A SYSTEM OF MORPHOMETRY

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ABSTRACT

This system is based on a square lattice superimposed on a map of the lake. Area, volume, depth distribution, length of shoreline and other morphometric concepts are derived simply. The system has the advantage that the precision of the measurements can be estimated. The method is simple, rapid and requires no special instruments.

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Morphometry begins with a map of a body of water which can be an ocean, a lake, pond, bay, river, or any part of one of these. For convenience in description we shall use the term lake to denote the water body under consideration whether this be a puddle or an ocean. From the map of the lake certain abstractions are obtained such as area, mean depth or length of shoreline. These abstractions serve various purposes, chief of which are the aid in characterizing or describing the lake and the aid in comparing the given lake with other lakes. Obviously, these two purposes are closely related.

There is a parallel between morphometry and statistics, the proper understanding of which can be of value in assessing the techniques used in morphometry. Just as the statistician, in comparing two sets of data, is careful to determine the conditions under which the data were taken, so should the worker in morphometry take pains to be aware of, to specify, or to describe the origin, accuracy and precision of the data with which he works.

The system of morphometry proposed here is believed to have the advantage that the accuracy and precision of the derived concepts can be estimated, thus leading to a more meaningful and useful comparative morphometry. It has the further advantage that no special instruments, such as planimeter or map measurer, are required and that the appplication is likely to be easier, quicker and less susceptible to *blunders* than conventional methods. One further advantage is that it is adaptable to punch card methods.

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This system was first suggested by the author in 1951 and in 1952 it was distributed in mimeograph form as Contribution 21 from the Oceanographic Institute, Florida State University.

CERTAIN WEAKNESSES OF PRESENT MORPHOMETRIC METHODS

The great diversity of the procedures involved in a morphometric study constitutes a troublesome and embarrassing weakness. The area can be determined by polar planimeter, hatchet planimeter, Simpson's Rule, counting squares, etc. Each of these methods has its own accuracy and precision in addition to the personal error of the operator. The length of shore can be estimated by a *map measurer, thread and pins* method, stepping off segments with dividers, etc. Here too, each method has its inherent precision, accuracy and liability to personal error. However, none of the methods used for measuring shoreline is related to a method of measuring area. Since in any serious work, the expected errors should be estimated and stated, it will be necessary to do so separately for each morphometric *statistic*. The fact that this is practically never done in published works attests to this weakness of present methods.

An even more serious source of difficulty is the effect of map scale. This is most easily demonstrated in computing length of shoreline. As the scale decreases, more and more irregularities appear and the estimated length of shoreline will increase. As the scale decreases, eventually boulders, rocks and even grains of sand will be represented on the map and obviously a halt must be made some place. The exact position of the level of detail which must be ignored should depend upon the purpose of the investigation at hand. A real-estate operator interested in selling lakeshore property will probably want details down to 5 or 10 feet. At the other extreme might be a railroad cartographer. A limnologist working on the productivity of littoral regions may demand high detail; but if interested only in benthic regions, he may be satisfied with the crudest approximation.

A workable, consistent system of morphometry must recognize the varying requirements of accuracy and precision demanded by workers in various fields, for it is manifestly impracticable and unfair to demand the same accuracy and precision of overyone. On the other hand, it is essential that each worker state the accuracy and precision of his published results.

PROCEDURE

It is assumed that a map of the lake is available, that depths are shown either as contours or as representative points, and that the scale of the map is known.

Step 1. — Lay a square grid upon the map. This can be done by drawing equally spaced horizontal and vertical lines on the map, by laying on the map a sheet of tracing paper with a grid ruled on it; by laying both map and a sheet of rectangular cross-section paper on a tracing table; or by use of a map template made of clear plastic sheet with uniformly spaced holes.

The matter of grid spacing requires some judgment. In practice, the spacing may vary from 0.2 to 0.5 inch. For elongate lakes, it is preferable that the axis of the lake be inclined to the grid lines and not parallel to one set.

