REMARKS ON THE COMPUTATION
OF EQUAL-ALTITUDE OBSERVATIONS

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1) In a recent article published in the *International Hydrographic Review* [*], I showed that the observational equation in the equal-altitude method could be expressed in linear form in terms of the unknown values without recourse to use of an approximate solution. This characteristic facilitates calculation of the series of observations by means of modern computing machines.

Referring back to the notations of the latter article, the observational equation is written :

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos (H - G)$$

The introduction of three auxiliary unknowns

$$X = \frac{\cos \varphi \cos G}{\sin h} \qquad Y = \frac{\cos \varphi \sin G}{\sin h} \qquad Z = \frac{\sin \varphi}{\sin h}$$

enables the observational equation to be expressed in linear form :

$$X \cos \delta \cos H + Y \cos \delta \sin H + Z \sin \delta = 1$$

It struck me as odd at the time that this type of solution had not been indicated during the early years of the XIXth century, when the equal-altitude method was devised and developed.

This remark was erroneous, and I am pleased to note that the French astronomer DELAMBRE showed at this period that the observational equation in the equal-altitude method could be made linear in a simple manner [**].

2) The method described by this astronomer for solving a system of three equal-altitude observations actually amounts to selecting three auxiliary unknowns

$$\xi = \cot \varphi \cos G \qquad \eta = \cot \varphi \sin G \qquad \zeta = \frac{\sin h}{\sin \varphi}$$

which enables the observational equation to be written :

$$\xi \cos \delta \cos H + \eta \cos \delta \sin H - \zeta = - \sin \delta$$

(*) A. GOUGENHEIM. A new method of computation of equal-altitude observations, *International Hydrographic Review*, Nov. 1958.

(**) DELAMBRE. Sur la solution nouvellement donnée par M. GAUSS, d'un problème d'astronomie sphérique, dans lequel on se propose de déterminer tout-à-la-fois la latitude, la correction de la pendule et celle d'un instrument, par les hauteurs égales de trois étoiles connues, *Connaissance des Temps pour l'an 1812*, Paris, July 1810.

The author considers that this type of solution by three linear equations with three unknowns is « easy and complete », but adds that this « most natural of all » procedures is also the longest in that 39 logarithms are required to supply the values of the three unknowns. Hence he gives preference to trigonometric methods in which auxiliary arcs are used, and suggests a slightly shorter method than the one indicated by GAUSS.

However, he does not fail to note that the use of the linear form of the observational equation, despite the disadvantage of lengthy calculation, offers the valuable asset of applying to an indeterminate number of observations. « For », he writes, « there is nothing to prevent observation during one night of the equal altitudes of 12 or 15 stars, each supplying its equation. These equations could be assembled in three separate groups, which would form the three equations required to solve the problem, and the three unknowns would be obtained at the same time from the totality of observations. This would be considerably shorter than combining all stars observed in sets of three, regardless of the method that might be selected. »

By using the least squares method to form the three final equations indicated by DELAMBRE, a type of solution quite similar to the one developed by me is arrived at.

3) There is, however, an appreciable difference between the linear form suggested by DELAMBRE and my own. DELAMBRE obtains the auxiliary unknowns ξ, η, ζ by dividing both members of the observational equation by sin φ, whereas I form simultaneously the unknowns X, Y, Z by dividing by sin h. Now the altitude of observation h is generally taken fairly high : it is never less than 15 degrees, and is generally at least equivalent to 45 degrees, so that the value of $1/\sin h$ and hence the absolute values of X, Y, Z are at most equivalent to 1.4. In lower latitudes, however, $1/\sin \varphi$ can take on very high values and those of the unknowns ξ, η, ζ can extend over a wide range, which is less advantageous for calculation purposes.

The linear form adopted by DELAMBRE is also less convenient with respect to the interpretation of residuals and consideration of variations of refraction. Thus the coefficients tan h and cot h that we found in order to multiply respectively the residuals and refraction variations must be replaced by $\dfrac{\sin \varphi}{\cos h}$ and by its inverse value. Such coefficients involve not only the altitude of observation used, but the latitude at the place of observation, and are thus no longer constant for a given equal-altitude instrument.

4) In a relatively early article, written in collaboration with Madame CHANDON and likewise published in the *Hydrographic Review* [*], DELAMBRE's research into the calculation of equal-altitude observations has already been mentioned :

« DELAMBRE », we wrote, « gave a detailed analysis of GAUSS's method, but he confined himself almost entirely to criticism and simplification of the formulae set up by his illustrious contemporary, without, it appears, realising the importance of this new method in positional astronomy ».

The present article is additional confirmation of our appreciation of

DELAMBRE'S work. The reservation contained in the final clause of the preceding sentence is particularly due to the following passage in his paper :

« During a night when three stars might be observed at an identical altitude, some known star might very probably be seen passing across the meridian which would supply latitude with less trouble and greater accuracy, and chronometer time by a single altitude. But such questions are curious and are a useful exercise for those who like observations and calculations ».

DELAMBRE was far from suspecting that scarcely a century later competition would be established between meridian observations and equalaltitude observations, or that owing to a particularly successful instrument, DANJON's impersonal prismatic astrolabe, the equal-altitude method would become, at least for a time, the most accurate of positional astronomy methods.