NOTE ON VARIATIONS IN TIDAL DIFFERENCES

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INTRODUCTION

The practice of publishing tidal differences for « secondary » ports, based on « standard ports », is an old but invaluable method of concisely representing tidal data. Differences have commonly been taken at Full and Change of the moon and also at Quadrature for time differences, and at springs and neaps for height differences, and it has been generally assumed that for intermediate and extreme conditions interpolation and extrapolation respectively are satisfactory. Such differences are generally acknowledged to be only approximate, and this investigation examines the nature of the approximation and reveals the fallacy of assuming that interpolation and extrapolation are satisfactory tools.

Two approaches are used, one depending upon the mathematical expressions for the combined constituents M_2 and S_2, and the other the numerical method of combining these constituents given in Admiralty Tide Tables Part III. The mathematical method shows that time differences are not simply related to H.W.F. & C. or to springs and neaps, and the same holds for height differences. Both methods show that both sets of differences are uniquely determined in terms of high water time but not in terms of high water height. In fact, if the differences are plotted in terms of height then approximately oval curves are obtained, and for any given height of tide the differences depend upon whether the tide is increasing from neaps to springs or decreasing from springs to neaps.

The investigation is then extended to include a third harmonic constituent (N_2). The mathematical method then becomes extremely complex, and only the second method is used. The results show that there is no simple law of variation as regards the use of either high water time or height as variable. The general conclusion is that the best approximation for both time and height differences is obtained by using high water time as a basis, though the author is well aware that such a change in standard procedure is unlikely to be readily adopted.

The difference variations caused by these constituents can nevertheless be uniquely represented without great difficulty or labour by using the phase differences between two pairs of the corresponding equilibrium tide constituents as co-ordinates, and plotting the time and height differences as contours. An example is given, and it is considered that this method might well be adopted, in the absence of a tide predicting machine, if considerations of accuracy require the inclusion of N_2 in variable differences.

The difficulties arising from shallow water conditions or from variations between standard and secondary ports in the diurnal tide have not been considered in this paper. Their existence serves to emphazise the need for caution in the use of differences.

INVESTIGATION FOR TWO CONSTITUENTS

Let time be measured from syzygy.

Then

$$M_2 = M \cos(mt - M_2^\circ)$$
$$S_2 = S \cos(st - S_2^\circ) = S \cos(mt - M_2^\circ + \Phi)$$

where

$$\Phi = (s - m)\, t - (S_2^\circ - M_2^\circ)$$

Then the compound tide is given by

$$\begin{aligned} Z_2 &= (M + S \cos \Phi) \cos(mt - M_2^\circ) - S \sin \Phi \sin(mt - M_2^\circ) \\ &= Z \cos(mt - M_2^\circ - \eta) \end{aligned} \tag{1}$$

where

$$\tan \eta = - \frac{S \sin \Phi}{M + S \cos \Phi} \tag{2}$$

$$\frac{Z}{M} = \left(1 + 2 \frac{S}{M} \cos \Phi + \frac{S^2}{M^2}\right)^{\frac{1}{2}} \tag{3}$$

In (1) Z is the apparent amplitude of tide, and $\frac{M_2^\circ + \eta}{m}$ is the time of high water. η is the phase difference between M_2 and Z_2.

It can be shown that

$$\eta = - r \sin \Phi + \frac{1}{2} r^2 \sin 2\Phi - \frac{1}{3} r^3 \sin 3\Phi + \ldots..$$

$$\frac{Z}{M} = 1 + r \cos \Phi + \frac{1}{2} r^2 \sin^2 \Phi - \frac{1}{2} r^3 \sin^2 \Phi \cos \Phi + \ldots..$$

where $r = \frac{S}{M}$.

We shall use the symbol δ to denote differences between the standard and secondary ports, and by the ordinary process of taking differentials we have

$$\begin{aligned} \delta\eta = (&- \sin \Phi + r \sin 2\Phi - r^2 \sin 3\Phi + \ldots)\, \delta r \\ &+ (- \cos \Phi + r \cos 2\Phi - r^2 \cos 3\Phi + \ldots)\, r\delta\Phi \end{aligned} \tag{4}$$

$$\delta\left(\frac{Z}{M}\right) = (\cos \Phi + r \sin^2 \Phi + \ldots)\, \delta r + \left(- \sin \Phi + \frac{1}{2} r \sin 2\Phi + \ldots\right) r\delta\Phi \tag{5}$$

$$\text{and } \delta Z = M \cdot \delta\left(\frac{Z}{M}\right) + \frac{Z}{M} \cdot \delta M \tag{6}$$

These results at once show that time and height differences are not simply related to H.W.F. & C. (which is proportional to Φ).

At spring tides $\Phi = 0$ at the standard port, and (4), (5) and (6) give

$$\left.\begin{aligned} &\delta\eta = (- r + r^2 - r^3 + \ldots)\, \delta\Phi = - \frac{r}{1 + r} \cdot \delta\Phi \\ &\delta\left(\frac{Z}{M}\right) \doteqdot \delta r \quad , \quad \frac{Z}{M} \doteqdot 1 + r \\ &\delta Z \doteqdot M \cdot \delta r + (1 + r)\, \delta M \end{aligned}\right\} \tag{7}$$

Similarly, *at neap tides* ($\Phi = 180°$) we have

$$\left.\begin{aligned} \delta\eta &= (r + r^2 + r^3 + ...)\,\delta\Phi = \frac{r}{1-r}\,\delta\Phi \\ \delta\left(\frac{Z}{M}\right) &\doteq -\delta r \quad , \quad \frac{Z}{M} \doteq 1 - r \\ \delta Z &\doteq -M\cdot\delta r + (1-r)\,\delta M \end{aligned}\right\} \qquad (8)$$

These show clearly the changes in time and height differences for springs and neaps.