Step 2. — Determine the grid spacing. Denote this by g. From the scale of the map, determine the distance the grid spacing represents. Denote this by u. Determine the area represented by a unit square of the grid. Denote this by a. Obviously, $a = u^2$.

For example, if the grid spacing is 0.5 inch and one inch on the map represents 4 miles, $g = 0{,}5$ inch, $u = 2$ miles, $a = 4$ square miles.

Step 3. — Prepare the Depth Matrix. The grid as placed on the map consists of lines, points of intersection of horizontal and vertical lines, and unit squares. It is a characteristic of this method that neither the lines nor the squares are employed, only the points of intersection. These points will henceforth be called *lattice points*. Counting points inside a boundary is far easier than counting squares since there is no problem of estimating fractions of the unit squares along the boundary. To be sure, some accuracy is lost in so doing but there is likely to be a compensating reduction in *blunders*.

At each lattice point the depth is noted and recorded in a table or *matrix* so that the order of the numbers in the matrix corresponds to the order of the corresponding lattice points. A hypothetical depth matrix constructed purely for illustrative purposes is given in table 1.

TABLE I

Hypothetical Depth Matrix

Depths in feet. $u = 2$ miles, $a = 4$ sq. mi.

0	0	0	0	0	0	0	0
0	1	0	2	2	1	0	0
0	1	2	4	6	3	1	0
0	0	1	2	5	2	0	0
0	0	0	1	4	3	0	0
0	0	1	2	2	0	0	0
0	0	0	0	1	0	0	0

The concept of Depth Matrix is introduced for the purpose of making the set of operations precise, thus not only permitting more rigorous mathematical methods used in further treatment of the data but also facilitating the adaption of the procedure to punch card technique. As a practical matter, depths can be written directly on the map or tracing paper. The numerous zero depths corresponding to regions outside the lake boundaries can be ignored.

In practice, depths must be obtained by interpolation from contours or representative points.

Step 4. — Determine length of shoreline. This is done by counting the number of changes from or to zero depth in each row and column of the matrix. For example, in table 1, in the first row there are no changes. In the second row there are four changes : from 0 to 1 ft, from 1 to 0 ft, from 0 to 2 ft, and from 1 to 0 ft. In the 3rd, 4th, 5th, 6th, and 7th rows there are 2 changes in each. Thus, in horizontal rows there are 14 changes.

Vertically there are no changes in the first column, 2 in the 2nd, 4 in the 3rd, 2 in the 4th, 1 in the 5th, 2 in the 6th, 2 in the 7th and none in the 8th. Thus, in the columns, there are 13 changes in all. The total number of changes is then $14 + 13 = 27$.

The length of shoreline S is found by multiplying the total number of changes by 0.785 u. In our example, we have

$$S = 27 \times 0.785\, u = 27 \times 0.785 \times 2 = 42.4 \text{ miles.}$$

As in Step 3, this seemingly cumbersome procedure is introduced for convenience in mathematical analysis. However, for most purposes we can ignore the changes from zero depth and simply record the number of times the shoreline intersects the horizontal lines and the vertical lines. The sum of these intersections is multiplied by 0.785 u to obtain the length of shoreline.

One troublesome feature of the method of counting intersections is the problem of how to count a point of tangency. Logically, this should be half an intersection, but practically it becomes cumbersome to try to carry along half numbers in counting. This can be avoided by the artifice of rounding off to the nearest even integer. Thus, after counting 137 intersections, a point of tangency will be counted as one point, raising the count to 138. A point of tangency following 142 intersections is ignored.

Step 5. — Prepare depth and volume distribution tables. The procedure is illustrated by using the example in table 1. Results are given in table 2.

Column A merely lists the depths occurring in table 1 and column B lists the frequency of occurrence. Column C is obtained by multiplying the entries in A by the entries in B. Column D is obtained by multiplying the entries in B by the area a of the unit square. This gives the area-depth distribution. Thus, 32 sq mi have a depth of 1 ft, 28 sq mi a depth of 2 ft, etc. Depth-volume distribution is computed in column E. The factor $5\,280^2$ converts the units of a into square feet so that the numbers in the E column are cubic feet. Columns F and G are cumulative totals of columns D and E, respectively. Columns H and I are the cumulative totals of columns F and G expressed in per cent.

