AZIMUTH FROM OBSERVATIONS OF CLOSE CIRCUMPOLAR STARS

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ABSTRACT

In this paper suggestions have been made to modify considerably the existing time-azimuth and altitude-azimuth formulae to obtain more easily and quickly the first-order and second-order azimuths from observations of close circumpolar stars using lesser numbers of figures in both logarithmic and machine computations with the help of some small correction tables only. A simple chart has also been designed in order to read immediately from the chart third-order or less precise azimuths from observations of the above stars without recourse to elaborate computations. First-order azimuths are used in geodetic surveys while second-order and third-order azimuths have ample applications in topographic and hydrographic surveys including navigation.

INTRODUCTION

The astronomical azimuth of a heavenly body is defined as the angle counted clockwise from the north, between the meridian plane of the observer and the vertical plane passing through the observer and the heavenly body. The determination of azimuth at a survey station therefore consists in obtaining the azimuth or true bearing of any line emanating from the station so that the azimuths of all the survey lines meeting there may be deduced. In primary determinations, the observed azimuth should preferably be that of a side of the triangulation. Otherwise a convenient reference mark is often established for the purpose. An essential feature of the observation is thus the measurement of the horizontal angle between a heavenly body and the survey line or the reference mark so that when the azimuth of the heavenly body becomes known, that of the survey line or the reference mark may be immediately obtained by simple addition of angles. The precision of azimuth observations is however dependent on the quality of the angular work of the survey and is accordingly classified into the following three categories :

(i) first-order azimuths giving a probable error for the result of $0''.5$ or better, obtained from observations on a single night;

(ii) second-order azimuths giving a probable error for the result of $2''.0$ or less and

(iii) third-order azimuths giving a probable error of nearly $1'.0$.

To provide a geodetic framework in a country is the main responsibility of a geodetic survey organization. What is required of a topographer or a hydrographer is just to fill up the big gaps inside the existing geodetic framework falling within his jurisdiction of land survey or sea survey, by fixing minor control points in order to produce accurate maps of his area. A hydrographer is seldom required to extend any geodetic framework and is therefore not much concerned with precise observations of first-order azimuths. The third-order azimuths on the other hand being of great help in both detail survey and navigation, a hydrographer is normally interested in second- and third-order azimuths only.

There are however various methods in existence for determining azimuths from observations of close circumpolar stars. Among these, the more simple and in common use now are :

(i) the method of time-azimuth by noting chronometer times (L.S.T.) and taking horizontal circular readings; and

(ii) the method of altitude-azimuth by recording simultaneous observations for both horizontal and vertical circular readings of intersections of close circumpolar stars by means of a theodolite. In older hydrographic surveys sun azimuths were the general rule, but these hardly provide sufficiently accurate results for satisfactory use in modern work. Moreover star azimuths are definitely much easier to observe with a modern theodolite than sun azimuths though working at night is a slight disadvantage.

According to the method of time-azimuth, the hour angle of a star measured the shorter way from the upper transit, is deduced from correct chronometer times (L.S.T.) and then the astronomical triangle P Z S solved for the azimuth angle A between the elevated pole and the star, giving :

$$\tan A = \frac{\sin t}{\cos \varphi \tan \delta - \sin \varphi \cos t} \qquad (1)$$

where φ the latitude of the observer's position is known and the azimuth of the star counted clockwise from the north is A or $180° - A$ in the case of north latitudes and $360° - A$ or $180° + A$ in the case of south latitudes according as the star is east or west of the meridian of the observer.

The formula is however absolute, involving no approximations, and as such can give azimuth values to any desired degree of accuracy, provided the observed and the given data are sufficiently accurate. For first-order azimuths chronometer times are required to be noted to the nearest 0. s 1 or more depending on the magnitudes of φ, δ and t, with the help of a chronograph or simply a stopwatch, and the horizontal circular readings to the nearest $0.''1$ by means of a precise theodolite, e. g., Wild 1_3, Wild 1_4, Geodetic Tavistock, etc. For second-order azimuths, however, chronometer times are to be noted only to the nearest unit of a second by means of a stopwatch, and the horizontal circular readings to the nearest unit of a second by means of an ordinary glass arc theodolite. Also t being known in the present case, there should not be any difficulty in determining beforehand the approximate position, e. g. azimuth and altitude of any of the close circumpolar stars , for easily bringing them within the field of view of the theodolite, with the help of the chart appended at the end. Difficulties occur only in practically working out the formula. It is particularly inconvenient for logarithmic computations. Albrecht's modified form :

$$\tan A = \cos \delta \sin t \sec \varphi \left(\frac{1}{1 - a}\right)$$

where $$a = \tan\varphi \cot \delta \cos t \qquad (2)$$

is a great improvement on the original formula (1), and in conjunction with the bulky table of log $\left(\frac{1}{1-a}\right)$, is in general use now. Even then the computation is quite lengthy requiring at least 6-figure logarithmic tables for first-order azimuths and 5-figure tables for second-order azimuths. In case of machine computations also the existing formula is not very satisfactory, as it involves a lot of time-consuming arithmetical operations with trigonometrical functions of similar numbers of figures for first-order and second-order azimuths respectively.

According to the method of altitude-azimuth, the altitude of the star is observed at least two hours before or after the times of its transits, and then the astronomical triangle PZS solved for the azimuth angle A between the elevated pole and the star, measured the shorter way, east or west, giving

$$\tan \frac{A}{2} = \sqrt{\sec S \sin (S - h) \sin (S - \varphi) \sec (S - \Delta)} \qquad (3)$$

where $\Delta = 90° - \delta$, Δ being measured from the elevated pole, $S = \frac{h + \varphi + \Delta}{2}$ being always treated as positive and the azimuth of the star counted clockwise from the north is A or $180° - A$ in the case of north latitudes and $360° - A$ or $180° + A$ in the case of south latitudes according as the star is east or west of the meridian.

This formula also is absolute as before, involving no approximations. Yet it does not normally give an accuracy better than second-order azimuths as the altitude values used in the formula are generally not accurate enough to always warrant azimuths of first-order precision. Even in obtaining second-order azimuths by the above formula, there are some practical difficulties :

(a) It requires use of at least 5-figure tables in both logarithmic and machine computations.

(b) In logarithmic computations it needs special care in evaluating the small angle $\frac{A}{2}$ and also in finding the values of log sin $(S - h)$ or log sin $(S - \varphi)$ when the latter reduce to small quantities.

(c) In machine computations also there are time-consuming factors like extracting square roots to the nearest 5th place of decimal, etc.

Even in obtaining azimuths of third-order precision by the above methods, surveyors cannot help following the long way of working out the entire formula though slightly less rigorously than in the case of second-order azimuths. But all the same, considering the degree of precision of the final results required of third-order azimuths, the existing procedure appears most unsatisfactory.

An attempt has therefore been made here to suggest simplified alternative formulae or modifications of the existing ones so as to make them equally useful for both logarithmic and machine computations of first- and second-order azimuths, requiring far fewer arithmetical operations — a matter of primary consideration in survey computations. A simple chart has also been added at the end to read azimuth values of third-order precision by the above methods without recourse to elaborate computations.