At mean tide between springs and neaps : $\Phi = 90°$

$$\left.\begin{aligned} \delta\eta &= (-1 + r^2 - ....)\,\delta r \;- r^2\,\delta\Phi \\ \delta\left(\frac{Z}{M}\right) &\doteq r\cdot\delta r - r\cdot\delta\Phi \qquad \frac{Z}{M} \doteq 1 + \frac{1}{2}r^2 \\ \delta(Z) &\doteq M(r\delta r - r\delta\Phi) + \left(1 + \frac{1}{2}r^2\right)\delta M \end{aligned}\right\} \qquad (9)$$

At mean tide between neaps and springs : $\Phi = 270°$

$$\left.\begin{aligned} \delta\eta &= (1 - r^2 + ...)\,\delta r - r^2\,\delta\Phi \\ \delta\left(\frac{Z}{M}\right) &\doteq r\cdot\delta r + r\cdot\delta\Phi \qquad \frac{Z}{M} \doteq 1 + \frac{1}{2}r^2 \\ \delta Z &\doteq M(r\delta r + r\delta\Phi) + \left(1 + \frac{1}{2}r^2\right)\delta M \end{aligned}\right\} \qquad (10)$$

If the time and height differences were functions of the height (Z) of the compound tide at the standard port, we should expect results (9) and (10) to agree. They do not, the time differences varying by $\pm (1 - r^2)\,\delta r$ and the height differences by $\pm\, Mr\,\delta\Phi$ ($= \pm\, S\cdot\delta\Phi$). Hence the departures from linearity with Z depend upon δr for times and $\delta\Phi$ for heights, and upon r for both. Both, however, are independent of time, and we may therefore conclude that *there is no justification for the use of heights as a reference for differences, but that differences should be expressed in terms of time.*

Tables I(a) and I(b) afford some idea of the errors which can be incurred for average tides by assuming a linear law of variation between springs and neaps.

TABLE I

(a) Maximum time errors $\pm (1 - r^2)\,\delta r$ in minutes of time.				*(b)* Maximum height errors $\pm\, S\cdot\delta\Phi$ in feet.				
$\frac{\delta r}{r}$	5 %	10 %	15 %	$\frac{\delta\Phi}{\Phi}$	5 %	10 %	15 %	20 %
r				S				
0.2	1	2	3	1 ft	0.1	0.2	0.3	0.3
0.4	2	4	6	3 ft	0.3	0.5	0.8	1.0
0.6	2	4	7	5 ft	0.4	0.9	1.3	1.7

These and later points which will arise may be advantageously illustrated by numerical examples. We take two ports « A » (the secondary port) and « B » (the standard port), and assume values of the harmonic constituents for M_2 and S_2 as in Table II. The values approximate to those at Portsmouth (A) and Devonport (B).

By using Table 3 of Admiralty Tide Tables Part III, we may combine M_2 and S_2 into a compound constituent at intervals of 30° in Φ. This is equivalent to intervals of 1.2304 day, and we can cover the complete spring/neap cycle with twelve applications of the table for each port. To reduce confusion, it should be pointed out that in Admiralty Tide Tables Part III the compound tide is referred to S_2 as a standard, whereas in the foregoing exposition M_2 has been the reference constituent. In consequence, we have the following alterations in notation and hence in procedure :

Admiralty Tide Tables Part III		*This paper*	
d	$\equiv$	Φ	(11)
$Z_2 = F \cos (st - S_2^\circ - e)$	$\equiv$	$Z_2 = Z \cos (mt - M_2^\circ - \eta)$	
$D = M/S$		$r = S/M$	
$\tan e = \dfrac{D \sin d}{1 + D \cos d}$		$\tan \eta = - \dfrac{r \sin \Phi}{1 + r \cos \Phi}$	
$E = F/S$		Z/M	

It should be added that the original calculations for Table 3 were used for the numerical examples, so affording an extra decimal place in the calculations.

Table II illustrates the calculations for Ports A and B. We commence the calculations with $t = 0$ at syzygy, i.e. $d = M_2^\circ - S_2^\circ$. δe and δF represent the time and height differences for Port A on Port B. These are plotted in figures 1(a) - 1(d) both as functions of high water time at B (which differs from e by only a constant amount) and amplitude of tide at B. It is at once apparent that there is a simple law of variation in terms of e, but not in terms of F.

These simple variations approximate to sine curves, and an important feature of each is that its maximum and minimum do not coincide with either spring or neap tides. Indeed, it may be shown mathematically that in general neither $\delta\eta$ nor δZ have their maxima or minima at springs or neaps.

It should further be noted that the practice of giving the ratio of secondary port ranges to standard port ranges suffers from the defect that has been remarked with regard to the use of F as a reference. This may be seen from an examination of the values of F at each port in Table II.

Reference to Tables I(a) and I(b) and figures I(c) and I(d) shows agreement between theory and the numerical indications in so far as the possible errors in differences for average tides are concerned.

