GRAVITY ACCELERATION MEASUREMENTS AT SEA BY GRAVIMETERS

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It is common knowledge that given the value g of gravity acceleration at numerous points on the earth (*), it is possible first to obtain a precise value of the flattening of the spheroid, which is the mathematical datum surface on which triangulations are computed, and secondly to determine the height of the geoid at any point above this datum surface by Stokes's formula. But a requisite is that the measurements applied of g shall have been made at evenly distributed points on the earth's surface, hence at sea as well as on land, since over three fifths of the earth's surface are submerged.

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Measurements of gravity on land at first were carried out solely by pendulum methods, but remarkable progress has been made since the development of gravimeters. These enable the interpolation of measurements between the values obtained by pendulum.

A gravimeter is an instrument whose indications vary with the value of g in accordance with some unspecified law *which need not be known.* It suffices to calibrate the instrument by noting its indications at places where gravity acceleration has been previously determined by pendulum methods, to construct a calibration diagram, and then transfer the gravimeter to the measurement site, where the value of g can be derived from the gravimeter indications and the calibration diagram.

Numerous types of gravimeters have been devised in recent years, and a considerable number of interpolation measurements take place annually. This is due to the fact that such measurements, besides being easy to effect, are used in mine and oil prospection.

Many gravimeters, at least in theory, are identical with the spring-type balance. At one end of the balance beam is a mass varying in weight with the value of g; at the other end is a spring whose tension balances the weight of the mass.

The beam deviates more or less from the horizontal position according to the value of g, and measurement consists in either recording the amount of deviation, or better, in restoring horizontality by introducing an additional force. This is supplied either by the traction of an additional spring or by an electric current, or by any other factor, whose value gives the

(*) In the C.G.S. unit system, the value of g is approximately 981 gals, the gal (an abbreviation of GALILEO) being the acceleration in a uniformly accelerated motion having a velocity increase of 1 centimetre per second.

value of g provided the gravimeter calibration diagram is available. The accuracy normally obtained on land with modern gravimeters is one-hundredth of a milligal.

At sea, gravity measurements present special difficulties owing to lack of stability. To local gravity must be added the accelerations due to the motion of the ship; accelerations may ordinarily reach 50 000 milligals, and as much as 500 000 milligals in heavy weather, i.e. 50 % of g, whereas g is to be determined to within 1 milligal.

Meanwhile a large number of accurate measurements (nearly 4 000) have been made at sea since 1923 by a pendulum method developed by Dr. Vening Meinesz, the Netherlands geodesist (*). It will be remembered that in Vening Meinesz's method the action of the horizontal component of acceleration due to ship's motion — a preponderant horizontal acceleration — is eliminated by observing two practically isochronous pendulums which oscillate on an identical support and in the same vertical plane. It can be shown that under such conditions the difference in the amplitudes of the pendulums is independent of horizontal perturbation and follows the same law of variation as the amplitudes of a fictitious pendulum unaffected by perturbation. The value of g is derived from the period of oscillation of the fictitious pendulum measured on record diagrams. The mean value of the vertical component of acceleration is obtained by a recording process of adequate duration (30 minutes).

But the Vening Meinesz method is only applicable when accelerations due to ship motion do not exceed 1/150 gal, or 7 000 milligals. Hence it may only be used when roll and pitch are moderate, thus leading to operation in a submarine beneath the surface. This is a serious drawback, since submarines are seldom made available to geophysicists for gravity measurements. Even if this should occur, the cramped quarters limit the choice of space for the instruments, and the trips to sea may be wearing for the observer.

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May gravimeters designed for land observations be used at sea, and interpolation measurements be obtained ?

Land gravimeters were first used in sea operations by placing them in waterproof containers on the bottom. A remote-control device is then required to release the apparatus and bring the beam into a horizontal position by means of a control device located on the surface. The method is of adequate accuracy, but owing to the necessity of anchoring the container, it can normally only be used inshore. Actually it has been possible to obtain numerous measurements of gravity by this method down to a depth of 150 metres. By working from a bathyscaph resting on the bottom, quite satisfactory results have been obtained at a depth of 820 metres.