A MODIFIED TIME-AZIMUTH FORMULA

Let t be the hour angle measured the shorter way from upper transit of à close circumpolar star of declination δ, and φ the latitude of the place of observation. Then if h denotes the altitude and A the azimuth angle between the elevated pole and the star, we have from the astronomical triangle PZS, the following relation :

$$\frac{\sin A}{\cos \delta} = \frac{\sin t}{\cos h}$$

or :

$$\sin A = \frac{\sin \Delta \sin t}{\cos h}, \ \Delta = 90^\circ - \delta \tag{4}$$

Since :

$$h = \varphi + \Delta \cos t - \frac{\Delta^2}{2} \sin 1'' \sin^2 t \tan h + \ldots.$$

we have from (4) :

$$\sin A = \frac{\sin \Delta \sin t}{\cos \left(\varphi + \Delta \cos t - \frac{\Delta^2}{2} \sin 1'' \sin^2 t \tan h + \ldots.\right)}$$

$$= \frac{\sin \Delta \ \sin t}{\cos(\varphi + \Delta \cos t) \cos\left(\frac{\Delta^2}{2} \sin 1'' \sin^2 t \tan h \ldots\right) + \sin(\varphi + \Delta \cos t) \ \sin\left(\frac{\Delta^2}{2} \sin 1'' \sin^2 t \tan h \ldots\right)}$$

$$= \frac{\sin \Delta \ \sin t}{\cos (\varphi + \Delta \cos t)} \left[1 + \tan (\varphi + \Delta \cos t) \tan \left(\frac{\Delta^2}{2} \sin 1'' \sin^2 t \tan h\right)\right]^{-1},$$

neglecting smaller terms :

$$= \frac{\sin \Delta \ \sin t}{\cos (\varphi + \Delta \cos t)} \left(1 - \frac{1}{2} \tan^2 (\varphi + \Delta \cos t) \sin^2 \Delta \sin^2 t\right)$$

approximately,

$$= \frac{\sin \Delta \ \sin t}{\cos (\varphi + \Delta \cos t)} - \frac{1}{2} \sin^3 A \ \sin^2 (\varphi + \Delta \cos t)$$

or :

$$A = \Delta \sin t \ \sec (\varphi + \Delta \cos t) - \frac{A^3 \sin^2 1''}{2} \sin^2 (\varphi + \Delta \cos t) +$$

$$+\frac{A^3}{6}\sin^2 1'' - \frac{A\Delta^2}{6}\sin^2 1''$$

$$= \Delta \sin t \sec (\varphi + \Delta \cos t) +$$

$$+\frac{A}{6}(A^2 - \Delta^2)\sin^2 1'' - \frac{A^3}{2}\sin^2 1'' \sin^2 (\varphi + \Delta \cos t)$$

$$\text{or : } A = A' + B \tag{5}$$

where A' (principal term) $= \Delta \sin t \sec (\varphi + \Delta \cos t)$
and :

$$B = \frac{A'}{6}(A'^2 - \Delta^2)\sin^2 1'' - \frac{A'^3}{2}\sin^2 1'' \sin^2 (\varphi + \Delta \cos t).$$

Except for the term B, the above formula is evidently a simple one and can be worked out more quickly and conveniently than the existing one with calculating machines and logarithmic tables, using only 4-figure and 5-figure tables for second-order and first-order azimuths respectively, unless, of course, the azimuth values are of the order of 3° or greater, in which case one more figure needs to be considered in the computations in order to attain the desired degree of precision.

However, B is a very small quantity, less than even a second, provided observations are made within the latitude belt of 50° N and 50° S (except up to about 8° north and south of the geographical equator) and limited to close circumpolar stars of N.P.D. up to about 1°, and as such, is always negligible in the case of second-order azimuths. For first-order azimuths, however, B is not always inappreciable and therefore requires to be computed using only 2- or 3-figure tables in order to be used as a sort of small correction to the principal term of the formula (5). A specimen correction table given below, has been prepared for observations of a close circumpolar star having $\Delta = 1°$ within the latitude belt of 50° N and 50° S except for the area limited by the latitude lines 8° N and 8° S. This table can also be applied to any other close circumpolar star having $\Delta = x^0$ (x being not much greater than 1) on multiplying the interpolated tabular values with x^3.

Correction table B
for $\Delta = 1°$
(corrections always negative)

H. A'. s	Hours 6	Hours 5 or 7	Hours 4 or 8	Hours 3 or 9	Hours 2 or 10
Latitudes N or S					
0°	0".0	0".0	0".0	0".0	0".0
10	0. 0	0. 0	0. 0	0. 1	0. 1
20	0. 1	0. 1	0. 1	0. 1	0. 1
30	0. 1	0. 1	0. 1	0. 1	0. 1
35	0. 2	0. 2	0. 2	0. 1	0. 1
40	0. 3	0. 3	0. 3	0. 2	0. 2
45	0. 5	0. 5	0. 4	0. 3	0. 2
50	0. 8	0. 8	0. 6	0. 4	0. 2

A MODIFIED ALTITUDE-AZIMUTH FORMULA

Let h be the observed altitude corrected for refraction, of a close circumpolar star of declination δ, and φ the latitude of the place of observation. Then if A denotes the azimuth angle between the elevated pole and the star, we have from the astronomical triangle P Z S, the following relation :

$$\cos A = \frac{\cos \Delta - \sin h \sin \varphi}{\cos h \cos \varphi}$$

or :

$$\sin A = \left[1 - \left(\frac{\cos \Delta - \sin h \sin \varphi}{\cos h \cos \varphi}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{(\cos a - \cos \Delta)^{\frac{1}{2}} [\cos (h + \varphi) + \cos \Delta]^{\frac{1}{2}}}{\cos (h + \varphi) + \cos a}$$

putting $h - \varphi = a$:

$$\sin A = \frac{2(\cos a - \cos \Delta)^{\frac{1}{2}} [\cos(h + \varphi) + \cos \Delta]^{\frac{1}{2}}}{\cos (h + \varphi) + \cos a}$$

$$= \frac{2(\cos a - \cos \Delta)^{\frac{1}{2}} \left[2 \cos^2 \frac{h + \varphi}{2} - 1 + \cos \Delta\right]^{\frac{1}{2}}}{2 \cos^2 \frac{h + \varphi}{2} - 1 + \cos a}$$

$$= \frac{2(\cos a - \cos \Delta)^{\frac{1}{2}} \left[1 - \frac{1 - \cos \Delta}{2} \sec^2 \frac{h + \varphi}{2}\right]^{\frac{1}{2}} \sec \frac{h + \varphi}{2}}{1 - \frac{1 - \cos a}{2} \sec^2 \frac{h + \varphi}{2}}$$

$$= \Delta \sqrt{1 - \frac{a^2}{\Delta^2}} \sec \frac{h + \varphi}{2} \left[1 - \frac{1}{24} (\Delta^2 + a^2) \sin^2 1'' - \frac{1}{8} (\Delta^2 - 2a^2) \sin^2 1'' \sec^2 \frac{h + \varphi}{2}\right],$$

neglecting smaller terms after simplification.

