TRAVERSE IN FRENCH HYDROGRAPHIC SURVEYS BY GEODIMETER AND TELLUROMETER

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In hydrographic surveying, when there is no pre-established triangulation, the determination of the coordinates of the signals to be used as control points for operations at sea and for the topographic survey of the shore area requires observation of a chain of triangles along the coast. This is a lengthy, difficult and costly operation in countries where communications are inexistent. An example is certain tropical areas, where circulation is difficult, and the transportation of equipment for setting up signals and observational instruments even harder. The necessity for the economic development of these areas and the encouraging prospects afforded by many of them have resulted in numerous requests for hydrographic surveys. It was essential that such demands be met rapidly, as the surveys largely affected subsequent projected investigations. The Hydrographic Office has consequently been led in recent years to search out methods for reducing the time and cost of standard geodetic operations, while preserving with the new procedures an acceptable accuracy for surveying purposes and for the work scheduled to be carried out later by other organizations, in particular those concerned with oil prospecting.

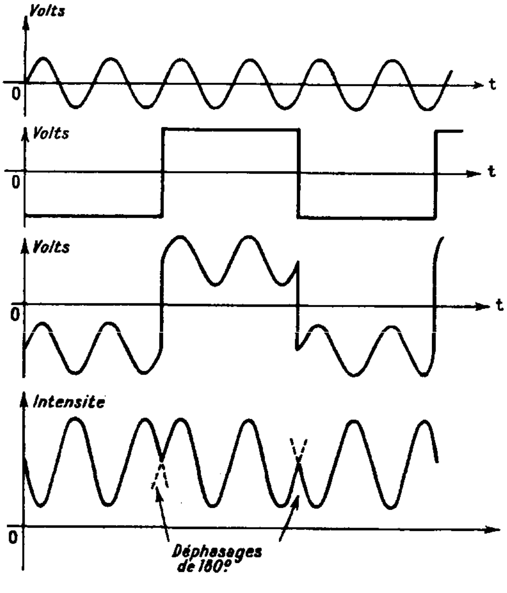
Utilization of the geodimeter and tellurometer in the French hydrographic missions answers such needs. It was discovered that such instruments could be used in the rapid measurement of lines several tens of kilometres in length with sufficient accuracy, and that geodetic work could be lightened by combining standard measurements of horizontal angles with length measurements, or even by effecting an actual traverse over several hundred kilometres, by taking advantage of the possibility of proceeding along the coast, walking the beaches or landing on the shore.

I. — TRAVERSE BY GEODIMETER

(a) *Principle of geodimeter*

The concept and realization of the geodimeter are due to a Swedish geodesist, Bergstrand, who adapted and modernized the method known as the *cogwheel method* used formerly by Fizeau to measure the velocity of light. It is well known that this method consists in determining the time taken by the light wave to travel along a base of given length. The reverse process is accomplished with the geodimeter : assuming that the velocity of light is known, the length of the base is derived by measuring, indirectly, it is true, the travel time.

Operations take place at night. The instrument is placed at one end of the distance to be measured, whence the other end must necessarily be seen (*). From a light source L a parallel set of rays is emitted, the intensity of which is modulated twice, first at a high frequency *f* of about 1 500 kcps, then at a low frequency F of 50 cps. Both modulations are superimposed, and under the influence of the slow modulation, which in theory is of rectangular shape (⊓⊔⊓⊔⊓), the rapid modulation is dephased 100 times per second by 180°.



Alternating current producing rapid modulation

Alternating current producing slow modulation

Current at Kerr cell terminals produced by two superposed modulations

Intensity of light as function of square of voltage at Kerr cell terminals

The double modulation is accomplished through the agency of a Kerr cell placed between two polarizers N_1 and N_2 crossed at right angles. Alternating currents of appropriate frequency and amplitude are applied to the terminals. The rapid modulation is produced by an oscillator Cr, stabilized by a crystal placed in a casing maintained at constant temperature (50° C) by thermostat.