TABLE II

Depth and Volume Distribution

A	B	C	D	E	F	G	H	I
Depth	Frequency	$A \times B$	$B \times a$	$C \times a \times 5280^2$	D cumulative	E cumulative	% H	% G
1	8	8	32	8.92×10^8	32	8.92×10^8	38.1	16.7
2	7	14	28	16.73×10^8	60	25.65×10^8	71.4	47.9
3	2	6	8	6.69×10^8	68	32.34×10^8	80.1	60.4
4	2	8	8	8.92×10^8	76	41.26×10^8	90.5	77.1
5	1	5	4	5.58×10^8	80	46.84×10^8	95.2	87.5
6	1	6	4	6.69×10^8	84	53.53×10^8	100.0	100.0

Modification for non-linear maps. — For the average-sized lake, the distortion produced by mapping a part of a spherical surface upon a flat surface can be ignored. When the distortion may be appreciable, as in the Gulf of Mexico or the Baltic Sea, draw the grid on the map so that the grid spacing represents a constant length. Thus, in an example taken from the Gulf of Mexico, u was 20 km; at 30° N, g was 8.6 mm and at 20° N, g was 8.5 mm. With the grid drawn in this fashion, the procedure is exactly the same as in Steps 1 to 5.

NOTE. — At this point all the basic data have been obtained. The length of shoreline was found in Step 4, the total area is the last entry in column F and the total volume is the last entry in column G. The remaining part of this procedural outline will be devoted to various derived concepts.

Hypsographic curve : depth-area curve. — This is simply obtained by plotting the cumulative areas in column F against the depths in column A (table 2). The per cent hypsographic curve is obtained by converting the areas in column F into per cent as in column H. In the example, 84 sq mi will correspond to 100 %.

Median depth. — The depth corresponding to 50 % of the area is the median depth. It is found immediately on the per cent hypsographic curve. In our example, it is about 1.7 ft.

Depth-volume curve. — This is allied to the hypsographic curve. It is obtained by plotting the cumulative volumes in column G against the depths in column A. By expressing the volumes in per cent as in column I, the depth-per cent volume curve is obtained.

Volume-median depth. — This is found directly from the 50 % volume point on the depth-per cent volume curve. Care must be taken to distinguish the mean depth from the volume-median depth.

Mean depth. — This is simply the volume divided by the area.

Shore development. — For lakes, the shore development is computed from the equation :

$$s = \frac{S}{2\sqrt{\pi A}}$$

where s = shore development, S = length of shoreline and A = area. Both S and A must be expressed in consistent units. That is, if S is given in miles, A must be expressed in square miles.

For bays, this equation is not well defined and it is suggested that the method proposed by OLSON (1952) be used.

Mean slope of entire lake bottom. — There are several methods for estimating the mean slope. Those based on measuring the lengths of the depth contours (WELCH, 1948) are tedious to apply and involve the drawing and measuring of many contour lines as well as computing areas between the contours. The following suggested method has the advantage of ease and simplicity and does not seem to be so greatly affected by *pathological* situations (e. g., very abrupt drops in the lake bottom). Further, it is based on the simplest definition of average slope : the average change in depth per unit horizontal distance.

Step 1. — The method will be illustrated by the example in table 1. The first line is obviously not on the lake so it will not be used. In the second line there is a change of 1 ft in going from 0 to 1, a change of 1 ft in going from 1 to 0 (for slope calculations, all changes are considered positive), a change of 2 ft in going from 0 to 2, a change of 0 ft in going from 2 to 2 (this indicated a flat place on the lake bottom and it *must* be included in slope calculations), a change of 1 ft in going from 2 to 1, a change of 1 ft in going from 1 to 0. Continuing to the final lattice point of 0, we note that this region is *not* in the lake and it is not counted. The changes in the second line are thus 1, 1, 2, 0, 1 and 1 In the third line we have 1, 1, 2, 2, 3, 2 and 1. The procedure is continued for all horizontal lines.