or :

$$A = \Delta \sqrt{1 - \frac{a^2}{\Delta^2}} \sec \frac{h + \varphi}{2} - \frac{A}{24} (\Delta^2 + a^2) \sin^2 1'' - \frac{A}{8} (\Delta^2 - 2a^2) \sin^2 1''. \sec^2 \frac{h + \varphi}{2} + \frac{A^3}{8} \sin^2 1''.$$

$$= \Delta \sqrt{1 - \frac{a^2}{\Delta^2}} \sec \frac{h + \varphi}{2} + \frac{A}{24} \Delta^2 \sin^2 1'' \tan^2 \frac{h + \varphi}{2} + \frac{Aa^2}{24} \sin^2 1'' \left(1 + 2 \tan^2 \frac{h + \varphi}{2}\right),$$

after further simplification.

$$= \Delta \sin t \sec \frac{h+\varphi}{2} + \frac{A\Delta^2}{24} \sin^2 1'' \tan^2 \frac{h+\varphi}{2} + \frac{Aa^2}{24} \sin^2 1'' \left(1 + 2\tan^2 \frac{h+\varphi}{2}\right),$$

putting $\sin t$ for $\sqrt{1 - \dfrac{a^2}{\Delta^2}}$

$$\text{or } A = A' + C \ldots \tag{6}$$

where A' (principal term) $= \Delta \sin t \sec \dfrac{h+\varphi}{2}$

and :

$$C = \frac{A'}{24} \Delta^2 \sin^2 1'' \tan^2 \frac{h+\varphi}{2} + \frac{A'a^2}{24} \sin^2 1'' \left(1 + 2\tan^2 \frac{h+\varphi}{2}\right)$$

Except for the term C, the formula (6) is evidently a simple one and can be worked out as quickly and conveniently as the previous formula (5) with both calculating machines and logarithmic tables using only 4-figure tables in the case of second-order azimuths, provided observations are carried out at least two hours before or after the time of the star's transits, unless, of course, the azimuth values are of the order of 3° or greater, in which case one more figure needs to be considered in the computations in order to attain the desired degree of precision. When observations are made within a short distance of the star's elongation, either east or west, it is possible to obtain first-order azimuths also with the formula (6), for when the star is near elongation, either east or west, the small errors in observed altitudes of the star due to uncertainties in the computed values of refractions, become less effective in the final result. Moreover the effect of latitude error, normally quite significant in the present circumstances, can be easily minimised by observing a pair of stars near east and west elongations respectively, differing by nearly 12 h in their R.A'.s and satisfying as nearly as conveniently possible the relation : $\sin A_e \tan h_e = \sin A_w \tan h_w$, where the suffixes e and w refer to stars near east and west elongations respectively. C is again a small quantity, less than even a second, provided observations are made within the latitude belts of 60° N and 60° S (except up to about 10° north and south of the geographical equator) and limited to close circumpolar stars of N.P.D. up to about 1°, and as such, is quite inappreciable in the case of second-order azimuths. This however is not the case with circumpolar stars of N.P.D. greater than 1°. A specimen correction table, given below, has therefore been prepared as before for observations of close circumpolar stars of N.P.D. upto 3 ½° within the latitude belt of 50° N and 50° S except for the area limited by the latitude lines 10° N and 10° S, where observed altitudes of stars are likely to be very unreliable due to many uncertainties in the computed values of refractions at such low altitudes. The corresponding correction table for first-order azimuths requires to be computed to the nearest 0″.1 instead of only 1″.0.

Correction table C
for Δ : 1° to $3\frac{1}{2}$°
(corrections always positive)

Latitudes N or S		10°	20°	30°	35°	40°	45°	50°
$\Delta = 1°$	$a = 0°$	0″	0″	0″	0″	0″	0″	0″
	$a = 1/2°$	0	0	0	0	0	0	0
$\Delta = 1{,}5°$	$a = 0°$	0	0	0	0	0	0	0
	$a = 1°$	0	0	0	0	0	0	1
$\Delta = 2°$	$a = 0°$	0	0	0	0	0	1	1
	$a = 1°$	0	0	0	0	0	1	1
$\Delta = 2{,}5°$	$a = 0°$	0	0	0	0	1	1	2
	$a = 1°$	0	0	0	1	1	1	2
$\Delta = 3°$	$a = 0°$	0	0	0	1	1	2	3
	$a = 1°$	0		1	1	2	2	3
	$a = 2°$	0	1	1	2	3	3	4
$\Delta = 3{,}5°$	$a = 0°$	0	0	1	1	2	3	4
	$a = 1°$	0	0	1	1	2	3	5
	$a = 2°$	1	1	2	2	3	5	7

Table R
Change per day : 4 minutes

Date		R		Date		R	
		h	m				
Jan.	6	7	00	July	8	19	02
	22	8	03		23	20	01
Feb.	6	9	02	Aug.	7	21	00
	21	10	02		23	22	03
Mar.	8	11	01	Sept.	7	23	02
	23	12	00		22	00	01
April	8	13	03	Oct.	7	1	01
	23	14	02		22	2	00
May	8	15	01	Nov.	7	3	03
	23	16	00		22	4	02
June	8	17	03	Dec.	7	5	01
	23	18	03			6	00

Upper transit of a star

(1) L.M.T. of transit = R.A. minus R.
(2) S.T. of transit = L.M.T. of transit plus (standard long. minus local long.) in time.

N.B. — The table is correct to 2 minutes for all years. For better accuracy, however, the tabular values may be taken to be true for

(i) 1950, 1954, 1958, 1962, 1966... i.e. two years after the leap years
(ii) all leap years after increasing the values by 2 minutes except for the months of January and February in which case the tabular values are to be decreased by 2 minutes
(iii) all years following the leap years after increasing by 1 minute
and (iv) the remaining years after decreasing by 1 minute.

The local mean time of a star's transit can also be easily determined with the help of the table R provided below. As observations are required at least two hours before or after the time of the star's transits, a knowledge of the local mean time or the standard time of the star's transit, is certainly of very great help to the observer lacking the local sidereal time, in deciding quickly which stars can be conveniently observed within the time (his own watch time) at his disposal. Again the H.A. of the star at the time of observation being thus roughly known to the nearest few minutes, there should not be any difficulty in determining beforehand its approximate position e. g., azimuth and altitude for easily bringing it within the field of view of the theodolite, by taking help of the chart appended at the end. Again, once the local mean time or the standard time of a star's transit becomes known from table R, it is very easy to obtain the local mean time or the standard time at east or west elongation of the star with the help of the table X provided below. A table Y giving the places of principal circumpolar stars has also been added for ready reference.