The Kerr cell consists of a parallelipiped bowl with glass sides, which contains nitrobenzine in which a flat condenser is immersed. The light passes between the armatures of the condenser, which are connected with the cell terminals. When the instantaneous voltage at the terminals is zero, the light at L, first polarized by N_1, is then turned off by N_2; when the voltage is other than zero, the liquid becomes birefringent and the light is transmitted at a voltage which is a function of the squared instantaneous

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(*) The following description and numerical values refer to the type NASM III geodimeter.

voltage. The voltage of the very-high-frequency beam can thus be modulated, without inertia.

By means of an appropriate optical arrangement, the beam is directed onto a reflector M (a plane, spherical or prism reflector) placed at the other end of the distance to be measured; it is reflected by M and returns to the geodimeter where it strikes the cathode of a photoelectric cell Ph.

The anode voltage of this cell, and hence its sensitivity, are modulated at the same high frequency f as the beam, but with a phase difference that may be varied by actuating a variable delay line R inserted in the anode circuit. A direct-current microammeter μ on the anode circuit measures, after amplification, the output current of the photocell.

The expression which gives in terms of time t the intensity of the light emitted by the geodimeter is a complex one. In simplified theory, we can assume that a pure sinusoidal modulation (*) is involved, represented by an expression of the form $A \pm B \sin 2\pi ft$, in which the + sign corresponds to the emission in the positive half-cycle of the slow modulation, and the — sign to the emission in the negative half-cycle, when the fast modulation is dephased by 180°.

Under these conditions the beam reflected by M which reaches the cathode of cell Ph has an intensity equivalent at time t to

$$J = A \pm B \sin 2\pi f \left(t - \frac{2D}{v}\right),$$

v being the velocity of light, D the length to be measured. In addition, the sensitivity of cell Ph at time t is of the form $S = C + D \sin 2\pi f(t - t_0)$, t_0 representing the phase difference between the sensitivity of Ph and the intensity of the light upon emission, a phase difference which is due to the inserted circuits, and in particular to the delay line R.

The instantaneous output current of cell Ph may be assumed proportional to the product J.S., i.e. to

$$\left[A \pm B \sin 2\pi f \left(t - \frac{2D}{v}\right)\right] \left[C + D \sin 2\pi f (t - t_0)\right]$$

A sine product appears in this expression and may be broken down to a difference in cosine. It will be seen that the expression is the sum of a constant term, of sine and cosine terms varying rapidly with time, and of a cosine term not dependent on time and of argument $2\pi f \left(\frac{2D}{v} - t_0\right)$. The D.C. microammeter μ of the anode circuit is unaffected by the current which the terms varying rapidly with time represent. The difference of two direct currents of the form :

$$i_+ = I_0 + I_1 \cos 2\pi f \left(\frac{2D}{v} - t_0\right)$$

$$\text{and } i_- = I_0 - I_1, \cos 2\pi f \left(\frac{2D}{v} - t_0\right),$$

the first corresponding to the positive half-cycle, and the second to the negative half-cycle of the slow modulation, alone is recorded.

(*) This amounts to neglecting the harmonics in the FOURIER series giving the intensity as against time. Actually the same conclusions are reached when they are taken into account.

Finally the intensity indicated by the microammeter μ is written by introducing the wavelength $\lambda = \frac{v}{f}$:

$$I = 2\,I_1 \cos \frac{2\pi}{\lambda} (2D - vt_0) .$$

It is ascertained that this intensity depends on D; it is zero if

$$D = \frac{vt_0}{2} + \frac{\lambda}{4}\left(k + \frac{1}{2}\right),$$

k being any integer.

This being granted, the delay line is adjusted at the factory (the operator can perfect the adjustment before beginning the measurements) so that $\frac{v\,t_0}{2} + \frac{\lambda}{8} = 0$. Under such conditions the intensity I cancels out whenever D is an integer k of one-quarter wavelength.

