

ACCURACY OF SPEED TRIALS ON THE MEASURED MILE

by J. Th. VERSTELLE

Hydrographic Office, Royal Netherlands Navy, April 1958

Reproduced from *Nieuwsbrief Hydrografie No. 28*

SUMMARY

Speed over the ground S is determined from observations of the time interval between the perpendicular crossing of two parallel lines of beacons, usually exactly one nautical mile apart.

The combined effect of observational errors is investigated. It is shown that — contrary to a widely held opinion — these effects are not negligible.

The required speed through the water V is determined by the usual method of taking the *mean of means* of at least 4 determinations of S on opposite courses. The reliability of this method of elimination of drift is not a subject of discussion in this News Letter; it is however shown that the standard error in V is considerably smaller than in each of the 4 determinations of S .

Trials in daytime are to be preferred, because night trials are likely to be considerably less accurate.

Other methods of speed trials are discussed in News Letter No. 27, 1958 (*).

1. Notations

S = hourly speed over the ground
 V = hourly speed through the water } same unit as s
 Δt = time interval in seconds between perpendicular crossing of two parallel lines of beacons
 s = length of measured mile = perpendicular distance between the extensions of two lines of beacons; when S and V are expressed in (nautical) miles, $s = 1$.

Note : In this News Letter the unit used in the computations is the metre. The final value of V is always converted into nautical miles, the British mile being 1 853 m and the continental mile equalling 1 852 m.

(*) This paper will be inserted in the next issue of the Review.

2. Computation of S and V

$$S = \frac{3600}{\Delta t} \times s \quad (1)$$

V is computed by the method of determining the so-called *mean of means* of S (at least 4 runs, alternatively in opposite directions).

Example :

S ₁	38003	40791	
S ₂	43578	40393	40592
S ₃	37207	40893	40643
S ₄	44578		

40618 m/h = 21.93 n. mi./hour

The formula for the mean of means (4 runs) reads :

$$V = \frac{S_1 + 3S_2 + 3S_3 + S_4}{8} \quad (2)$$

3. Systematic errors

Any systematic error in the perpendicular distance s between the two extensions of the lines of beacons (the *mile* being too short or too long)

has a $\frac{3600}{\Delta t}$ times as large effect on S and on V.

As the user has no possible means of control, in actual practice a measured mile is always assumed to be correct; systematic errors therefore will not be discussed in this News Letter.

The stopwatch used for measuring Δt should be well calibrated and a rate exceeding 0.2 second in, say, 10 minutes cannot be tolerated.

4. Effect of random errors

From differentiation of formula (1) :

$$dS = \frac{3600}{\Delta t} ds - \frac{3600 s}{\Delta t^2} d\Delta t$$

s and Δt are non-correlated quantities, hence :

$$m_S^2 = \left(\frac{3600}{\Delta t} \right)^2 m_s^2 + \left(\frac{3600 s}{\Delta t^2} \right)^2 m_{\Delta t}^2 \quad (3)$$

Differentiating formula (2) :

$$dV = \frac{1}{8} dS_1 + \frac{3}{8} dS_2 + \frac{3}{8} dS_3 + \frac{1}{8} dS_4$$

$$m_V^2 = \frac{1}{64} m_{S_1}^2 + \frac{9}{64} m_{S_2}^2 + \frac{9}{64} m_{S_3}^2 + \frac{1}{64} m_{S_4}^2$$

$$m_V^2 \approx \frac{1}{64} m_S^2 + \frac{1}{7} m_{S_2}^2 + \frac{1}{7} m_{S_3}^2 + \frac{1}{64} m_{S_4}^2 \quad (4)$$

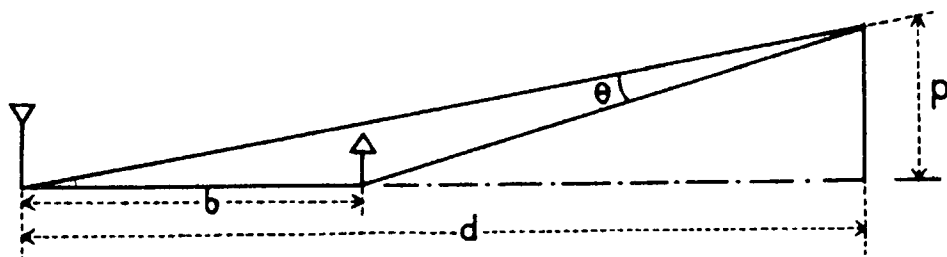
In these formulae, m = standard error.

5. Estimation of magnitude of random observational errors

5.1. The observer makes a certain error in his estimate of the exact moment of being on the extension of the line of beacons, or in other words in the realization of this line at sea.

The error is dependent on the *sensitivity* of the line and a measure for this sensitivity is the smallest observable angle θ (fig. 1). From a paper presented by the author to the 1955 International Conference on Lighthouses and other Aids to Navigation it follows that — *using binoculars* of about 6 times magnification — in day time the smallest observable angle θ is of the order of 10 seconds of arc (*), provided we have :

- (a) beacons of suitable design;
- (b) good visibility;
- (c) an experienced observer.