EXTENSION OF THE INVESTIGATION TO INCLUDE OTHER HARMONIC CONSTITUENTS

We have seen that even if we postulate only the simplest possible tidal combination, that of M_2 and S_2, the commonly accepted method of referring

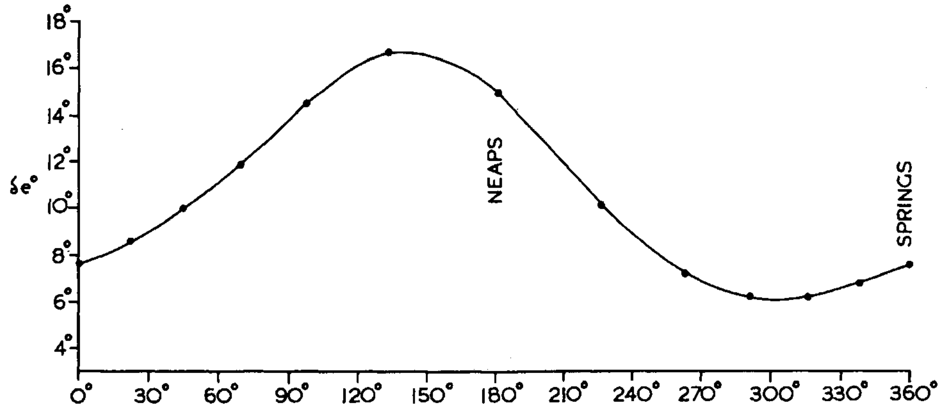

FIGURE 1(a)

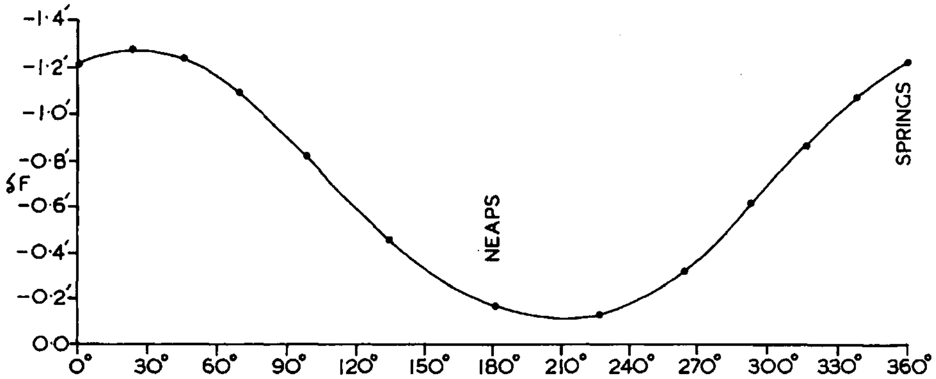

FIGURE 1(b)

height differences to tidal height is unreliable if a linear interpolation between springs and neaps is employed. In practice, of course, it is seldom possible to ignore N_2 or K_2, and the question arises as to how a third constituent affects the typical diagrams of figures 1(*a*) - (*d*).

The extension of the theoretical analysis to these constituents becomes extremely complicated, and it is much simpler to consider three constituents by an extension of the numerical method of combination. We shall use M_2, S_2 and N_2. It has been stated (11) that if we have M_2 and S_2 represented by

$$M_2 = M \cos (mt - M_2^\circ)$$
$$S_2 = S \cos (st - S_2^\circ)$$

then Table 3 of the Admiralty Tide Tables Part III gives the compound tide as

$$Z_2 = F \cos (st - S_2^\circ - e)$$

where

$$\frac{F}{S} = \{1 + 2D \cos d + D^2\}^{\frac{1}{2}} \quad , \quad \tan e = \frac{D \sin d}{1 + D \cos d}$$

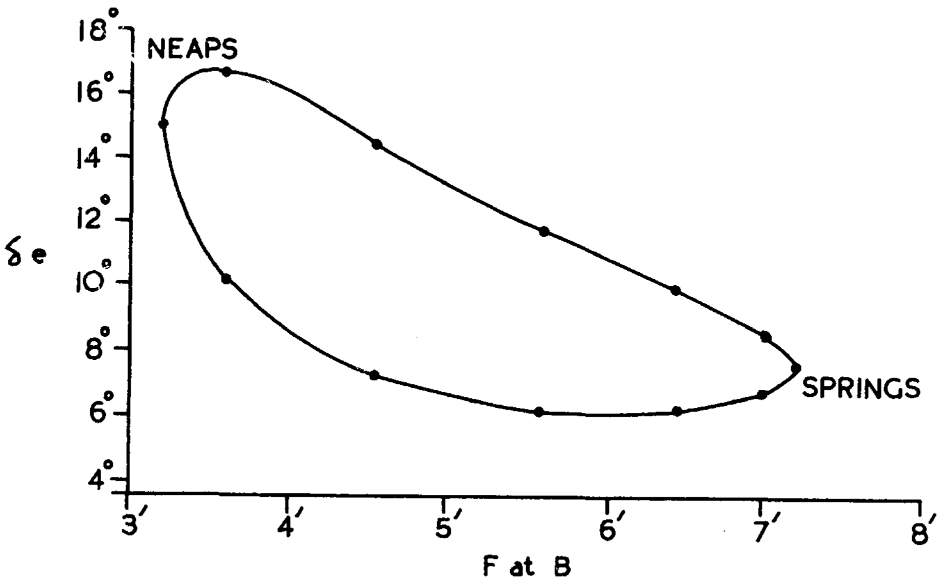

FIGURE 1(c)