If D be any distance, it may be assumed that it is equal to an integer k of one-quarter wavelength plus a fraction a of $\frac{\lambda}{4}$. The current I is then other than zero, and theoretically its measurement should enable the fraction a to be determined. In actuality a zero method is used which consists in varying the delay line until I cancels out; the dephasing required to be applied in order to obtain this result then gives the fraction a being sought.

It should be noted that the measurement thus effected is very accurate, as around zero value the intensity of the current at the microammeter varies rapidly.

Measurement of a supplies the distance D if the latter is known to within $\frac{\lambda}{4} = 50$ metres, which is frequently the case. When it is not so known, k, the one-quarter wavelength integer contained in D, is determined by taking a second measurement with a rapid modulation f_1 slightly different from f. The f_1 frequency, which in the NASM III geodimeter is in the neighbourhood of 1 540 kc ($\lambda_1 \simeq 195$ metres), may easily be introduced into the instrument by changing the stabilizer quartz in the oscillator Cr.

Designating the one-quarter wavelength integer and the fraction relating to the second measurement respectively as k_1 and a_1, we get :

$$(k + a)\frac{\lambda}{4} = (k_1 + a_1)\frac{\lambda_1}{4}$$

written as follows :

$$k = (k_1 - k)\frac{\lambda_1}{\lambda - \lambda_1} + \frac{a_1\lambda_1 - a\lambda}{\lambda - \lambda_1}$$

a relationship which supplies k provided its approximate value to within $\frac{\lambda}{\lambda - \lambda_1} = 40$ units is known, or, which amounts to the same thing, provided D to within $1/4\,\frac{\lambda\lambda_1}{\lambda - \lambda_1} = 2\,000$ metres approximately is known.

In order to measure D with accuracy, λ must be known, and hence the velocity of light v under the particular test conditions.

We know that $v = \frac{C}{n}$, C being the velocity of light *in vacuo* : (C = 229 792.1 ± 0.2 km per sec) and n the refraction index of the atmosphere. The value of n is given by an empirical formula involving absolute temperature, atmospheric pressure and water vapour pressure (*).

It will be observed that accuracy of measurement depends on the indications of the delay line, which must be carefully constructed in order to ensure stability, and perfectly calibrated. This circuit is in two sections: the first contains ten identical fixed elements of 5 metres *length* which are successively introduced by push buttons; the second is a variable constituent from 0 to 5 metres in *length* supplying the fine measurement, which is read from a dial.

The operator himself can check the accuracy of the indications of the variable element, by using an optical calibration track included in the equipment.

(b) *Measurements with the geodimeter*

In the type of geodimeter used, the makers indicate that the error (in metres) in the measurement of D (in metres) is :

$$\varepsilon = 0.1 + 4 \cdot 10^{-6} \mathrm{D} .$$

In order to check this claim, the base at Dakar, which is 5 878.36 metres long as measured by invar tape, was measured by the instrument in April 1958. The length found by geodimeter was 5 878.25 metres, showing an absolute error of 0.11 metre and a relative error of approximately 1 part in 60 000, whereas the makers' formula indicates a relative error of 1 in 50 000.

Six other comparative measurements were made along 10-kilometre sides of a triangulation carried out by a petroleum company in Gabon. Agreement is comparable, but here comparison only, instead of calibration, is involved, as the length of the sides in the triangulation were not known to within an accuracy above 1 in 50 000.

As a result of these tests, it was assumed that an accuracy of 1 part in 50 000 for lengths approaching 10 kilometres was ensured. It should be noted that work carried out in Sweden in 1957 and 1958, which included 496 ranges representing a total length of 2 950 km, gave a relative accuracy varying between 1 in 100 000 and 1 in 140 000.

The geodimeter was then systematically used in hydrographic surveying, first at the mouth of the Casamance River where measurements of horizontal angles were combined with range measurements along a traverse line twice oriented by azimuths taken on Polaris.