But the real practical value of gravimeters consists in using them on surface ships. The advantages are then obvious. One can work from any type of vessel (a naval vessel, liner, freighter, tanker, etc.) and thus count on rapidly obtaining a large number of observations well distributed over the ocean surface. In most cases the assignment of space for the instruments raises no problem, adequate thermal isolation of compartment space is

(*) In 1957 a pendulum instrument for measuring g, essentially consisting of a well-damped bifilar pendulum, mounted on a support on gimbals, was tried out in Japan by Dr. Tsuboi and Y. Tomado on a surface vessel.

easily effected, and operational costs are low since no special trips to sea need be organized.

We have already mentioned, however, that accelerations are considerable on the surface due to heaving seas. They may average ten times as much as accelerations under water, and thus require special planning.

The answer to the problem as given by Doctor Graf is as follows.

In the Graf method no attempt is made to keep the gravimeter beam horizontal, which would be impractical, and it is allowed instead to oscillate about its balanced position. Amplitudes are then recorded, and by plotting the mean oscillation curve the local value of g is obtained.

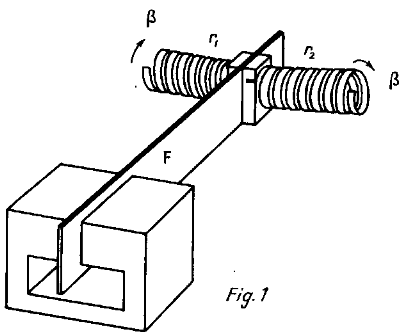

Fig. 1

Graf's gravimeter (fig. 1) consists of two taut spiral springs r_1 and r_2 arranged horizontally, with their common part supporting an aluminium beam F constituting the mass.

The springs are bent in an appropriate direction of angle β such that the axis of the beam be horizontal, this being the instrument's zero position. The free end of F, as it oscillates, breaks a strong magnetic field, and is subjected to considerable damping from the resulting eddy current. Such damping is necessary in order to subject the mean value of the recorded amplitude to the local gravity.

But in order that the gravimeter may properly operate at sea, strong damping does not suffice : the free end of the beam must not be able to move otherwise than vertically. As previously stated, horizontal accelerations predominate on a vessel at sea, and might distort the measurements should the beam respond to them. Experience shows that this can be prevented by lashing the beam with 8 sections of very fine steel wire (f in fig. 2), in such a way that the loose axes of the springs act as rigid rotation axes and the beam is only free to move in a single plane. The amplitude

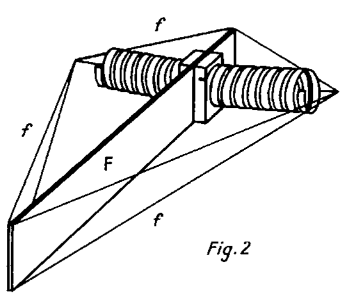

Fig. 2

of the free end of the beam is recorded as follows : a screen at the free end breaks a beam of light which strikes the plates of a differential photo-electric cell. In the beam's horizontal position (zero position) the two plates are equally lighted and the cell's output current is zero. Any variation about the zero position, however, causes an output current other than zero to appear, which is amplified and recorded.

The apparatus possesses several measurement ranges, and a range dependent on the value of g should be used. Transition from one range to another is effected by means of a draw spring actuating the balance beam and whose length is varied by a micrometer screw.

Special precautions are taken to free the instrument's indications from atmospheric pressure changes, and a double thermostat to a large extent ensures constancy of temperature.

The system of measurement adopted in the Graf gravimeter is based on the assumption that amplitude in the area of measurement is a linear function of the vertical acceleration to which the beam mass is subjected. The theoretical conditions here involved were studied by Dr. Graf and the theoretical conclusions were checked in the laboratory by placing the gravimeter on a table which artificially reproduced varying periods of roll and pitch. Such conditions enable computation of the accelerations to which the instrument is subjected; these are in inverse ratio to the square of the periods. It will be found that the difference with respect to the linear law is dependent on the square of the amplitude a of the vertical acceleration superimposed on the local gravity g_0 and that the resulting error in g_0 is

$$\Delta g = a^2 \beta^2 / 4 g_0$$

β being the angular amount of twist required by the springs to maintain the beam in a horizontal position.

By increasing the amount of damping, i.e. by decreasing a, Δg is reduced and the range within which amplitude may be regarded as a linear function of vertical acceleration is thereby increased. Actually the computations and laboratory studies showed that this range is extensive : $\pm$ 60 000 milligals for a period of 6 seconds, and $\pm$ 30 000 milligals for a period of 10 seconds.

