STEREOTRIANGULATION WITH PHOTOGRAPHS FROM 10 000 m ALTITUDE

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Scope : Investigation of the possibilities of using very high altitude-photography for increasing density of control net, sextant fix determination and shore line plotting for hydrographic survey work.

Acknowledgment : For the investigation, of which the first steps are described in this interim report no. 1, most valuable help has been received from the Geographical Survey Office, the Air Forces and the Photogrammetric Division of the Institute of Technology in Stockholm.

Pre-test : One 10 000 m altitude stereomodel from the southern part of the Stockholm archipelago was oriented and measured in the Wild A7 Autograph. The 16 control points gave, according to table 1, a radial standard error of 2.1 m.

Main test area : The coast (55 km) between the cities of Gävle and Söderhamn with an archipelago of limited width (average 5-6 km). Isolated islets, shoals etc. rather far out.

Geodetic control net : Within the area there are 43 geodetic triangulation stations. For an earlier photomapping of this area, the Geographical Survey Office several years ago determined terrestrially the coordinates for some details (small trees, rocks, etc.) around each triangulation station and identified these details in aerial photographs from 1945 at scale 1/20 000. As difficulties identifying these details in the high altitude photographs were foreseen, 20 of the stations were signalled in early July with thin white plastic sheets. The positions of the eccentric signals (13 points) were determined. A rather simple method, however, was used, giving a standard error of magnitude 1.0 m.

Aerial photography : The photography of the two strips was performed with a military aircraft and an Eagle camera (type Eagle 9 MK II) from an altitude of 9 700 and 9 300 metres. Due to poor visibility downwards from the pilot seat the planned strip could not be followed better than within 2 kilometres, which made the stereo-orientation more difficult and less adequate. Due to unfavourable weather conditions the photography was delayed about three months until the middle of October. During this delay of the photography all signals but nine were destroyed. The remaining signals were inspected by airplane late in the autumn.

Diapositives : These were made at the Geographical Survey Office with the special compensation plate for elimination of radial distortion and influence of refraction and earth curvature.

Stereo measurements : These were performed by state hydrographer A. Thunberg in the Wild A7 no. 362 Autograph at the Geographical Survey Office. The Ekelünd method for model triangulation with known heights was followed. In the common area for adjacent models natural details as well as five crosses marked in the diapositives (for monocular pointing with the floating mark) were measured. Due to the above-mentioned difference between planned and actual strip position, with a lack of orientation points in the normal positions as a consequence, the relative orientation was difficult to perform for some models. The mean residual parallaxes are given in the table 1. When pointing the control point details and signals with the floating mark, identification was found to be rather difficult and dubious in several cases.

Results : This step of the investigations shows that it is possible to obtain transformation results with a radial standard error of about, or less than, 2 metres when using photographs from about 10 000 m altitude and a precision plotting instrument. The conditions were not favourable in this test but rather representative for the future work of the Hydrographic Office.