It was then widely used on the Gabon coast where traverse was undertaken which, upon completion, will connect Port Gentil with Pointe Noire over an approximate 600-kilometre distance. At the present time 155 kilo-

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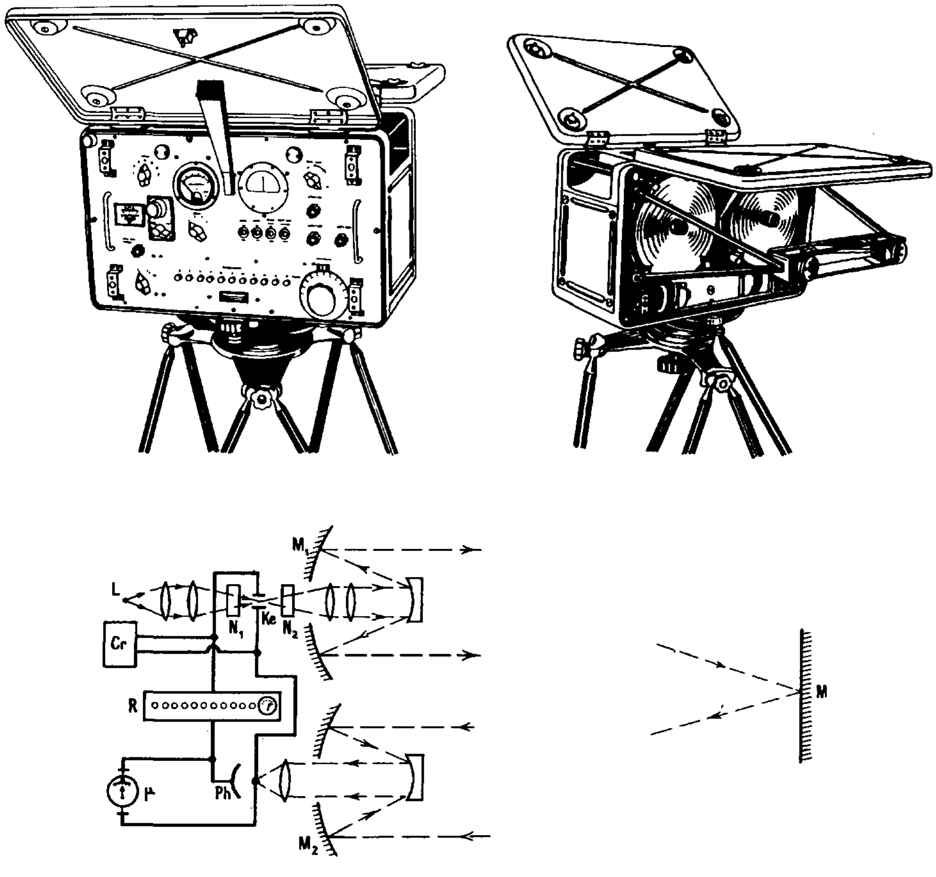
(*) SMITH and WEINTRAUB give the empirical formula :

$$(n - 1)\, 10^{6} = \frac{77.6}{\mathrm{T}} \left(p + 4.81 \times 10^{3} \frac{e}{\mathrm{T}}\right)$$

(T absolute temperature; p atmospheric pressure in millibars; e water vapour pressure in millibars).

metres have been measured from Port Gentil to Iguela. The extremities of the azimuth lines, which number thirty, are marked by concrete pillars used as supports for the instruments. The angles at the extremities have been carefully measured with a Wild T3 theodolite, and several measurements of astronomical azimuth have been scheduled throughout the work. In the north, the figures will be connected by triangulation with Libreville, and southwards with Pointe Noire. In the middle it will, in addition, be connected with an astronomical position observed in 1957.