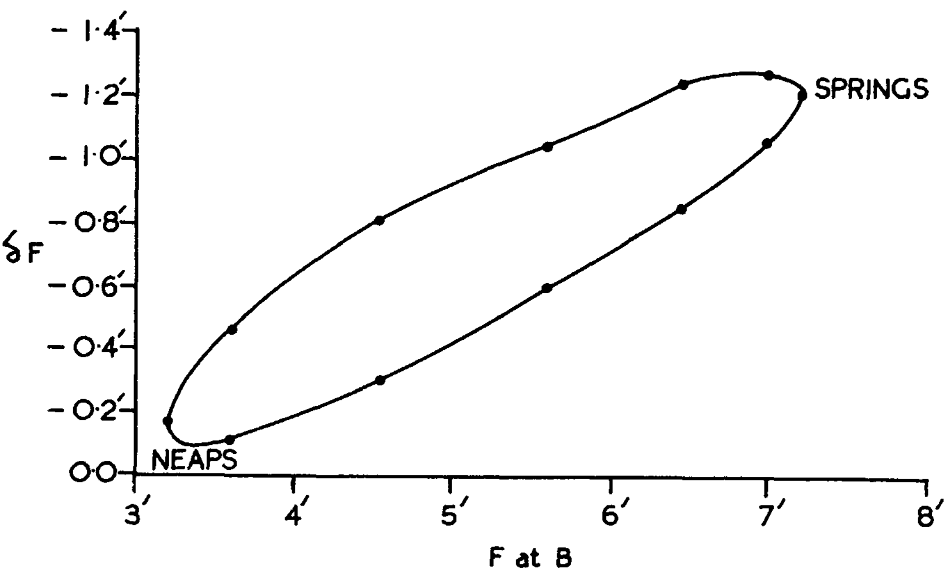

FIGURE 1(d)

Now with

$$N_2 = N \cos (nt - N_2^\circ + \alpha) \qquad (12)$$

where the phase of N_2 at syzygy is $\alpha - N_2$, put

$$d' = (s - n)\,t + (N_2^\circ - S_2^\circ) - e - \alpha \qquad (13)$$

and the compound tide $(M_2 + S_2 + N_2)$ is given by

$$F \cos (st - S_2^\circ - e) + N \cos (st - S_2^\circ - e - d')$$
$$= F' \cos (st - S_2^\circ - \overline{e + e'}) \qquad (14)$$

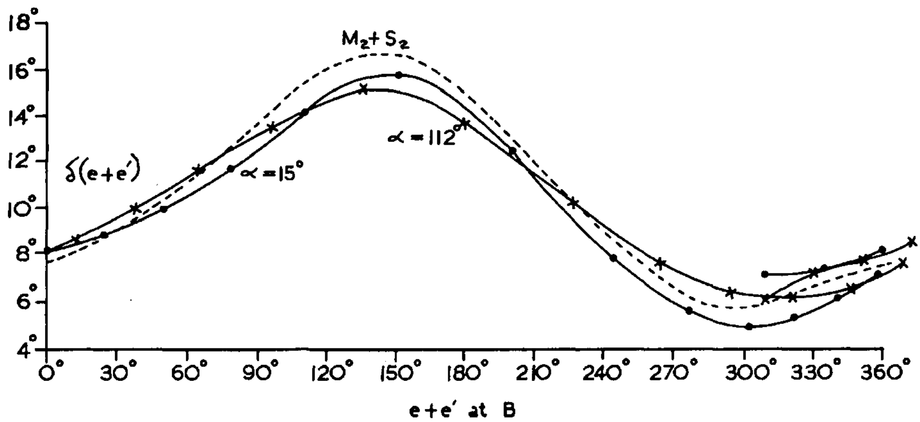

FIGURE 2(a)

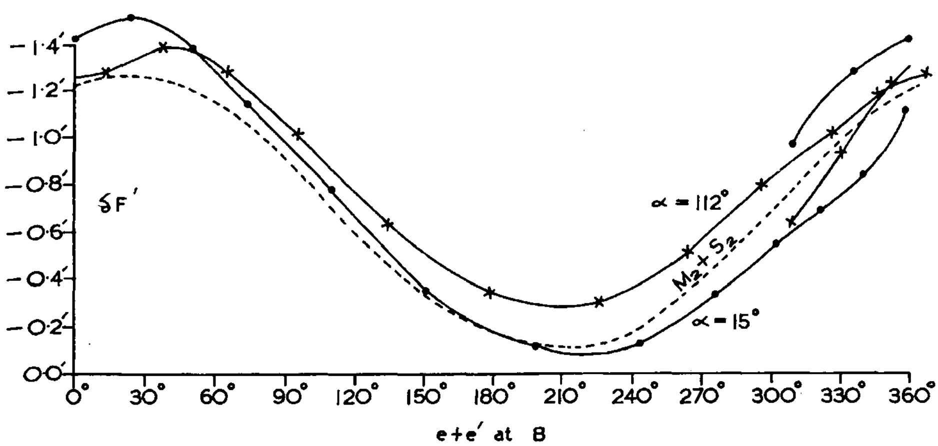

FIGURE 2(b)

where

$$\left.\begin{aligned} \frac{F'}{F} &= \{1 + 2D' \cos d' + D'^2\}^{\frac{1}{2}} = E' \\ \tan e' &= \frac{D' \sin d'}{1 + D' \cos d'} \end{aligned}\right\} \qquad (15)$$

$$D' = \frac{N}{F} \qquad (16)$$

From the above, it is apparent that by calculating D' and d' and using Table 3 we may obtain $\delta(e + e')$, which represents the difference between ports A and B of the phase shift of the compound tide on S_2, and $\delta F'$, the height difference of the compound tides at A and B. The computations are given in Table III. Commencing with the first line of the $M_2 + S_2$ computation, at $t = 0$, increments of $(s - n)t$ are computed, and hence $d' + \alpha$ for each port is obtained. Two examples are given, one for $\alpha = 15°$ corresponding to equinoxial spring tides (M + S + N) at port B ($d' = 0°$ when $d = 0°$), and one for $\alpha = 112°$, i.e. the combination (M — S + N) at port B