Horizontal Section

FIGURE 1

In the above-mentioned paper, the following formula is derived for the distance p (fig. 1) :

$$|p| \approx d \operatorname{tang.} \theta \left(\frac{d}{b} - 1 \right) \tag{5}$$

Assuming :

$$b = 400 \text{ m (437 yards)}$$

$$d - b = 2 \text{ n. mi. (Br.)} = 3\,706 \text{ m, hence } d = 4\,106 \text{ m}$$

$$\theta = 10''$$

it follows from (5) that :

$$|p| = 1.9 \text{ m}$$

Assuming :

$$b = 400 \text{ m}$$

$$d - b = 3 \text{ n. mi.} = 5\,559 \text{ m, hence } d = 5\,959 \text{ m}$$

$$\theta = 10''$$

it follows that :

$$|p| = 4.1 \text{ m}$$

Conclusion :

p increases rapidly with increasing distance to the shore.

(*) At night this angle is considerably larger, dependent on the vertical angle between the two beacon lights; 30'' seems to be a fair estimate (using binoculars), which would increase p by a factor 3.

Note :

1. For shorter distance b between beacons, p increases rapidly.
2. Only a very careful observer can detect an opening angle of $10''$.

The realization at sea of each of the two extensions therefore must be expected to be uncertain to an amount p and consequently $p\sqrt{2}$ is a fair estimate of the standard error in the distance s . Consequently, in the two assumptions made above, we find :

$$\begin{array}{l}
 m_s = 1.9 \times 1.4 = 2.66 \text{ m} \\
 m_s^2 = 7 \text{ m}
 \end{array}
 \left. \begin{array}{l}
 2 \text{ miles} \\
 \text{from} \\
 \text{shore}
 \end{array} \right\}
 \begin{array}{l}
 m_s = 4.1 \times 1.4 = 5.74 \text{ m} \\
 m_s^2 = 33 \text{ m}
 \end{array}
 \left. \begin{array}{l}
 3 \text{ miles} \\
 \text{from} \\
 \text{shore}
 \end{array} \right\}$$

5.2. The interval of time Δt is determined by means of a stopwatch. Any observer has a personal error, being in fact a reaction time in pressing the knob of the stopwatch at the moment when the beacons are observed to be in line, which — strange as it may seem — may be either positive or negative. For any particular observer, part of his reaction time is constant and consequently does not affect the *difference* in time Δt . The reaction time, however, is partly also of a randomly variable character, and elaborate trials have shown that this random error for an *experienced* observer is of the order of ± 0.4 second in Δt .

$$\begin{aligned}
 m_{\Delta t} &= 0,4 \text{ s} \\
 m_{\Delta t}^2 &= 16 \times 10^{-2} \text{ s}
 \end{aligned}$$

6. Examples

6.1. Distance to shore 2 n. miles.

$$S = 18 \text{ miles (Br.)} = 33354 \text{ m/h}$$

$$b = 400 \text{ m}; \theta = 10''; \Delta t = 200 \text{ seconds.}$$

Substituting the standard errors of sections 5.1 and 5.2 in formula (3) :

$$m_s^2 = 2268 + 4450 = 6718$$

$$m_s = 82 \text{ m/h} = 0.044 \text{ n.mi./h} = \frac{82}{333.54} = 0.25 \% \text{ of } S.$$

Substituting equal values of m_s for the 4 runs in formula (4) :

$$m_v^2 = 2099$$

$$m_v = 46 \text{ m/h} = 0.025 \text{ n.mi./h} \approx \frac{46}{333.54} \approx 0.14 \% \text{ of } V.$$

6.2. Distance to shore 3 n. mi.

$$S = 18 \text{ miles (Br.)} = 33354 \text{ m/h}$$

$$b = 400 \text{ m}; \theta = 10''; \Delta t = 200 \text{ seconds}$$

$$m_s^2 = 10692 + 4450 = 15142$$

$$m_s = 123 \text{ m/h} = 0.066 \text{ n.mi./h} = \frac{123}{333.54} = 0.37 \% \text{ of } S$$

$$m_v^2 = 4732$$

$$m_v = 69 \text{ m/h} = 0.037 \text{ n.mi./h} \approx \frac{69}{333.54} \approx 0.21 \% \text{ of } V.$$

Note : It should be realized that actual conditions often are less favourable than the assumptions made here, i.e. b may be shorter and θ larger, especially at night.

7. Conclusions

1. Observations must be made with great care and every possible precaution has to be taken to keep errors as small as possible.

2. Accuracy decreases rapidly with increasing distance of trial runs to the shore.

3. Although no general figure can be given as to accuracy to be expected, even for an experienced observer and for fairly *sensitive* lines of beacons, the effect of unavoidable observational errors in speed trials on the measured mile is not negligible.

4. Trials during daytime must be preferred, as night trials are likely to be considerably less accurate; a decrease by a factor 3 may be expected.

8. Consulted literature

1. Paper No. 443, International Conference on Lighthouses and other Aids to Navigation, Scheveningen 1955 : J. Th. VERSTELLE, « Lateral sensitivity of leading lights (ranges) ».

2. Netherlands Hydrographic Office News Letter No. 27, 1958 : J. Th. VERSTELLE, « Ships' acceptance trials ».