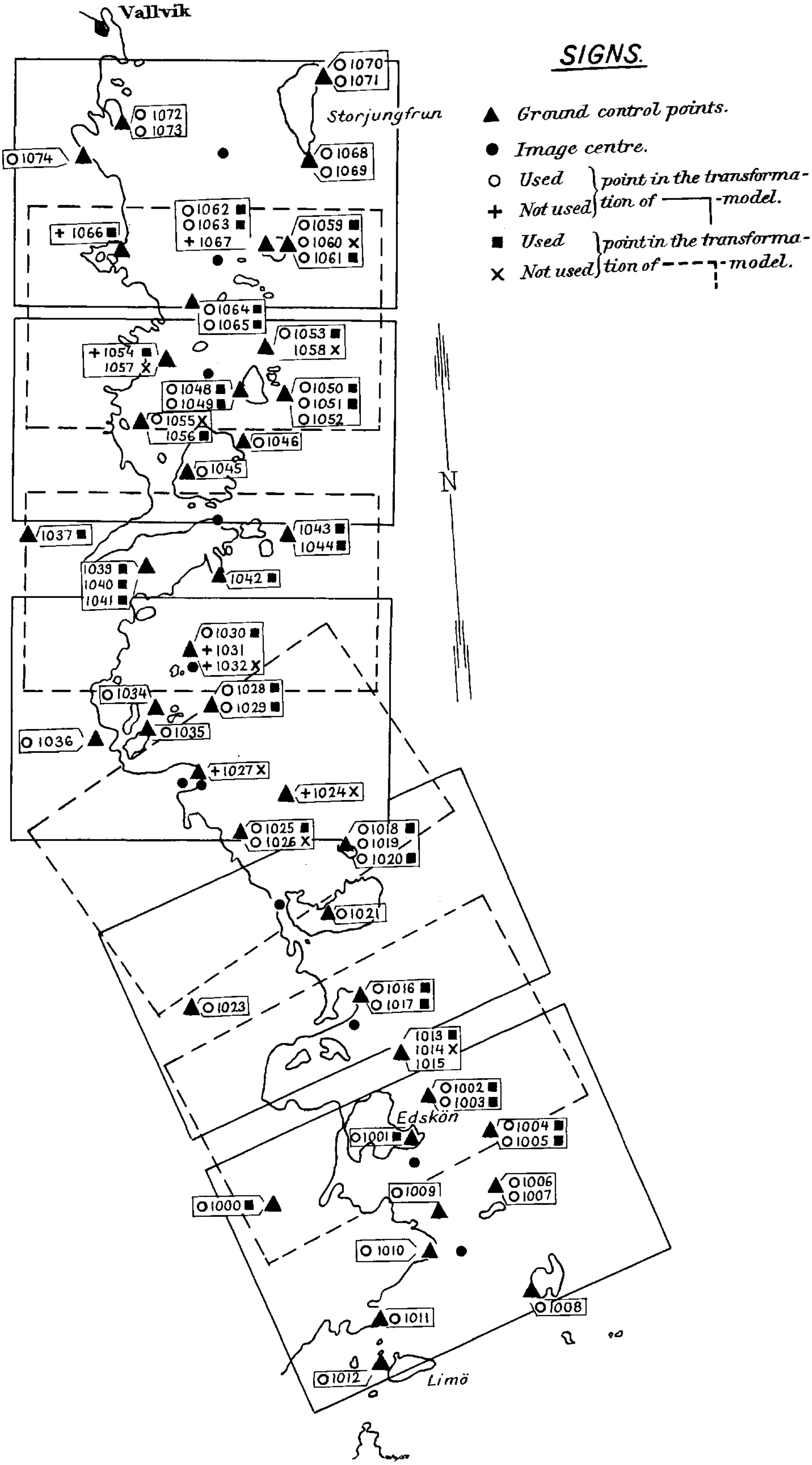
Coordinate transformation : Each model contained several more or less accurately identified control points. With a special program developed by A. Håkansson, MSC Eng., each model was transformed in the BESK electronic computer. The procedure made it possible to exclude immediately dubious points showing great errors and recalculate the transformation data. The finally accepted transformation results (errors) are shown in the tables 2-10.

Discussion of the results : The mean square value of the residual y-parallaxes, as computed from the unreduced measured parallaxes, amounted as an average to 0.0125 mm. The radial standard error of the coordinates to be expected from this value of the y-parallaxes would in this case amount to about 1.3 m. The higher empirical value is probably caused by the lack of identification. It should also be noted that certain residual y-parallaxes in this case are caused by the fact that the original photographs were projected into diapositives through a correcting glass plate, which also corrected for the influence of earth curvature. By this procedure the horizontal parallaxes are corrected for the influence of earth curvature, but the y-parallaxes will simultaneously become unfavourably influenced.

Further work : The next step is to join the various models to each other through transformations with the BESK and to determine the errors when using various points for the absolute orientation of the strip.

Thereafter, and in cooperation with the Institute of Technology, the Board of Roads and Waterways, the Geographical Survey Office, the State Power Board, and Mr. A. Håkansson, the models will be further adjusted according to Hallert's method (*). The computations will be performed with a program for BESK on which Mr. Håkansson is now working. This may be considered an attempt to get the highest possible accuracy from aerial triangulation with very high altitude photographs.

(*) Hallert, B. : The principles of numerical corrections in aerial photogrammetry *Photogrammetric Engineering*, April 1956.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Ground control points.
Image centre.
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point in the transformation of -model.
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TABLE I