TABLE X

giving H.A.'s of stars at elongation

Latitude N	15°				45°			
Δ	West		East		West		East	
	h	m	h	m	h	m	h	m
1°	5	59	18	01	5	56	18	04
2	5	58	18	02	5	52	18	08
3	5	57	18	03	5	48	18	12
4	5	56	18	04	5	44	18	16
5	5	55	18	05	5	40	18	20

TABLE Y

giving places of principal circumpolar stars

Names of stars	Mag.	Right Ascension		Annual variation	Declination		Annual variation
		h	m	m	°	′	′
43 H. Cephei	4.52	1	03	+ 0.14	86	02	+ 0.32
α Ursae Minoris ..	2.12	1	54	+ 0.71	89	04	+ 0.29
51 H. Cephei	5.26	7	22	+ 0.46	87	07	— 0.12
δ Ursae Minoris ..	4.44	17	46	— 0.32	86	36	— 0.02
32 H. Cephei	5.38	22	17	— 0.08	85	54	+ 0.30
ζ Octantis	5.38	9	03	— 0.15	—85	30	— 0.24
ι Octantis	5.38	12	50	+ 0.11	—84	54	— 0.33
χ Octantis	5.22	18	31	+ 0.59	—87	39	+ 0.04
σ Octantis	5.48	20	25	+ 1.21	—89	07	+ 0.20
τ Octantis	5.56	23	22	+ 0.14	—87	44	+ 0.33

N.B. — The table is true for the year 1958. The same may be used for several years without any appreciable error provided the tabular values are corrected for the annual variations.

AN ALTERNATIVE ALTITUDE-AZIMUTH FORMULA

Let A be the azimuth angle of a close circumpolar star, measured the shorter way, east or west, and h the corresponding altitude. Let A_0 and h_0 be the azimuth angle and altitude of the star at elongation. Then if φ be the latitude of the place of observation, we have from the astronomical triangle PZS, the following relations :

$$\sin A_0 = \sin \Delta \sec \varphi \tag{7}$$

$$\sin h_0 = \sin \varphi \sec \Delta \tag{8}$$

$$\text{and } \cos A = \frac{\cos \Delta - \sin h \sin \varphi}{\cos h \cos \varphi} \tag{9}$$

Since A_0 and Δ are small, we have from (7), after simplification,

$$A_0 = \Delta \sec \varphi + \frac{\Delta^3}{6} \sin^2 1'' \tan^2 \varphi \sec \varphi$$

$$\text{or : } A_0'' = (\Delta'' + 0.235 \cdot 10^{-10} \Delta''^3 \operatorname{tg}^2 \varphi) \sec \varphi$$

$$\text{or : } A_0'' = (\Delta'' + D'') \sec \varphi \; ..... \tag{10}$$

$$\text{where } D'' = 0.235 \cdot 10^{-10} \Delta''^3 \operatorname{tg}^2 \varphi$$

D is obviously a small quantity and therefore can be easily computed using only 2 or 3-figure tables, and expressed for general use in a convenient tabular form.

Since $h_0 - \varphi$ is also a small quantity, we have from (8),

$$\frac{\sin h_0 - \sin \varphi}{\sin h_0 + \sin \varphi} = \frac{\sin \varphi \sec \Delta - \sin \varphi}{\sin \varphi \sec \Delta + \sin \varphi}$$

$$\text{or } \tan \left(\frac{h_0 - \varphi}{2}\right) = \tan \left(\frac{h_0 + \varphi}{2}\right) \tan^2 \frac{\Delta}{2}$$

or :

$$h_0'' = \varphi'' + \frac{\Delta''^2}{2} \sin 1'' \tan \varphi$$

$$h_0'' = \varphi'' + 0{,}242 \cdot 10^{-5} \Delta''^2 \tan \varphi$$

$$h_0'' = \varphi'' + E \; ..... \tag{11}$$

$$\text{where } E = 0.242 \cdot 10^{-5} \Delta''^2 \tan \varphi.$$

E being a small quantity, it can be easily computed using 3-figure tables only and expressed for general use in a convenient tabular form. E need not be very accurate. For first-order determinations E requires to be correct to only a second. In the case of second-order azimuths, it is enough if E is correct to only 10 seconds.

Again since $h = h_0 + x$ or $x = h - \varphi - E$, where x is a small quantity, we have from (9),

$$\cos A = \frac{\cos \Delta - \sin (h_0 + x) \sin \varphi}{\cos (h_0 + x) \cos \varphi}$$

$$= \frac{\cos \Delta - \sin h_0 \sin \varphi}{\cos h_0 \cos \varphi} + \frac{\cos \Delta - \sin (h_0 + x) \sin \varphi}{\cos (h_0 + x) \cos \varphi} - \frac{\cos \Delta - \sin h_0 \sin \varphi}{\cos h_0 \cos \varphi}$$

$$= \cos A_0 + \frac{2 \sin^2 \frac{x}{2} \cos \Delta}{\cos \varphi \cos (h_0 + x)}$$

$$= \cos A_0 + \frac{x^2}{2} \frac{\sin^2 1'' \cos \Delta}{\cos \varphi \cos h}$$

$$\text{or } \cos A = \cos A_0 + y \tag{12}$$

where $$y = \frac{x^2}{2} \frac{\sin^2 1'' \cos \Delta}{\cos \varphi \cos h}$$

The series (12) may now be expanded in terms of ascending powers of Y and will converge rapidly when x is small.

Mac Laurin's formula applied to this case is as follows :

$$A = A_0 + y \left(\frac{dA}{dy}\right)_0 + \frac{y^2}{2} \left(\frac{d^2A}{dy^2}\right)_0 + \frac{y^3}{6} \left(\frac{d^3A}{dy^3}\right)_0 + \ldots\ldots \tag{13}$$

Differentiating (12) and observing that when $y = 0$, $A = A_0$, we find the following values of the differential coefficients :

$$\left(\frac{dA}{dy}\right) = -\frac{1}{\sin A_0}; \qquad \left(\frac{d^2A}{dy^2}\right)_0 = -\frac{\cot A_0}{\sin^2 A_0};$$

$$\left(\frac{d^3A}{dy^3}\right)_0 = -\frac{1 + 3 \cot^2 A_0}{\sin^3 A_0}$$

Substituting these values in (13) and restoring the values of y, we find :

$$A = A_0 - \frac{x^2 \sin 1'' \cos \Delta}{2 \cos \varphi \cos h \sin A_0} - \frac{x^4 \sin^3 1'' \cos^2 \Delta \cot A_0}{8 \cos^2 \varphi \cos^2 h \sin^2 A_0} \ldots\ldots$$

or :

$$A'' = A''_0 - \frac{1}{2} \left(\frac{x}{\Delta}\right) x'' \sec \varphi - \frac{1}{8} \left(\frac{x}{\Delta}\right)^3 x'' \sec \varphi$$

or :

$$A'' = (\Delta'' + D'') \sec \varphi - \frac{1}{2} \left(\frac{x}{\Delta}\right) x'' \sec \varphi - \frac{1}{8} \left(\frac{x}{\Delta}\right)^3 x'' \sec \varphi, \text{ from (10)}$$

or :

$$A'' = (\Delta'' + D'' + F'') \sec \varphi \tag{14}$$

where $D'' = 0.235 \cdot 10^{-10} \Delta''^3 \tan^2 \varphi$ and $F'' = \frac{1}{2} \left(\frac{x}{\Delta}\right) + \frac{1}{8} \left(\frac{x}{\Delta}\right)^3 x''$.

