the components that have been selected are (1) the vertical bow (*BZ* curve) of the flight axis and (2) the twist of the ribbon of models. These corrections are determined by considering pairs of vertical-control points located on opposite sides of the axis. The vertical bow Δz is represented by a curved line (theoretically quadratic) whereas the twist $\Delta z'$ usually is a nearly-straight line.

The value of an ordinate on the twist-curve is the quotient of the difference in the z-corrections required for a pair of vertical-control points divided by the graph paper (model) distance between them. (This distance is reckoned perpendicular to the flight axis if the two points are not exactly opposite each other.) The value of an ordinate on the bow-curve is the value prorated from one of the control points onto the axis, utilizing the twist value already determined. Consequently, if the control points are equal distance from the axis, the bow ordinate is the average of the two corrections. The algebraic sign of the bow correction is obvious; that for the twist is determined by comparing the corrections required at the two control points. The sign for the twist is plus on one side of the axis and minus on the other, and reverses if the twist-curve crosses the axis. The twist correction for any other image is the product of the ordinate of the twist-curve at the point and the distance the point is from the axis.

These six corrections are summarized:

Corrected x = (instrument x) $\pm$ (ordinate of the x-scale curve) $\pm$ (product of ordinate of swing-curve and the distance the point is from the axis).

Corrected y = (instrument y) $\pm$ (ordinate of the azimuth-curve) $\pm$ (product of the ordinate of the y-scale curve and the off-axis distance).

Corrected z = (instrument z) $\pm$ (ordinate of the vertical bow-curve) $\pm$ (product of ordinate of twist-curve and the off-axis distance).

As the graphic procedure is relatively new, it is not performed in a well-established routine. Each analyst conceives and solves the problem in a slightly different manner, but utilizes the same fundamental ideas given here. Undoubtedly future developments will simplify, streamline and systematize the procedure so that it will be easier for others to apply.

Drawing the curves consists of a successive approximation procedure, involving redrawing at least one of the quadratic curves. One method is to plot first the quadratic horizontal-correction curve having the greatest magnitude, and its derived slope curve next. The slope curve gives corrections to be applied to the second quadratic curve as explained above. After the second curve and its derivative are plotted, the first curve can be redrawn to incorporate the second slope correction. Further approximations usually have an insignificant effect. In practice, a preliminary vertical bow curve is also frequently constructed prior to the twist curve, and finally the bow curve is redrawn taking both vertical components into consideration.

THE COMPUTATIONAL LINEAR TRANSFORMATION PROCEDURE

The computational procedure is used to adjust the coordinates in a uniform (linear, proportional) manner so that the instrument values are converted into correct geodetic values at the two horizontal control points at opposite ends, and at two vertical control points near one end and one at the other. No further corrections are then required at these end points, but all the other control points between the ends require the corrections by the graphic procedure already explained.

Three transformation equations are employed:

$$X = ax - by + c \qquad (1)$$

$$Y = bx + ay + d \qquad (2)$$

$$Z = e(x - x_3) + f(y - y_3) + gz + h \qquad (3)$$

Here x, y, z are instrument-coordinates for a point, X, Y, Z are the corresponding ground-coordinates for the same point, and a ..., h are the constants of transformation.

Equations showing how to determine the constants of transformation are given where: x, y, z, are instrument-coordinates of a horizontal-control point near one end of the strip; x_2, y_2, z_2 are for a horizontal-control point at the opposite end of the strip; x_3, y_3, z_3 are the instrument-coordinates of a *vertical*-control point near one end of the strip (x_3, y_3, z_3 is considered for convenience here as different from x_1, y_1, z_1, but a single horizontal and vertical-control point may well serve both roles if Z_1 is known); and x_4, y_4, z_4 and x_5, y_5, z_5 are the coordinates of two vertical-control points located on opposite sides of the axis at the opposite end of the strip (x_4, y_4, z_4 may well be identical to x_2, y_2, z_2 if Z_2 is known).

$$a = \frac{(X_1 - X_2)(x_1 - x_2) + (Y_1 - Y_2)(y_1 - y_2)}{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad (4)$$

$$b = \frac{(x_1 - x_2)(Y_1 - Y_2) - (y_1 - y_2)(X_1 - X_2)}{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad (5)$$

$$c = X_1 - ax_1 + by_1 \qquad (6)$$

$$d = Y_1 - bx_1 - ay_1 \qquad (7)$$

$$= m\sqrt{a^2 + b^2} \qquad (8)$$

(m, if needed, is a factor for the conversion of units, such as 3.2808 for converting metric units into English units.)

$$h = Z_3 - gz_3 \qquad (9)$$

$$k_4 = Z_4 - gz_4 - h \qquad (10)$$

$$k_5 = Z_5 - gz_5 - h \qquad (11)$$

$$e = \frac{k_4(y_5 - y_3) - k_5(y_4 - y_3)}{(x_4 - x_3)(y_5 - y_3) - (x_5 - x_3)(y_4 - y_3)} \qquad (12)$$

$$f = \frac{k_5(x_4 - x_3) - k_4(x_5 - x_3)}{(x_4 - x_3)(y_5 - y_3) - (x_5 - x_3)(y_4 - y_3)} \qquad (13)$$

The inverse transformation may be required, especially for the analysis of diagonally-flown strips:

$$x = \left(\frac{a}{a^2 + b^2}\right) X + \left(\frac{b}{a^2 + b^2}\right) Y - \left(\frac{a}{a^2 + b^2}\right) c - \left(\frac{b}{a^2 + b^2}\right) d \qquad (14)$$

$$y = \left(\frac{-b}{a^2 + b^2}\right) X + \left(\frac{a}{a^2 + b^2}\right) Y - \left(\frac{a}{a^2 + b^2}\right) d + \left(\frac{b}{a^2 + b^2}\right) c \qquad (15)$$

$$z = (1.g)\ [Z - e(x - x_3) - f(y - y_3) - h]. \qquad (16)$$

Outline of procedure

Two complete procedures are outlined for utilizing the graphic and computational operations described heretofore. In the first procedure, the instrument-coordinates are transformed into ground units of measurement *before* the graphic adjustment.