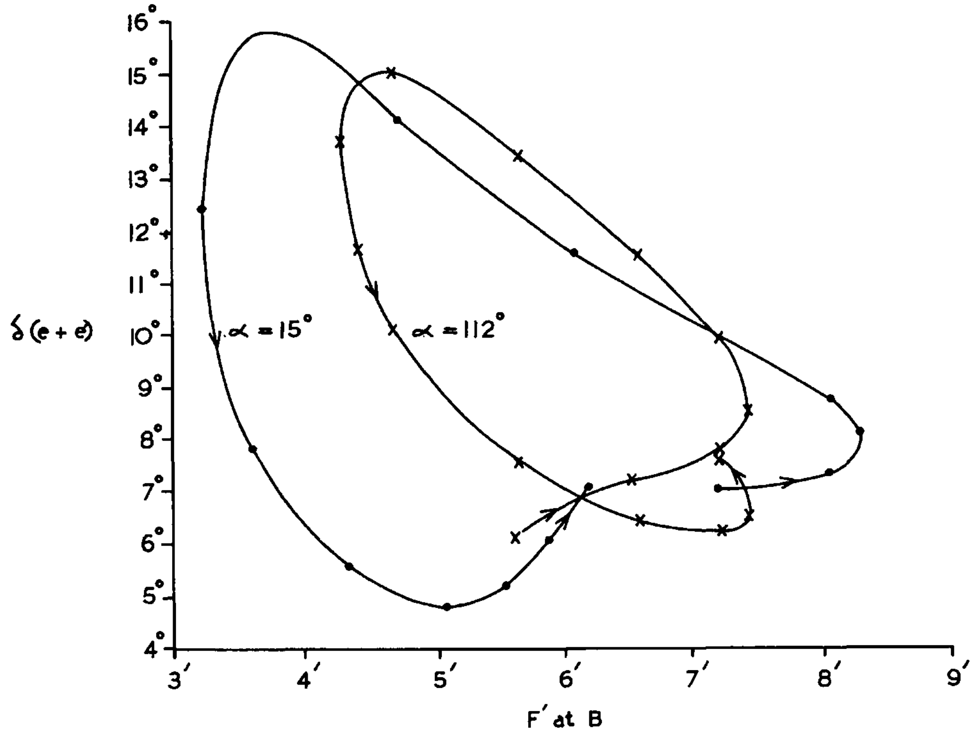

FIGURE 2(c)

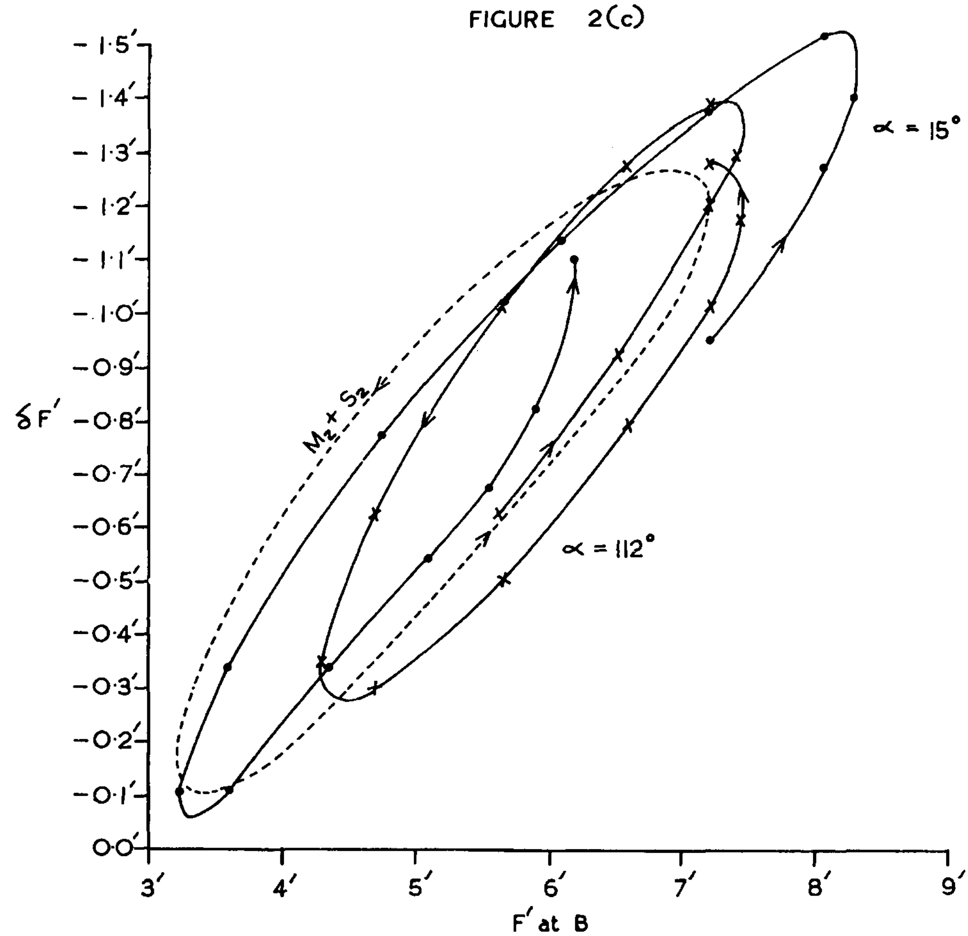

FIGURE 2(d)

($d' = 0°$ when $d = 180°$). The calculations of Table II covered a complete cycle of $M_2 + S_2$; those of Table III have been slightly extended beyond this cycle to give a better idea of the results for $M \pm S + N$, which have been plotted in figures 2(*a*) - 2(*d*). For comparison purposes figures 1(*a*), 1(*b*) and 1(*d*) have been superposed on figures 2(*a*), 2(*b*) and 2(*d*).