The formula (14) is evidently a simple one and can be worked out with the help of the correction tables : D, E and F, as quickly and conveniently as the previous formulae (5) and (6) with both calculating machines and logarithmic tables using only 4-figure and 5-figure tables for second-order and first-order azimuths respectively, unless, of course, the azimuth values are of the order of 3° or greater, in which case one more figure needs to be considered in the computations in order to attain the desired degree of precision. The only difficulty with this formula is that unless observations are carried out at lower latitudes, say, below about 45° N or 45° S and limited to a short distance of about 20 minutes of the star's elongation, the correction tables : E and F are likely to become very large. Specimen tables : D, E and F have been given below for first-order azimuths from observations of close circumpolar stars of N.P.D. up to $3\frac{1}{2}°$ within the latitude belt of 40° N and 40° S except for the area limited by the latitude lines 10° N and 10° S. For second-order azimuths, however, the corresponding correction tables : D and F require to be computed to the nearest 1''0 instead of 0''1 while the table E to the nearest 10''0 instead of 1''0.

CIRCUMPOLAR STAR CHART

In the adjoining figure let 0 be the celestial pole, and the semi-circle PSQ with 0 as centre be the apparent path of an east or west circumpolar star as it appears when viewed from the earth's equator, so that its radius represents the N.P.D. of the star drawn on a large convenient scale, say, $1'' = 1°$ or $^1/_2°$ or $^1/_4°$, etc. Let there be a number of such semi-circles having 0 as centre and representing apparent paths of east or west circumpolar stars with N.P.D.'s of different magnitudes preferably not greater than 5°. Let the meridian plane lying at right angles to the apparent paths of the stars, intersect the concentric semi-circles along POQ so that any star passing through it will have its H.A. zero on OP and 12 h or 180° on OQ, measured the shorter way, east or west. Let OS be drawn at right angles to POQ so that any star passing through OS will have its H.A. 6 h or 90° on OS. Let the semi-circle PQS be equally sub-divided into small arcs and the radial lines be drawn from O to these points of division so that each radial line may signify a particular H.A. of any star passing through it, which is equal to the angle it subtends with OP. Let these radial lines be produced to intersect the tangent S'SS'' drawn to the outermost concentric semi-circle, the points of intersection being marked as the difference between 90° and the H.A.'s as signified by the corresponding radial lines. Let the semi-circle PQS be further subdivided by straight lines drawn parallel to both OP and OS.

Let M_1 be the position of a circumpolar star of N.P.D. : Δ_1, its H.A. being denoted by the radial line OM_1. If OX_1 and OY_1 denote the abscissa and ordinate of M_1 along OS and OP respectively, and X_1M_1 when produced if necessary meet the radial line corresponding to the mark on the tangen line S'SS'', equal to $\varphi + OY_1$ at M_1 where $OY_1 = \Delta_1 \cos t$, then the azimuth angle of the star measured the shorter way, east or west is OM'_1 on the scale of drawing, where

$$OM'_1 = OX_1 \sec(\varphi + \Delta_1 \cos t) = \Delta_1 \sin t \sec(\varphi + \Delta_1 \cos t).$$

Again when instead of the H.A., the altitude : h of a star of N.P.D. : Δ_2 becomes known, one can obtain its azimuth angle as follows :

Let $OY_2 = h - \varphi = a = \Delta_2 \cos t'$ and $OX_2 = \Delta_2 \sin t'$. Let OX_2 when produced if necessary meet the radial line at M'_2 corresponding to the mark on the tangent line S'SS'', equal to $\frac{\varphi + h}{2}$. Then the azimuth angle of the star is OM'_2 on the scale of drawing, where :

$$OM'_2 = \Delta_2 \sin t' \sec \frac{h + \varphi}{2} \text{ and } \Delta_2 \cos t' = (h - \varphi).$$

Given H.A. and N.P.D. of a circumpolar star, one can also determine its altitude as follows :

Let an arc of a circle be drawn with O as centre and radius OM'_1 equal to the azimuth angle as obtained in the first case, to intersect at M''_1 the radial line corresponding to the mark on the tangent line S′SS″, equal to $\varphi + \dfrac{\Delta_1 \cos t}{2}$ instead of $\varphi + \Delta_1 \cos t$. Let the ordinate of M''_1 when produced if necessary intersect the concentric circle of radius Δ_1 at M″. Then the altitude of the star is $\varphi + a$ where a is the ordinate of M″.

Similarly when the altitude and N.P.D. of a circumpolar star are given, one can also determine its H.A., proceeding in a similar manner with the second case.

It is to be noted that after the azimuth angle A of a star is known, one has to reduce it to the azimuth of the star counted clockwise from north, which is A or $180° - A$ in the case of north latitudes and $360° - A$ or $180° + A$ in case of south latitudes according as the star is east or west of the meridian of the observer. Also for better accuracy it is always preferable to use the N.P.D. of the star after multiplying it with a convenient factor > 1 so that the N.P.D. thus enlarged is only just less than or equal to the radius of the outermost concentric semi-circle, while determining the azimuth angle, altitude or H.A. of a star as the case may be, and then divide the results of the azimuth angle and $h - \varphi$ or a so obtained, by the same factor.

The conventions adopted here for convenience are as follows :

Δ : measured from the elevated pole
φ : always treated as positive
t : measured the shorter way from the upper transit
A : the azimuth angle between the elevated pole and the star — always treated as positive
a : treated as positive in the upper quadrant and negative in the lower quadrant of the circumpolar star chart.

This is in brief the main principle of the construction and use of the circumpolar star chart. A practical example has also been given at the end to make the method clearer to both the surveyors and the navigators with little or no knowledge of practical astronomy.

EXAMPLES

1. On 1958 May 15, the circumpolar star : Polaris was observed with a precise theodolite at latitude 39°52′30″ N and longitude 70°10′15″ E, and the recorded time (L.S.T.) of its intersection was 19 h 50 m 12.4 s. What was the azimuth (first-order) of the star at the time of intersection ?