Step 2. — The same procedure is used on the vertical lines. The first and last columns are not on the lake and are not counted. In the second column we have 1, 0 and 1.

Step 3. — Find the average change by adding all horizontal and vertical changes and dividing by the total number of changes (zeros included). Thus, horizontally :

1, 1, 2, 0, 1, 1
1, 1, 2, 2, 3, 2, 1
1, 1, 3, 3, 2
1, 3, 1, 3
1, 1, 0, 2
1, 1

total = 42-ft change in 28 steps

vertically :

1, 0, 1
2, 1, 1, 1, 1
2, 2, 2, 1, 1, 2
2, 4, 1, 1, 2, 1
1, 2, 1, 1, 3
1, 1

total = 39-ft change in 27 steps.

Therefore, there is a total of $42 + 39 = 81$ ft of change in $28 + 27 = 55$ steps, or $81/55 = 1.47$ ft per step. Since each step equals the grid spacing, or 2 miles, the average slope is 1.47 ft/2 miles or 0.74 ft per mile.

Mean slope of shore to a given depth. — In certain cases it may be desired to compute the average slope of the shore to a given depth (e. g., the compensation depth). Practically equivalent to this problem is the determination of the average distance from shore to this depth since the average slope can be defined as the given depth divided by the average horizontal distance to that depth.

The mean slope can be estimated with little difficulty and reasonably good accuracy if the following conditions hold : 1) the length of the contour at the given depth is no less than 80 % of the length of shoreline; and 2), there are no *isolated* shoals having depths less than the given depth.

If d is the given depth and S the length of shoreline, the volume V of water at depths less than or equal to d is given by

$$V = \frac{x\,S\,d}{2}$$

where x is the mean distance to the contour at depth d. Therefore,

$$x = \frac{2V}{S d}$$

Since d is given, S is known and V can be found directly from the depth-volume curve, x can be found with no difficulty. The mean slope to depth d is then

$$k = \frac{d}{x}.$$

SUGGESTED MANNER OF PRESENTING AND USING DATA

In order that morphometric data be presented in such form that meaningful comparisons can be made, it is suggested that they be presented as in table 3.

The first line gives the name of the lake and its location with sufficient detail so that one can find it in an atlas. This is hardly a stringent requirement, yet it is too frequently overlooked.

The second line tells what map was used and its date. The source of the soundings and their dates should either be given or some reference should be made to where such information can be obtained. In this case, the USCGS is a sufficient reference.

No comment seems necessary for the third line which merely gives the scale of the map. However, the datum level appearing in the fourth line is all important. Failure to take this statistic into account can cause much mischief. Since Crescent Lake joins the Atlantic, soundings are referred to mean low water level. Datum levels for inland lakes are usually referred to mean sea level. For example, depths in Lake Erie may be given in terms of the datum level 571.5 ft above mean sea level at New York. For smaller inland lakes and ponds, it may be neither convenient nor possible to determine the datum level with respect to sea level or any other known reference level. In that event, every effort should be made to find a suitable local reference level : a stake driven into the ground at a known location, the top of the foundation of a nearby house, a cross chiselled on a cliff or large rock — it matters little what it is so long as it is permanent and can be found when needed.

The grid spacing in the fifth line is a measure of the *precision* of the derived data. In this case, the spacing is 2.222 ft and we then know that shore features considerably smaller than this are ignored. For example, no effort is made to consider coves whose dimensions are of the order of 300 ft. To be sure, a lattice point may fall within such a cove and therefore must be counted, but such events are likely to be cancelled by a lattice point falling on a spit of the same magnitude. If we are making comparisons of absolute lengths of shoreline, it is important that the grid spacings are of the same magnitude.

The maximum length is not always clearly defined and sometimes the maximum effective length is the more reasonable datum. WELCH (1948) discusses these concepts.

TABLE III

Morphometry of Crescent Lake, Putnam County, Florida.