Type NASM III Geodimeter

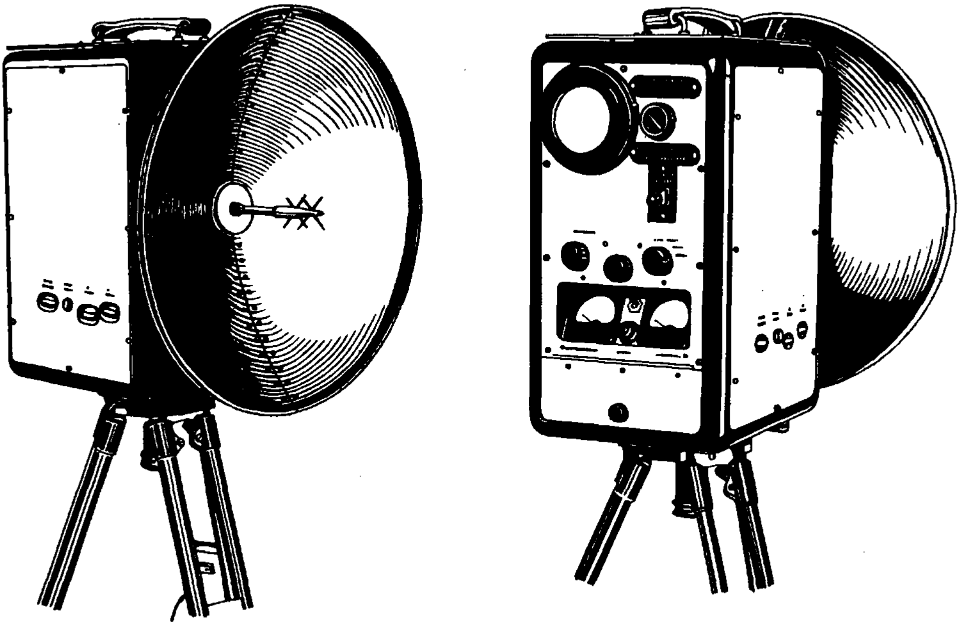
During all this work difficulties were encountered due to the poor adaptability of the equipment to operations in the tropics. Another difficulty was due to the poor visibility on the shore caused by the spray from waves.

Moreover, blue and violet radiations, those for which the photocell shows maximum sensitivity, are thoroughly absorbed, which reduces the range of the instrument. In fact the longest distance measured was 12 545 metres. The average relative accuracy of the distance measurements was 1 part in 90 000.

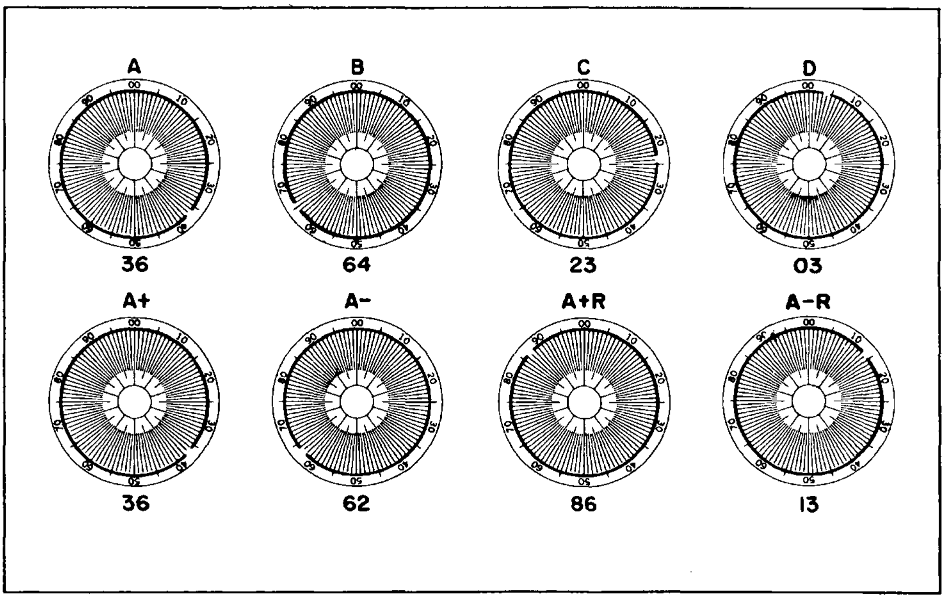
II. — TRAVERSE BY TELLUROMETER

(a) *Principle of tellurometer*

In this instrument also, distance is determined by measuring the travel time of light, assuming the velocity of light to be known at the time of measurement.