Formulae :

$$A' = \Delta \sin t \sec (\varphi + \Delta \cos t)$$
$$A = A' + B$$

L.S.T. of observation	= 19 h 50 m 12.4 s	
R.A. of Polaris	= 1 h 53 m 55.3 s	$\Delta = 3358''.9$
H.A. of Polaris	= 17 h 56 m 11.1 s	$t = 90°55'44''$

Machine computation	
$\sin t$	$= 0.99987$
$\cos t$	$= 0.01621$
Δ	$= 3358''.9$
$\Delta \cos t$	$= -54''.4$
φ	$= 39°52'30''.0$
$\varphi + \Delta \cos t$	$= 395135.6$
$\sec(\varphi + \Delta \cos t)$	$= 1.30274$
A′	$= 1°12'55''.2$
B (from table)	$= -0''.3$
A	$= 1°12'54''.9$
Azimuth	$= 1°12'54''.9$

Logarithmic computation	
$\log \cos t$	$= \bar{2}.20982$
$\log \Delta$	$= 3.52620$
$\log \Delta \cos t$	$= 1.73602$
$\Delta \cos t$	$= -54''.5$
φ	$= 39°52'30''.0$
$\varphi + \Delta \cos t$	$= 39°51'35''.5$
$\log \sec(\varphi + \Delta \cos t)$	$= 0.11486$
$\log \sin t$	$= \bar{1}.99994$
log A′	$= 3.64100$
A′	$= 1°12'55''.2$
B (from table)	$= -0''.3$
A	$= 1°12'54''.9$
Azimuth	$= 1°12'54''.9$

N.B. For second-order azimuth one less figure should be used in the computations.

2. On 1958 May 15, the altitude of the west circumpolar star : 51 H Cephei observed at latitude 39°52′30″ N, was found on applying refraction correction, to be 39°05′14″. What was the azimuth (second-order) of the star at the time of intersection ?

Formulae :

$$A' = \Delta \sin t \sec \frac{h+\varphi}{2}, \quad \cos t = \frac{a}{\Delta} = \frac{h-\varphi}{\Delta}$$

$$A = A' + C$$

$$h = 39°\ 05'\ 14'' \qquad h - \varphi = -2836''$$

$$\varphi = 39°\ 52'\ 30''$$

$$\frac{h+\varphi}{2} = 39°\ 28'\ 52'' \qquad \Delta = 10396''$$

Machine computation	
$\frac{a}{\Delta} = \cos t$	$= -0.27280$
t	$= 105°49'50''$
$\sin t$	$= 0.96207$
$\sec \frac{h+\varphi}{2}$	$= 1.29562$
A′	$= 3°35'58''$
C (from table)	$= 1$
A	$= 3°35'59''$
Azimuth	$= 356°24'01''$

Logarithmic computation	
$\log a$	$= 3.45271$
$\log \Delta$	$= 4.01687$
$\log \frac{a}{\Delta} = \log \cos t$	$= \bar{1}.43584$
t	$= 105°49'50''$
$\log \sin t$	$= \bar{1}.98321$
$\log \sec \frac{h+\varphi}{2}$	$= 0.11247$
log A′	$= 4.11255$
A′	$= 3°35'58''$
C (from table)	$= 1$
A	$= 3°35'59''$
Azimuth	$= 356°24'01''$

3. On 1958 May 15, the altitude of the east circumpolar star : σ Octantis observed at latitude 38°15′20″.0 S and longitude 75°05′20″ E, was found on applying refraction correction, to be 38°12′35″. What was the azimuth (first-order) of the star at the time of observation ?

Formula : $A'' = (\Delta'' + D'' + F'') \sec \varphi$

h	$= 38°12'35''$	Δ''	$= 3224''.8$
φ	$= 38°15'20''$	D'' (from table)	$= + 0.1$
E (from table)	$=$ 20	F'' (from table)	$= - 5.3$
$h - \varphi - E$	$=$ 0 03 05	$\Delta'' + D'' + F''$	$= 3219.6$

Machine computation		Logarithmic computation	
sec φ	$= 1.27347$	log sec φ	$= 0.10499$
A''	$= 4100.00$	log $(\Delta'' + D'' + F'')$	$= 3.50780$
A	$= 1°08'20''.0$	log A''	$= 3.61279$
Azimuth	$= 178°51'40''.0$	A''	$= 4100.1$
		A	$= 1°08'20''.1$
		Azimuth	$= 178°51'39''.9$

N.B. For second-order azimuth one less figure should be used in the computations.

4. Find the H.A., altitude and azimuth of the circumpolar star : δ Ursae Minoris at 20.00 hours I.S.T. on 1956 May 30 at lat. 30°20′ and long. 78°00′. At what time (I.S.T.) was the star at east elongation ? (Given R.A. and declination of the star, tables R, X and Y and the star chart).

R.A. of star from table Y	= 17 h 45 m
R. from table R	= 16 h 30 m
L.M.T. of transit of star	= 1 h 15 m
(st. long. — local long.) in time	= 0 h 18 m
I.S.T. of transit of star	= 1 h 33 m
I.S.T. of observation	= 20 h 00 m
H.A. of star	= 18 h 27 m
N.P.D. of star	= 3°24′
Altitude of star from chart	= 30°40′
Azimuth of star from chart	= 3°55′
Azimuth of star from north	= 3°55′
I.S.T. of transit of star	= 1 h 33 m
H.A. of star at east elongation from table X	= 18 h 08 m
I.S.T. at east elongation of star	= 19 h 41 m

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CORRECTION TABLE D (always positive)

Latitude N or S		8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	37°	38°	39°	40°
Δ °	′	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	50														0.0	0.0	0.1	0.1	0.1	0.1
1	00											0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1
1	10				0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
2	40		0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0	1.2	1.4	1.6	1.8	1.9	2.1	2.3	2.5
2	50	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0	1.2	1.4	1.6	1.9	2.2	2.3	2.5	2.7	2.9
3	00	0.1	0.2	0.2	0.3	0.5	0.6	0.7	0.8	1.0	1.2	1.4	1.6	1.9	2.3	2.6	2.8	3.1	3.3	3.5
3	10	0.1	0.2	0.2	0.3	0.5	0.6	0.8	0.9	1.2	1.4	1.7	1.9	2.3	2.7	3.0	3.3	3.6	3.8	4.1
3	20	0.2	0.2	0.3	0.4	0.6	0.7	0.8	1.0	1.3	1.6	1.9	2.3	2.7	3.1	3.6	3.8	4.2	4.4	4.7
3	30	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.3	1.5	1.8	2.2	2.6	3.1	3.6	4.1	4.4	4.7	5.1	5.5

CORRECTION TABLE E (always positive)