Model No.	Model scale	Negative scale	Number of measured vertical parallaxes n	Vertical parallaxes (*) Model scale $\sqrt{\frac{[p_v\,p_v]}{n-1}}$ 1/1000 mm	Vertical parallaxes (*) Model scale $\frac{[\lvert p_v \rvert]}{n}$ 1/1000 mm	Vertical parallaxes (*) Negative scale $\sqrt{\frac{[p_v\,p_v]}{n-1}}$ 1/1000 mm	Vertical parallaxes (*) Negative scale $\frac{[\lvert p_v \rvert]}{n}$ 1/1000 mm	Number of ground control points for transform.	$\sqrt{\frac{[f_x\,f_x]}{n-2}}$ m_x m	$\sqrt{\frac{[f_y\,f_y]}{n-2}}$ m_y m	$\sqrt{\frac{[f_x\,f_x]+[f_y\,f_y]}{n-2}}$ m_r m
XFV 5723 :											
01-03	1/25 900	1/63 900	35	± 33	28	± 13	11	13	± 1,3	± 1,1	± 1,7
03-06	1/24 100	1/63 900	40	± 23	18	± 9	7	9	± 1,5	± 1,2	± 1,9
06-09	1/23 200	1/63 900	37	± 27	22	± 10	8	7	± 1,1	± 1,2	± 1,6
09-12	1/25 900	1/63 900	41	± 17	13	± 7	5	5	± 0,7	± 1,1	± 1,3
XFV 5724 :											
01-03	1/21 600	1/61 450	35	± 17	14	± 6	5	8	± 0,8	± 0,9	± 1,2
03-06	1/23 450	1/61 450	34	± 31	24	± 12	9	8	± 1,0	± 1,0	± 1,4
06-09	1/23 450	1/61 450	32	± 28	21	± 11	8	9	± 1,2	± 1,1	± 1,6
09-11	1/20 350	1/61 450	42	± 25	20	± 8	7	14	± 1,6	± 1,2	± 2,0
11-13	1/20 800	1/61 450	39	± 52	40	± 18	14	12	± 2,2	± 2,0	± 3,0
Pretest model	1/30 060	1/65 000	24	± 21	18	± 10	8	16	± 1,6	± 1,4	± 2,1

(*) Before computing $[p_v\,p_v]$ the measured parallaxes have been reduced with their mean value. Consequently $[p_v] = 0$ after reduction.

Note. — In this and the following tables, read the comas as decimal points.

TABLE 2

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5723 : 01-03	1000	— 1,8	— 2,5	good	
	1001	— 0,7	+ 0,2	good	
	1002	+ 0,8	+ 0,3	accurate	
	1003	+ 2,1	+ 0,8	indistinct	
	1004	— 1,9	— 0,1	good	
	1005	+ 0,4	+ 1,1	indistinct	
	1006	± 0,0	— 0,4	good	
	1007	+ 0,9	— 1,6	rather good	doubtful measurement
	1008	— 1,3	— 1,1	not visible	presumptive position
	1009	— 0,7	+ 0,6	good	
	1010	— 0,6	+ 0,4	accurate	
	1011	+ 0,8	+ 1,2	good	
	1012	+ 1,1	+ 0,4	good	

$m_x = \pm 1{,}3 \quad m_y = \pm 1{,}1$

$m_r = \pm 1{,}7$

TABLE 3

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5723 : 03-06	1000	— 0,2	± 0,0	good	
	1001	+ 1,0	+ 0,5	good	
	1002	— 1,9	+ 1,0	rather good	doubtful measurement
	1003	+ 1,5	+ 1,4	rather good	doubtful measurement
	1004	— 2,2	— 1,6	good	
	1005	+ 1,3	— 0,6	good	
	1013	± 0,0	+ 1,3	uncertain	doubtful measurement
	1016	— 1,0	— 0,9	signal	
	1017	+ 1,5	— 1,0	good	

$m_x = \pm 1{,}5 \quad m_y = \pm 1{,}2$

$m_r = \pm 1{,}9$

	Points excluded from the trans-formation				
	1014	+ 0,7	+ 8,0	not visible	presumptive position

TABLE 4

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5723 : 06-09	1016	— 0,5	— 1,8	signal	
	1017	+ 1,4	— 1,3	good	
	1018	± 0,0	+ 0,4	signal	
	1019	+ 0,4	— 0,1	rather indistint	
	1020	+ 0,9	+ 1,1	uncertain	doubtful measurement
	1021	— 0,7	+ 0,9	indistinct	
	1023	— 1,6	+ 0,8	uncertain	presumptive position

$m_x = \pm 1{,}1 \quad m_y = \pm 1{,}2$

$m_r = 1{,}7$

TABLE 5

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5723 : 09-12					
	1018	— 0,6	+ 0,5	signal	
	1020	+ 0,5	— 0,7	rather good	
	1025	— 0,1	+ 0,3	accurate	
	1028	— 0,5	— 1,2	accurate	bad position in the model
	1029	+ 0,7	+ 1,1	rather uncertain	
		$m_x = \pm 0{,}7$ $m_y = \pm 1{,}1$ $m_r = \pm 1{,}3$			
	Points excluded from the trans-formation				
	1024	+ 1,1	+ 21,2	very incertain	
	1026	— 5,1	— 1,5	not visible	presumptive position
	1027	— 5,0	+ 1,6	very uncertain	