These calculations and diagrams make it quite clear that the inclusion of another constituent makes the law of variation of differences, with respect to either time or height of tide, extremely complicated. Referring, for example, to figure 2(*d*), we see that for any abscissa other than the two extremes represented by $M + S + N$ (= 8.3 ft) and $M - S - N$ (= 2.1 ft), there can never be less than two values of $\delta F'$, and in general there will be as many values as there are ways of phasing M_2, S_2 and N_2.

DIFFERENCES AS FUNCTIONS OF THE EQUILIBRIUM TIDE

This paper would not be complete without an indication as to how the effects of all possible combinations of M_2, S_2 and N_2 upon time and height differences can be uniquely represented, if so desired, in terms of the phases of the corresponding equilibrium tide constituents.

It is obvious that the compound tide $M_2 + S_2 + N_2$ at any place (and hence the time and height differences between two places), is a function of the phases of the individual constituents. If we plot time (or height) differences between A and B in the form of contours, with the differences in phase of S_2 and M_2 at the standard port (B) as abscissae and the differences in phase of S_2 and N_2 at B as ordinates, we then have a unique representation of all possible variations of these differences. We do not need to know the exact phases at any given time of any particular constituent; all that is required is the value of V, the phase of the corresponding equilibrium tide constituent for a given time.

In practice, given a series of time and height differences at high and/or low water, tabulations of (V of S_2) — (V of M_2) and (V of S_2) — (V of N_2) are required. These are readily available for zero hour of any day from Schureman's « Manual of Harmonic Analysis and Prediction of Tides » (U.S. Coast and Geodetic Survey Special Publication No. 98), and the corresponding increments appropriate to the time of high or low water at the standard port have to be added. It is considerably quicker, however, to use Tables 4 and 5 in Volume 2 of the Deutsches Hydrographisches Institut's « Gezeitentafeln », published annually, in which may be found (*a*) the values of V for the major harmonic tidal constituents for zero hour of each day of the year, and (*b*) the increment in each, at 10 minute intervals, up to 24 hours.

We may illustrate the type of result from the examples already given. At syzygy, V of M_2 = V of $S_2 = 0$. With this initial condition, values of (V of S_2) — (V of M_2) are equal to the values of $(s - m)t$. For any given value of α, values of (V of S_2) — (V of N_2) were computed from $(s - n)t - \alpha$.

Figures 3(*a*) and 3(*b*) illustrate the contours of $\delta(e + e')$ and $\delta F'$ respectively, with $(s - m)t$ and $(s - n)t - \alpha$ as abscissae and ordinates respectively. The decimal point of each entry indicates its position on the graph, and data for $\alpha = 195°$ and $292°$ have been included with those for $\alpha = 15°$ and $112°$ computed in the previous examples. Contour lines have been drawn at intervals of 2° in $\delta(e + e')$ and 0.2 ft in $\delta F'$, without great difficulty.

It is conceivable that in the absence of a harmonic tide predicting machine this type of diagram may be seriously considered for predicting tides by differences, to a greater accuracy than the normally accepted methods can permit. Reliable and sufficient observations are indispensable for any wholly satisfactory method of prediction, be it by differences or predicting machine, but, having stated this, let us examine how such a scheme could be applied in practice.

From the relative speeds of M_2, S_2 and N_2, we may deduce that the coverage obtained in figures 3(*a*) and 3(*b*) is equivalent to two sets of observations, each covering one month, and separated by about 3 months. These data would enable contours to be drawn, and the amount of calculation required in connexion with the phases of the equilibrium tide would be small. It is doubtful, however, whether contours based on two months' observations would be reliable enough to justify the labour involved in their production, for a sufficiently long span of records must be used to ensure that the effects of constituents other than M_2, S_2, and N_2 have been averaged out.