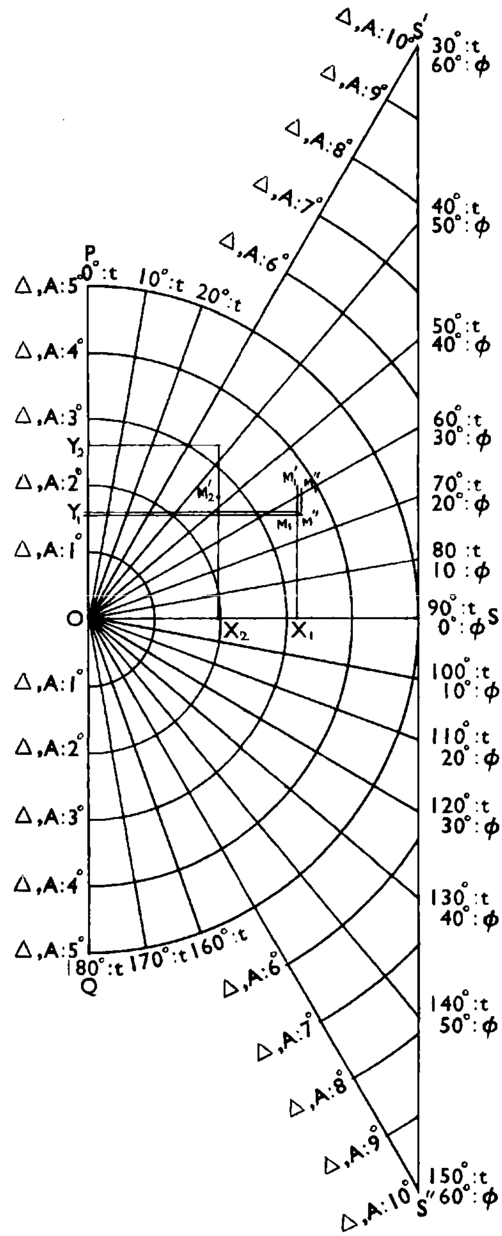
Latitude N or S		0°	4°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°
Δ °	′	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	50	0	1	3	4	5	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	00	0	2	4	6	7	8	9	10	11	13	14	15	16	18	19	21	23	25	26
1	10	0	3	6	8	9	11	12	14	15	17	19	21	23	25	27	29	31	33	36
2	40	0	16	32	39	47	56	64	73	81	90	99	109	119	129	140	151	162	175	187
2	50	0	18	36	44	54	63	72	82	92	102	112	123	134	146	156	170	183	197	212
3	00	0	20	40	50	60	70	81	92	103	114	126	138	150	163	177	191	205	221	237
3	10	0	22	45	56	67	79	90	102	115	127	140	154	167	182	197	212	229	246	264
3	20	0	25	50	62	74	87	100	113	127	141	155	170	186	202	218	235	254	273	293
3	30	0	27	55	68	82	96	110	125	140	155	171	188	205	222	240	260	280	301	323

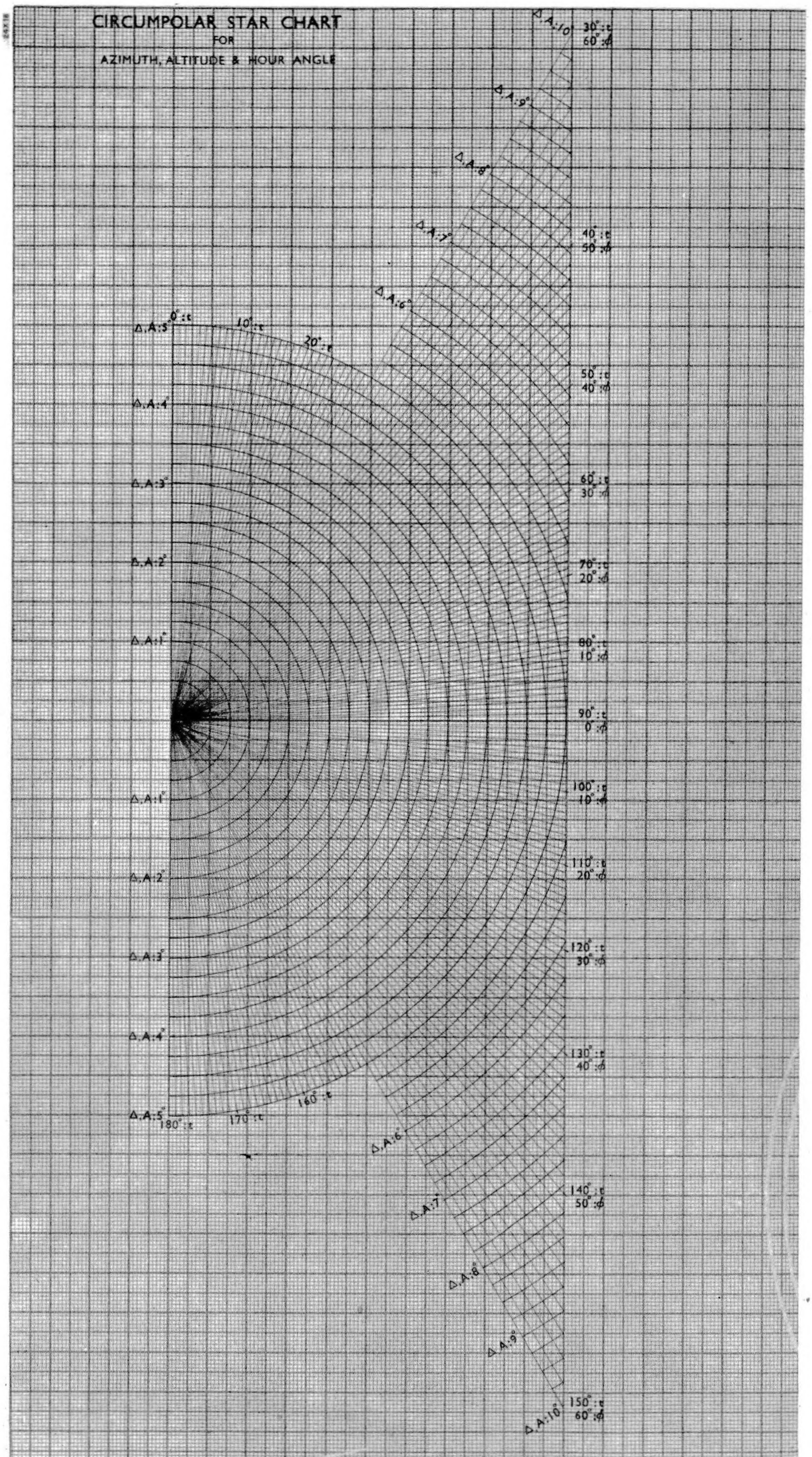
CORRECTION TABLE F (always negative)