TABLE 6

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5724 : 01-03					
	1025	+ 0,6	— 0,5	good	
	1026	— 0,4	— 0,5	not visible	presumptive position
	1028	+ 0,1	+ 0,1	good	
	1029	+ 1,1	+ 1,4	rather uncertain	
	1030	— 0,1	+ 0,4	signal	
	1034	— 0,3	— 0,3	accurate	
	1035	+ 0,3	— 1,2	accurate	
	1036	— 1,3	+ 0,5	good	
		$m_x = \pm 0{,}8$ $m_y = \pm 0{,}9$ $m_r = \pm 1{,}2$			
	Points excluded from the trans-formation				
	1027	— 6,5	+ 3,1	very uncertain	
	1024	— 3,0	+ 15,6	very uncertain	
	1031	+ 12,1	+ 4,4	uncertain	
	1032	+ 16,4	— 1,5	very indistinct	

TABLE 7

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5724 : 03-06					
	1030	+ 1,2	+ 0,4	signal	
	1037	+ 0,7	— 1,3	very uncertain	
	1039	— 0,3	+ 1,7	signal	
	1040	+ 0,7	+ 0,4	good	
	1041	— 1,2	— 0,3	rather indistinct	
	1042	— 1,4	— 0,3	indistinct	
	1043	+ 0,6	— 0,6	accurate	
	1044	— 0,3	± 0,0	good	
		$m_x = \pm 1,0$ $m_y = \pm 1,0$ $m_r = \pm 1,4$			
	Points excluded from the trans-formation				
	1032	+ 16,4	— 7,0	probably wrong	

TABLE 8

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5724 : 06-09					
	1045	+ 1,5	+ 1,6	very uncertain	
	1046	— 1,3	— 1,4	good	
	1048	± 0,0	— 0,5	good	
	1049	+ 1,8	— 0,8	rather indistinct	
	1050	+ 0,1	+ 0,8	signal	
	1051	+ 0,1	— 0,2	accurate	
	1052	— 0,1	+ 1,0	uncertain	
	1053	— 1,3	+ 0,4	rather uncertain	
	1055	— 0,9	— 1,0	not visible	presumptive position
		$m_x = \pm 1,2$ $m_y = \pm 1,1$ $m_r = \pm 1,6$			
	Points excluded from the trans-formation				
	1054	— 2,5	— 4,5	uncertain	

TABLE 9

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5724 : 09-11	1056	— 2,2	+ 0,3	probably right	
	1054	— 2,2	— 2,6	rather uncertain	
	1048	+ 1,3	+ 0,4	good	
	1049	+ 1,9	± 0,0	good	
	1050	+ 1,6	+ 0,4	signal	
	1051	+ 1,3	— 1,6	good	
	1053	— 2,4	— 0,6	possibly right	
	1059	+ 1,0	+ 0,5	accurate	
	1061	— 0,5	+ 0,7	rather uncertain	
	1062	+ 0,1	+ 1,1	signal	
	1063	+ 0,2	— 0,9	rather uncertain	
	1064	— 1,1	+ 0,1	accurate	
	1065	— 0,7	+ 0,4	good	
	1066	+ 1,7	+ 1,8	very indistinct	
	$m_x = \pm 1{,}6$ $m_y = \pm 1{,}2$ $m_r = \pm 2{,}0$				
	Points excluded from the trans-formation				
	1055	+ 3,7	+ 3,2	not visible	presumptive position
	1057	+ 0,8	— 3,3	uncertain	doubtful measurement
	1058	+ 2,9	+ 2,2	not visible	presumptive position
	1060	+ 3,7	+ 0,8	not visible	presumptive position

TABLE 10

Model no.	Trans-formation point no.	f_x m	f_y m	Identification	Note
XFV 5724 : 11-13	1064	— 0,1	— 0,9	accurate	
	1065	— 0,4	+ 0,6	very indistinct	
	1062	+ 1,1	+ 1,6	signal	
	1063	+ 1,1	— 1,1	good	
	1059	+ 1,3	+ 1,9	good	
	1060	+ 2,6	— 1,1	uncertain	
	1061	+ 2,1	+ 1,9	very indistinct	
	1068	— 4,0	+ 1,9	good	
	1069	— 4,1	+ 3,2	accurate	
	1070	— 1,5	— 2,8	uncertain	
	1071	— 0,3	— 1,5	good	
	1072	— 0,1	+ 0,5	rather indistinct	probably right
	1073	+ 2,6	— 0,9	not visible	presumptive position
	1074	— 0,3	— 3,4	good	
	$m_x = \pm 2{,}2$ $m_y = \pm 2{,}0$ $m_r = \pm 3{,}0$				
	Points excluded from the trans-formation				
	1067	— 4,8	+ 6,2	indistinct	
	1066	— 4,2	+ 0,2	not visible	presumptive position