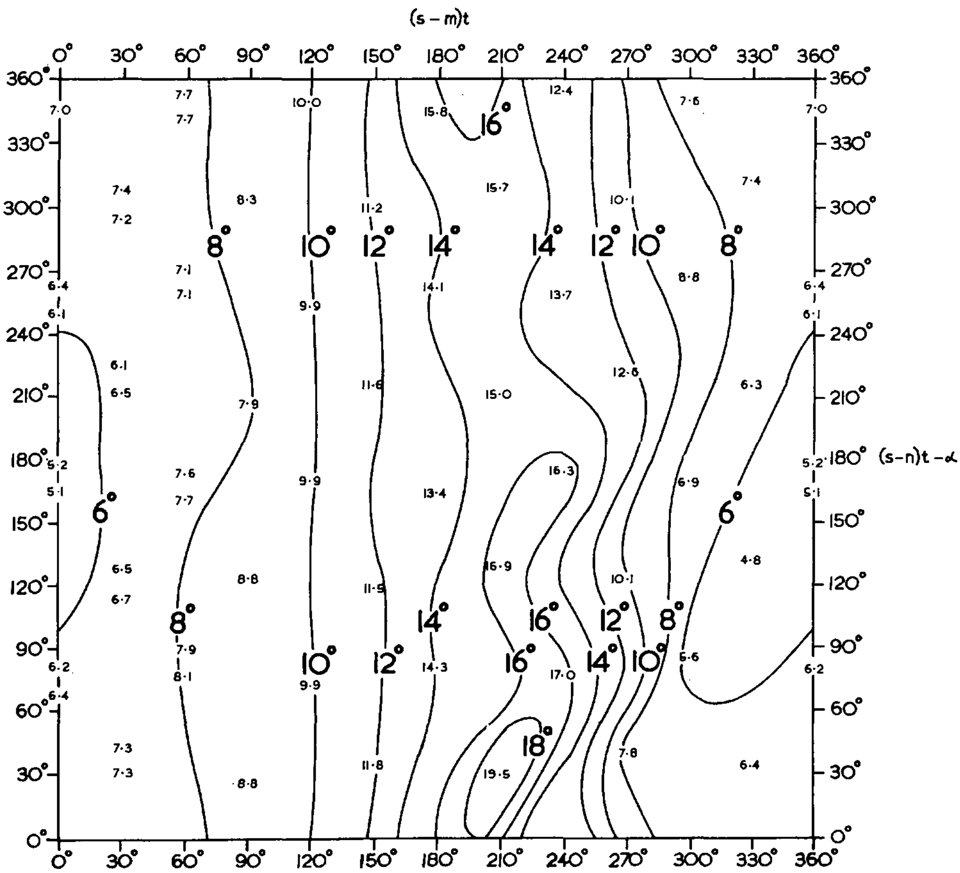

FIGURE 3(a)

CONTOURS of $\delta(e+e')$

Having obtained such contours, predictions could be readily prepared by using a transparent mask with dots marked at intervals of 12°.62 in abscissa and 19°.38 in ordinate. These values correspond to intervals of 12 hour 25 minutes, the period of M_2, which we assume to predominate. If we also assume that the period of the actual tide is the same, the differences for successive tides may be read off the contours against successive dots; the error involved in this last assumption is negligible. As the transparent mask can easily be dotted to cover one month's tides, and as only one calculation of the equilibrium phases is required each month (to fix the position relative to the contour sheet of the first dot on the mask), it will be appreciated that the use of such a method could be speedy and labour saving.

In the absence of shallow water effects only one contour sheet would be required for high and low water time differences, and another for high and low water height differences. Otherwise a separate sheet would be required for each.

The author wishes to acknowledge his indebtedness to Dr. A.T. Doodson for suggesting this paper and for making available results of earlier unpublished work on the combination of two harmonic constituents.

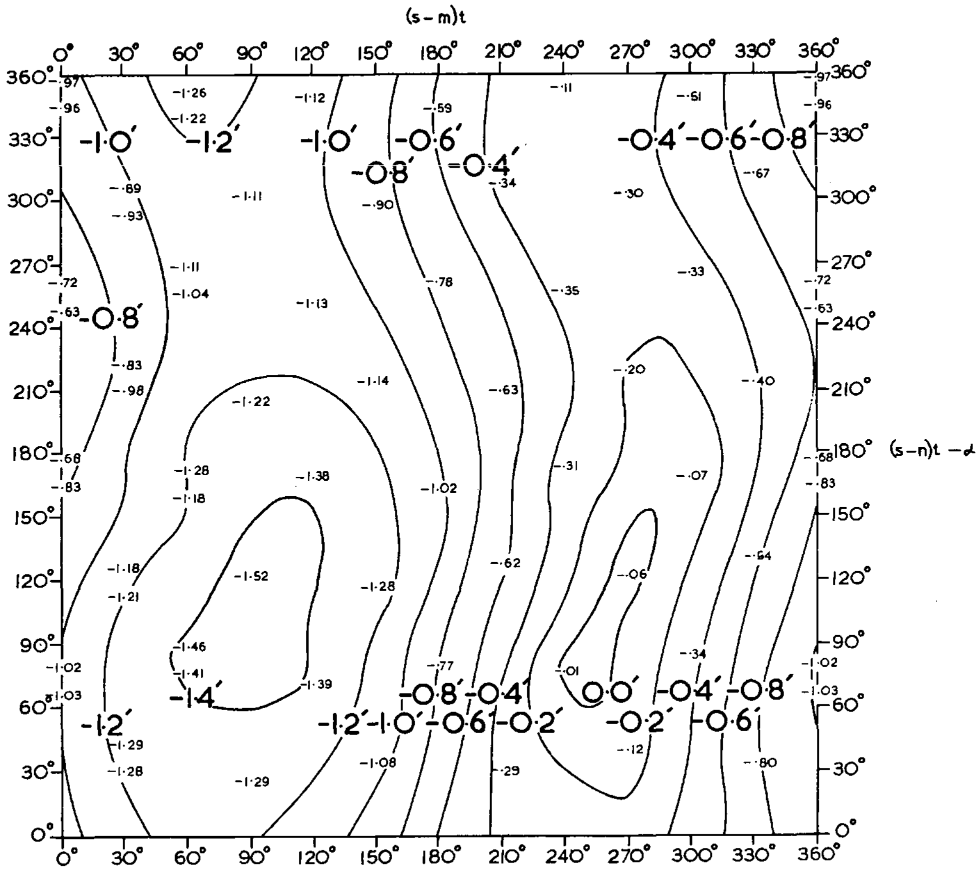

FIGURE 3(b)

CONTOURS of δF′

TABLE II

COMBINATION OF M_2 AND S_2

	M_2		S_2		N_2		D (= M/S)
PORT « A »	4.5′	320°	1.5′	10°	0.9′	303°	3.0
PORT « B »	5.2′	150°	2.0′	210°	1.1′	133°	2.6

PORT « A »				PORT « B »				« A » - « B »	
d	*e*	E	F	*d*	*e*	E	F	δ*e*	δF
—50°	—38.1	3.72	5.58	—60°	—44.3	3.22	6.44	6.2	—.86
—20	—15.0	3.96	5.94	—30	—21.8	3.50	7.00	6.8	—1.06
10	7.6	3.99	5.98	0	0.0	3.60	7.20	7.6	—1.22
40	30.3	3.82	5.73	30	21.8	3.50	7.00	8.5	—1.27
70	54.2	3.47	5.20	60	44.3	3.22	6.44	9.9	—1.24
100	80.8	2.99	4.49	90	69.0	2.79	5.58	11.8	—1.09
130	112.0	2.48	3.72	120	97.6	2.27	4.54	14.4	—.82
160	150.6	2.09	3.14	150	134.0	1.80	3.60	16.6	—.46
190	—165.1	2.02	3.03	180	—180.0	1.60	3.20	14.9	—.17
220	—123.9	2.32	3.48	210	—134.0	1.80	3.60	10.1	—.12
250	—90.4	2.82	4.23	240	—97.6	2.27	4.54	7.2	—.31
280	—62.8	3.32	4.98	270	—69.0	2.79	5.58	6.2	—.60