Map used : United States Coast and Geodetic Survey Chart # 686, corrected to 8 May 1950.
Scale : 1/40 000.
Datum level : mean low water.
Grid spacing : 2 222 ft.
Maximum length : 12.5 miles.
Average width : 2.07 miles.
Area : 25.86 square miles (146 points).
Volume : 6 183 $\times$ 10^6 cubic feet.
Maximum depth (on chart): 14 ft.
Mean depth : 8.6 ft.
Length of shoreline: 31.4 miles (52 + 43 points).
Shore development : 1.74.

The average width (in this table) is the area of the lake divided by the maximum length. Here too, there are many instances when the average width is meaningless. It is suggested that when this is the case, it should be so stated in the table.

When presenting data on the area, if islands are present, that fact must be mentioned. It should also be stated whether the area of the islands is included or not. If sizable bays are present, it is important to state whether or not they are included. If not, the boundaries between the lake and bays must be given unless obvious from the map. For example, in the Black Sea, the Sea of Azov forms a significant *gulf* as does the Gulf of Kara-Bugaz in the Caspian Sea. In both cases, the boundaries are fairly obvious and unless great accuracy is necessary, there is little need to define them. Hudson Bay presents a different situation. If James Bay is to be excluded, the boundary between it and Hudson Bay must be defined. The seaward boundaries of Hudson Bay must also be given.

The number of points used to determine the area is a measure of the *accuracy* of the determination. While it may be possible to derive mathematically from probability considerations the probable errors in the area measurements in terms of the number of points involved, such a procedure is likely to be more difficult and tedious than the results might justify. The writer has used an empirical method of measuring known areas of various sizes and shapes by the number of included lattice points. It was found that the probable error is inversely proportional to the number of points. That is, for 50 points the error is likely to be less than 2 %; for 1 000 points, the error is likely to be less than 0.1 %. The term *probable error* has been used loosely in this discussion but it is believed that the meaning has not been greatly violated.

The volume given in the table refers to the total volume of the lake below the datum level. In applications, it should be remembered that frequently the datum level chosen is a low water level and that most of the time the lake level is above this.

Maximum and mean depths are likewise referred to the datum level. The maximum depth in the table is the maximum depth on the chart. It may or may not be the maximum depth at the lattice points.

The length of shoreline is likewise determined to an accuracy dependent upon the number of points. In this case, the total number is 95 points and therefore the accuracy is about 1 %. The reason it is suggested that the number of intersections with vertical and horizontal lines be given separately is that the accuracy seems to fall off when the two numbers differ widely. From experience, the writer feels that as long as the ratio of the larger to the smaller number is less than 2/1, there is no cause for concern. If the ratio is between 4/1 and 2/1, the error may be double that estimated from the total number of points. The theoretical basis of this method of measuring length of shoreline has been given by OLSON (1950). The length of shoreline as computed in Step 4 will, in practice, be less than the length computed by the intersections of horizontal and vertical lines. While this is actually not a weakness of the method, care must be taken in preparing the summary table to indicate whether the shoreline is computed by *points* of intersection or by *zero changes* in the depth matrix.

Since the shore development is a measure of the distortion of the lake outline from a circular form, in making comparisons the lengths of shorelines of the two lakes should be computed to the same order of accuracy. For example, in comparing Reelfoot Lake with Lake Tanganyika, the relative detail should be the same in both cases. That is, if in Reelfoot Lake we ignore a cove whose width is 1/1 000 the length of the lake, in Lake Tanganyika we should ignore a bay whose width is 1/1 000 the length of the lake. Therefore, the total number of points used to measure the shoreline should be about the same in both cases.

It is conceivable, however, that the concept of shore development might be used in a different sense than a mere distortion from the circular form. For example, a limnologist may be interested in the productivity of the psammo-littoral zone and wishes to compare the length of shoreline per unit-length of the lake with that of another lake. In this case, the ratio of shore developments is applicable but here the shorelines must be measured with the same absolute detail. Therefore, the grid spacings and not the total number of points must be comparable.

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