Tellurometer



Example of readings at master station

The tellurometer consists of a master station and a slave station located at either end of the distance to be measured D.

Each station is a transceiver of decimetric hertzian waves with a directional aerial (dipoles at the centre of a parabolic reflector). A suitably modulated continuous wave is transmitted by the master station; it is received at the slave, immediately retransmitted, and returns to the master station which it reaches with a phase difference. Measurement of the phase difference supplies the travel time.

Outwardly the stations are almost identical in appearance. They consist of a cabinet set up on a tripod, with the controls, measurement apparatus and cathode screen in front; on the back are the dipole antenna and parabolic reflector. Duplex radiotelephone communication is included in the stations, operated by switching.

Transmission at the master station is by a carrier wave at an approximate frequency of 3 000 Mc, hence of about 10-cm wavelength. The carrier wave can be modulated to four different frequencies, equivalent in kilocycles to A = 10 000; B = 9 990; C = 9 900; D = 9 000.

All the frequencies are quartz-stabilized. The most important one, frequency A, has a carefully regulated quartz calibrated according to temperature, which is taken at each measurement by mean of a device built into the instrument. Any one the four modulations is readily applied by operation of a switch.

The slave station is also equipped with quartz crystals for frequencies A, B, C and D, and moreover possesses variable condensers enabling the accurate tuning of the quartz frequencies with those of the master station.

The guide for this is the shape of the curve described by the trace on the oscilloscope screen, which should be a stationary ellipse.

Measurement of the phase differences is effected at the master station by means of the cathodic oscilloscope. The trace on the screen describes a circle on which the phase is indicated by a notch in the circle, and which can be spotted by a radial graduation with 100 divisions.

With the A modulation = 10 000 kc, the wavelength is 30 metres, and a 360° phase difference corresponds to a variation in distance of 15 metres, recorded on the screen by a complete revolution of the notch. Each of the 100 divisions is therefore equivalent to 15 centimetres, hence to one millimicro-second (1mμs) of travel time.

Dephasing of modulation A therefore supplies in tens, units and decimals the travel time in mμs, the complete number, i.e. the total number of revolutions remaining unknown.

This unknown number is obtained by changing the scale on the screen three times, in such a way that a complete revolution of the notch no longer corresponds to 15 metres, but to 150 metres, 1 500 metres and 15 000 metres.

For this purpose frequencies B, C and D are successively applied, and the differences in readings between the A modulation phase and the B, C and D modulation phases are determined.

Let us consider for example the phase difference measured by the modulations A = 10 000 kc and B = 9 990 kc.

The frequency difference is 10 kc. For a phase difference of 360°, i.e. for one complete revolution of the notch, the corresponding distance is

15 000 metres. One of the hundred divisions is hence equivalent to 150 m, i.e. 1 000 mμs, so that the reading supplies the tens of thousands of mμs of travel time.

Similarly the phase difference between the A and C modulations supplies the thousands, and the phase difference between the A and D modulations the hundreds of thousands of mμs of travel time. For distances above 15 km, hence for a travel time longer than 100 000 mμs, the hundred-thousand figures must be known by pre-estimating the distance.

It should be noted that so-called *rough* measurements, or those obtained by the phase differences between frequencies A-B, A-C, A-D, are not subject to the zero errors due for example to a poorly adjusted scale, small phase defects, personal error of the observer, etc.

But the most important measurement for time determination, the *fine* measurement on frequency A, is subject to such errors. In order that they be eliminated, it is so contrived that all measurements are obtained by differences. For this the A measurement is actually the average of four measurements : first the A measurement properly so called (A +); then a measurement called *A negative* (A —), for which the slave station is equipped with a special quartz; and lastly the two measurements derived from those preceding by simple inversion of the phases (A + R, A — R). In the average of the four measurements the zero errors are eliminated.

An additional source of error consists in parasitic phase changes produced by random reflections of the beam on obstacles or on the ground itself. Water surfaces in particular are good reflectors and may have marked effects.