TABLE III

COMBINATION OF M_2 AND S_2 WITH N_2

Intervals in t are 1.2304 day. Corresponding increment in d' is $(s-n)t = 46°.07$

	PORT « A »								PORT « B »							« A » - « B »	
	$\alpha = 15°$																
	$(s-n)t$	$d'+\alpha$	D′	d'	e'	E′	F′	$e+e'$	$d'+\alpha$	D′	d'	e'	E′	F′	$e+e'$	$\delta(e+e')$	δ F′
$t=0$	0	331	.16	316	−5.6	1.12	6.24	−43.7	327	.17	312	−6.4	1.12	7.20	−50.7	7.0	−.96
	46.1	354	.15	339	−2.7	1.14	6.77	−17.7	351	.16	336	−3.2	1.15	8.05	−25.0	7.3	−1.28
	92.1	18	.15	3	0.5	1.15	6.87	8.1	15	.15	0	0.0	1.15	8.28	0.0	8.1	−1.41
	138.2	41	.16	26	3.5	1.14	6.53	33.8	39	.16	24	3.2	1.15	8.05	25.0	8.8	−1.52
	184.2	63	.17	48	6.4	1.12	5.82	60.6	63	.17	48	6.4	1.12	7.20	50.7	9.9	−1.38
	230.3	82	.20	67	9.7	1.10	4.94	90.5	84	.20	69	9.9	1.09	6.08	78.9	11.6	−1.14
	276.4	97	.24	82	12.9	1.06	3.94	124.9	102	.24	87	13.2	1.04	4.72	110.8	14.1	−.78
	322.5	105	.29	90	16.2	1.04	3.26	166.8	112	.30	97	17.1	1.00	3.60	151.1	15.7	−.34
	8.6	107	.30	92	16.8	1.03	3.12	−148.3	112	.34	97	19.3	1.01	3.23	−160.7	12.4	−.11
	54.6	112	.26	97	14.8	1.00	3.48	−109.1	112	.30	97	17.1	1.00	3.60	−116.9	7.8	−.12
	100.7	124	.21	109	12.0	.95	4.01	−78.4	121	.24	106	13.6	.96	4.35	−84.0	5.6	−.34
	146.8	143	.18	128	9.1	.91	4.53	−53.7	139	.20	124	10.5	.91	5.07	−58.5	4.8	−.54
	192.8	164	.16	149	5.5	.87	4.85	−32.6	160	.17	145	6.5	.86	5.53	−37.8	5.2	−.68
	238.9	187	.15	172	1.4	.85	5.05	−13.6	184	.16	169	2.1	.84	5.88	−19.7	6.1	−.83
	285.0	210	.15	195	−2.7	.85	5.08	4.9	208	.15	193	−2.2	.86	6.19	−2.2	7.1	−1.11
	$\alpha = 112°$																
			D′	d'	e'	E′	F′	$e+e'$		D′	d'	e'	E′	F′	$e+e'$	$\delta(e+e')$	δ F′
$t=0$			.16	219	−6.6	.89	4.97	−44.7		.17	215	−6.5	.87	5.60	−50.8	6.1	−.63
			.15	242	−8.1	.94	5.58	−23.1		.16	239	−8.5	.93	6.51	−30.3	7.2	−.93
			.15	266	−8.5	1.00	5.98	−0.9		.15	263	−8.6	1.00	7.20	−8.6	7.7	−1.22
			.16	289	−8.2	1.07	6.13	22.1		.16	287	−8.2	1.06	7.42	13.6	8.5	−1.29
			.17	311	−6.5	1.12	5.82	47.7		.17	311	−6.5	1.12	7.21	37.8	9.9	−1.39
			.20	330	−4.9	1.18	5.30	75.9		.20	332	−4.6	1.18	6.58	64.4	11.5	−1.28
			.24	345	−2.9	1.24	4.61	109.1		.24	350	−1.9	1.24	5.63	95.7	13.4	−1.02
			.29	353	−1.6	1.29	4.05	149.0		.30	0	0.0	1.30	4.68	134.0	15.0	−.63
			.30	355	−1.2	1.30	3.94	−166.3		.34	0	0.0	1.34	4.29	−180.0	13.7	−.35
			.26	0	0.0	1.26	4.38	−123.9		.30	0	0.0	1.30	4.68	−134.0	10.1	−.30
			.21	12	2.1	1.21	5.12	−88.3		.24	9	1.7	1.24	5.63	−95.9	7.6	−.51
			.18	31	4.6	1.16	5.78	−58.2		.20	27	4.4	1.18	6.58	−64.6	6.4	−.80
			.16	52	6.4	1.11	6.19	−31.7		.17	48	6.4	1.12	7.21	−37.9	6.2	−1.02
			.15	75	7.9	1.05	6.24	−7.1		.16	72	8.2	1.06	7.42	−13.6	6.5	−1.18
			.15	98	8.6	.99	5.92	16.2		.15	96	8.6	1.00	7.20	8.6	7.6	−1.28