1. The constants of transformation are derived by desk calculator based on the control points at the ends.

2. The instrument-coordinates of all the points of the strip are transformed into equivalent ground coordinates. (In practice, the control points are transformed in Step 1 by desk calculator, as they are but few in number, the results can be used immediately, and the other points will not be used until later. The other points are transformed by means of IBM.)

3. The *corrections* (not errors) that need to be applied to the transformed instrument-coordinates of control points are computed.

4. The six systematic correction curves are plotted using the corrections of Step 3.

5. The total correction of each image point is computed from the graphs.

6. The final photogrammetric ground-coordinates of all the image points are computed by applying the total correction of Step 5 to the transformed values of Step 2.

7. Using the final photogrammetric ground-coordinates, positions of all the images are plotted at a suitable scale, and on a suitable medium, for the use of the detailing instrument.

In the second procedure, total graphic corrections in millimeters are applied directly to the instrument-coordinates, after which the corrected values are transformed into the final photogrammetric ground-coordinates. (The vertical phase is treated in the same manner as in the first procedure at the present time.)

The second procedure is but slightly different from the first one:

1. The constants of transformation are determined as in the first procedure, except that the coefficients of the inverse transformation are also computed from Equations 14 and 15.
2. Using the inverse coefficients of Equations 14 and 15, the known ground-control coordinates are transformed into corresponding instrumental values in millimeters. (The vertical phase may be omitted.)
3. The corrections in millimeters that need to be applied to the instrument coordinates of ground control points are computed, regarding the transformed ground values as correct.
4. The corrections are used to construct the systematic corrections graphs.
5. The total corrections are computed for all the images as in the first procedure.
6. The total corrections are applied to the observed instrument-coordinates.
7. All the corrected instrument-coordinates are then transformed into their corresponding final photogrammetric ground-coordinates using the direct transformation constants and Equations 1, 2 and 3.
8. As in the first procedure, using the final photogrammetric ground-coordinates, the position of all the images are plotted at a suitable scale and on a suitable medium for the use of the detailing instrument.

CONTROL REQUIREMENTS

The basis for specifying a given number and distribution of control points is related to the requirements for the construction of the correction graphs and to the required map accuracy. Theoretically, only three horizontal-control points per strip are needed to construct the curves, where one point is near each end, and the third near the center of the strip.

Five points per strip are preferable as a fault in one of them is both indicated and isolated, and it is unlikely (although not impossible) that two mistakes will occur together. This means that five horizontal-control points plus five pairs of vertical points are desirable to yield a rigid analysis.

ACCURACY

Frankly, the accuracy of the system has not been well established. It has been demonstrated repeatedly that strips of 40 photographs taken at 30 000 feet can be aerotriangulated satisfactorily for 1/200 000 scale mapping by using only alternate strips. It is fairly evident that the residual errors are related to the flight altitude, the length of the strip, and the certainty of control identification.

Several other factors are considered to have pronounced effects on accuracy: (1) the quality of the adjustment of the instrument and its mechanical condition; (2) the work habits of the organization, such as designating one operator to be responsible for the entire triangulation of a specific strip; (3) the use of numerical relative orientation; (4) the application of non-systematic corrections to cross tilts and height-tilt factors mentioned previously, and (5) the practice of pre-marking control points.

A few numerical quantities are mentioned to give an idea of the accuracy that might be obtainable. If photographs are taken from 10 000 feet with a six-inch camera, the preferred model scale is double the negative scale, or 1/10 000. A first-order instrument is read in units of 0.01 mm, which corresponds to about four inches on the ground. Repeated readings on very sharp images usually do not differ more than ± 0.02 mm. The residual parallax after relative orientation is ordinarily less than 0.03 mm, where half the amount presumably derives from each photograph. The connection of one model to another may have a horizontal discrepancy of 0.02 mm, and 0.04 mm vertically. The standard error summation for horizontal points is still less than ± 0.03 mm, and ± 0.05 mm for vertical points in a single model. This corresponds with the findings in practice. These errors are of an accidental nature appearing haphazard in a single model, and they do not include the cumulative quadratic, systematic tendencies of the aerotriangulation procedure.

For the 1/200 000 scale project, the Bureau specified a horizontal-control point in not more than every tenth model (40 miles) and a pair of vertical-control points in not more than every eighth model (32 miles). The standard residual errors after adjustment were less than 50 feet, or 1/600 of the flying height.

On more recent work on photographs taken from 15 000 feet with a six-inch camera, with scattered control in shorter strips (10 to 20 models), the standard residual horizontal error was less than five feet, or 1/3 000 of the flight altitude without using pre-marked control. The very best results that one should have reason to expect ideally would be about two feet.

Experience is not yet sufficient to predict the standard error to be expected under any given set of conditions. In any event, the photogrammetric control requirements for Bureau mapping are considerably more lenient than formerly when a field elevation was required in the corner of each model for leveling and two horizontal points were always considered as necessary in the first model in order to adjust its scale.