For Δ : 50′ to 3° 30′
and x : 1′ to 30′

Δ		0°50′	0°55′	1°00′	1°05′	1°10′	2°40′	2°45′	2°50′	2°55′	3°00′	3°05′	3°10′	3°15′	3°20′	3°25′	3°30′
x ′	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
1	00	0.6	0.5	0.5	0.5	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1
1	30	1.3	1.2	1.1	1.0	1.1	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3
2	00	2.4	2.2	2.0	1.8	1.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6
2	30	3.7	3.4	3.1	2.9	2.7	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	0.9	0.9	0.9
3	00	5.4	4.9	4.5	4.2	3.9	1.7	1.6	1.6	1.5	1.5	1.5	1.4	1.4	1.3	1.3	1.3
3	30	7.3	6.7	6.1	5.7	5.2	2.3	2.2	2.2	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.7
4	00	9.6	8.7	8.0	7.4	6.9	3.0	2.8	2.8	2.7	2.7	2.6	2.5	2.5	2.4	2.3	2.3
4	30	12.1	11.0	10.1	9.3	8.7	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	3.0	2.9
5	00	15.0	13.6	12.5	11.5	10.7	4.7	4.5	4.4	4.3	4.2	4.0	3.9	3.8	3.7	3.7	3.6
5	30	18.0	16.5	15.1	14.0	13.0	5.7	5.5	5.3	5.2	5.0	4.9	4.8	4.6	4.5	4.4	4.3
6	00	21.5	19.5	18.0	16.6	15.4	6.7	6.5	6.3	6.2	6.0	5.8	5.7	5.5	5.4	5.3	5.1
6	30	25.2	22.9	21.0	19.5	18.1	7.9	7.7	7.4	7.2	7.0	6.8	6.7	6.5	6.3	6.2	6.0
7	00	29.3	26.6	24.4	22.5	19.9	9.2	8.9	8.6	8.4	8.2	7.9	7.7	7.5	7.3	7.2	6.0
7	30	33.5	30.6	28.0	25.9	24.0	10.5	10.2	9.9	9.6	9.4	9.1	8.9	8.6	8.4	8.2	8.0
8	00	38.2	34.7	31.9	29.4	27.3	12.0	11.6	11.3	11.0	10.7	10.4	10.1	9.8	9.6	9.4	9.1
8	30	43.0	39.2	35.9	33.2	30.9	13.5	13.1	12.7	12.4	12.0	11.7	11.4	11.1	10.8	10.6	10.3
9	00	48.2	43.9	40.3	37.2	34.6	15.2	14.7	14.3	13.9	13.5	13.1	12.8	12.4	12.1	11.8	11.6
9	30	53.6	48.8	44.8	41.4	38.5	16.9	16.4	15.9	15.5	15.0	14.6	14.2	13.9	13.5	13.2	12.9
10	00	59.4	54.0	49.7	45.8	42.6	18.7	18.2	17.6	17.1	16.7	16.2	15.8	15.4	15.0	14.6	14.3
10	30						20.7	20.0	19.4	18.9	18.4	17.9	17.4	16.9	16.4	16.1	15.7
11	00						22.7	22.0	21.3	20.6	20.1	19.6	19.1	18.6	18.1	17.7	17.3
11	30						24.8	24.0	23.3	22.6	22.0	21.4	20.9	20.3	19.8	19.3	18.9
12	00						27.0	26.2	25.4	24.7	24.0	23.3	22.7	22.1	21.6	21.0	20.5
12	30						29.3	28.4	27.5	26.8	26.0	25.3	24.6	24.0	23.4	22.8	22.3
13	00						31.7	30.7	29.8	28.9	28.1	27.4	26.7	26.0	25.3	24.7	24.1
13	30						34.0	33.0	32.1	31.2	30.3	29.5	28.7	28.0	27.3	26.6	26.0
14	00						36.6	35.5	34.5	33.5	32.6	31.7	30.9	30.1	29.4	28.6	28.0
14	30						39.3	38.1	37.0	35.9	34.9	34.0	33.0	32.3	31.5	30,7	30.0
15	00						42.1	40.8	39.6	38.4	37.4	36.3	35.4	34.5	33.7	32.9	32.1
15	30						44.9	43.5	42.3	41.0	39.9	38.8	37.8	36.8	35.9	35.0	34.3
16	00						47.9	46.4	45.0	43.7	42.5	41.4	40.3	32.2	38.3	37.3	36.4
16	30						50.9	49.4	47.9	46.5	45.2	44.0	42.8	41.7	40.7	39.7	38.7
17	00						53.9	52.4	50.9	49.4	48.0	46.8	45.5	44.3	43.2	42.1	41.1
17	30						57.2	55.4	53.9	52.3	50.9	49.5	48.2	47.0	45.8	44.7	43.6
18	00						60.5	58.7	56.9	55.4	53.8	52.4	51.0	49.7	48.4	47.3	46.1
18	30						63.9	62.0	60.1	58.4	56.8	55.3	53.9	52.5	51.2	49.9	48.7
19	00						67.4	65.4	63.5	61.6	59.9	58.3	56.8	55.4	54.0	52.7	51.4
19	30						70.9	68.9	66.8	64.9	63.1	61.4	59.8	58.3	56.9	55.5	54.2
20	00						74.6	72.4	70.3	68.3	66.4	64.6	62.9	61.3	59.8	58.4	57.0
20	30						78.4	76.0	73.8	71.8	69.8	67.9	66.1	64.4	62.8	61.2	59.9
21	00						82.2	79.8	77.5	75.2	73.2	71.2	69.4	67.6	65.9	64.3	62.7
21	30						86.2	83.6	81.2	78.9	76.7	74.6	72.7	70.8	69.1	67.4	65.7
22	00						90.3	87.5	84.9	82.6	80.3	78.1	76.0	74.2	72.3	70.5	68.9
22	30							91.6	88.9	86.3	84.0	81.7	79.5	77.5	75.6	73.8	72.0
23	00							95.6	92.9	90.2	87.7	85.4	83.1	81.0	79.0	77.1	75.3
23	30								96.9	94.2	91.6	89.1	86.8	84.6	82.4	80.4	78.6
24	00								101.1	98.1	95.5	92.9	90.4	88.2	86.0	83.9	81,9
24	30									102.3	99.5	96.8	94.3	91.8	89.6	87.4	85.3
25	00									106.5	103.6	100.7	98.2	95.6	93.2	91.1	88.9
25	30										107.8	104.8	102.1	99.5	97.0	94.6	92,5
26	00										112.0	109.0	106.1	103.4	100.9	98.4	96.0
26	30											113.2	110.3	107.4	104.7	102.2	99.8
27	00											117.5	114.4	111.5	108.7	106.1	103.6
27	30												118.7	115.6	112.8	110.0	107.5
28	00												123.0	119.9	116.9	114.1	111.4
28	30													124.1	121.1	118.1	115.4
29	00													128.5	125.3	122.3	119.4
29	30														129.7	126.5	123.6
30	00														134.0	130.8	127.7

CIRCUMPOLAR STAR CHART
FOR
AZIMUTH, ALTITUDE & HOUR ANGLE




CIRCUMPOLAR STAR CHART
FOR
AZIMUTH, ALTITUDE & HOUR ANGLE
Δ.A:10°
Δ.A:9°
Δ.A:8°
Δ.A:7°
Δ.A:6°
Δ.A:5°
Δ.A:4°
Δ.A:3°
Δ.A:2°
Δ.A:1°
0°:t
10°:t
20°:t
30°:t 60°:φ
40°:t 50°:φ
50°:t 40°:φ
60°:t 30°:φ
70°:t 20°:φ
80°:t 10°:φ
90°:t 0°:φ
100°:t 10°:φ
110°:t 20°:φ
120°:t 30°:φ
130°:t 40°:φ
140°:t 50°:φ
150°:t 60°:φ
160°:t
170°:t
180°:t