These reflection effects distort results, which deviate from the correct values depending on the difference in the length of the direct and indirect rays, on the more or less perfect reflection of the wave on the ground, and especially on the phase differences between the direct or indirect carrier waves.

To eliminate the influence of these parasitic phase changes, series of measurements (ten or twelve) are made on carrier waves spaced about 10 Mc apart around the mean value of 3 000 Mc. The parasite changes then take on different values in either direction, the average thus being compensated. The series are only observed for the fine measurement, as the rough measurements remain adequately constant regardless of the carrier frequency.

Theoretically there should be no obstructions between the two stations during measurement. But owing to diffraction processes, small obstacles can be skirted by the waves and are then of no importance.

At the halfway point no obstacle is troublesome unless large enough to intercept the major portion of the beam, which has a 10°-aperture and is of notable width a few kilometres away from the master station.

Haze, fog, clouds or light rainfall do not prevent measurement, but a heavy rain may interfere. Experience has shown, however, that ideal conditions consist in daylight operations in dry, sunny weather, and that it is best to avoid night measurements for accurate work.

In any case the temperature, pressure and humidity must be measured at each station to enable computation by empirical formula of the refraction index of the atmosphere, hence of the speed of light at the time of measurement.

b) *Measurements with the tellurometer*

The accuracy claimed by the makers is such that the error (in metres) for a distance D (in metres) is given to D = 56 km by the formula :

$$\varepsilon = 0.05 + 3 \cdot 10^{-6}\,D$$

The instrument was tried out by the Madagascar Hydrographic Mission on a geodetic baseline at Tanandava, about 50 km from Morombe. The base had been measured by invar tape by the Geographical Service.

Operational conditions were excellent :

— flat country covered with vegetation (bush, bramble, and baobab forest) ensuring good transmission of radio waves, hence negligible parasitic reflections;

— very dry sandy soil of limited conductibility;

— warm, dry weather. The very warm ground ensured good homogeneity by vertical convection; a slight breeze from the south ensured good horizontal homogeneity.

There was no danger of parasitic reflection near the aerials, as the stations were set up on two large wooden signals 20 metres high with observational platforms and instrument bases; the instruments were screwed to Wild T3 theodolite supports. All precautions had been taken for carrying out reductions to centre under optimum conditions.

The results obtained are the following :

— distance between supports derived from length of base measured by invar tape : 9 625.07 metres;

— distance between supports measured with tellurometer : 9624.84 metres, showing a difference of 0.23 metre.

Accuracy is consequently slightly under the figure given by the makers' formula. It must however be pointed out that the length of the measured base did not correspond to the optimum utilizable distance of the equipment, which is from 25 to 35 kilometres.

Temperature, pressure and humidity determinations had been, moreover, made under mediocre conditions, resulting in some uncertainty as to the value of the speed of light.

The tellurometer was later used in French Somaliland to carry out triangulation intended for a survey near Djibuti. Two distances were measured : one about 8 km long between the mast on the terrace of the Port Administration building and Bouet cairn southwest of the city, and the other 42 km long between Bouet cairn and the beacon of Aibat Island.

These distances were used concurrently with theodolite sights in computing the triangulation. In both cases the geometrical locus derived from the distance passed within 30 centimetres of the position resulting from the theodolite sights.

These results may be compared with those obtained by the French National Geographical Institute, which carried out systematic investigations of the tellurometer in the autumn of 1958 in Poitou and Brittany. The studies covered errors to be expected when the instruments are placed on metal geodetic signals, the influence of screening between stations, and the effects of wave absorption by the ground and of parasitic reflections.

During these investigations a distance of 62 km was measured without difficulty, and over the various measured lengths the accuracy amounted to at least 1 part in 100 000.

ERRATUM

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A SIMPLE DEVICE FOR OBSERVING BOTTOM CURRENTS

page 167 :

(1) End of paragraph 2 :

... coloured vegetable. (Fig. 1).

should read :

coloured vegetable oil. (Fig. 1).

(2) End of paragraph 5 :

... bottle to chill. (Fig. 2).

delete : (Fig. 2).