We have already noted that oscillations of the balance beam were damped by magnetic methods. Actually this initial damping does not suffice for measurements at sea, and before recording, an additional electric damping circuit must be resorted to in order to further reduce the amplitudes. Thus for a period of 6 seconds, magnetic damping reduces the amplitudes by 1/50 and electric damping by 1/200, so that finally an acceleration of $\pm$ 50 000 milligals is recorded as if it amounted to 5 milligals before damping. Owing to this twofold damping, the gravimeter will not indicate the average value of acceleration, i.e. local gravity, until a certain time interval (about 5 minutes) has elapsed after a change in measurement conditions (new ship heading, change in range, etc.).

A shipborne gravimeter is either hung on gimbals or placed on a gyro-stabilized support. If during measurement α_0 be the mean value of the slope of the support, it will readily be seen that a g_0 value with an error of g_0 $(1 - \cos \alpha_0) \simeq - g_0 \frac{\alpha_0^2}{4}$ is obtained.

To ensure accuracy to within one milligal, it will thus be seen that horizontality of the support must be $\pm 5'$.

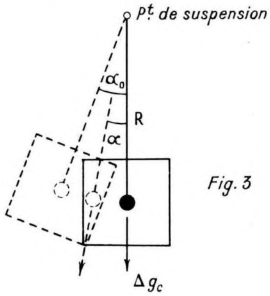

Fig. 3

In gimbal suspension, the entire apparatus swings about the point of suspension and effects oscillations with a period of T_0. This results (fig. 3) in two errors when measuring g : the first is the slope error, expressed, as we have seen, by $\Delta g_i = - g_0 \frac{\alpha_0^2}{4}$; the other is due to the appearance in the swinging motion of a centrifugal acceleration which affects g with a positive error Δg_e expressed as :

$$\Delta g_e = + \frac{R}{2} \alpha_0^2 \cdot \frac{4\pi^2}{T_0^2}$$

R being the distance from the beam's centre of gravity to the point of suspension.

As these two errors are of opposite sign, it is possible at a fixed distance R to compensate one by the other, by assigning an appropriate value to T_0. Two auxiliary masses m_1 and m_2 are placed for this purpose above the gravimeter case (fig. 4), and when displaced cause a variation in the period of oscillation, which in the laboratory attains a value of

$$T_0 = 2\pi \sqrt{\frac{2R}{g_0}}.$$

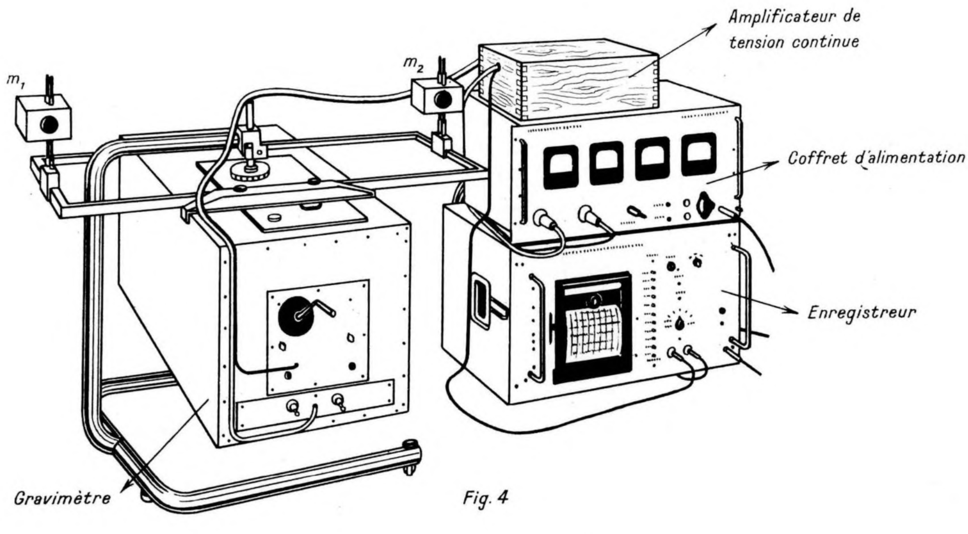

Fig. 4

The values of g obtained with Graf's shipborne gravimeter require several corrections in addition to the instrumental corrections. These include *first order effect* and *second order effect* corrections, which occur by reason of the instrument's subjection to accelerations and its consequent indication of an apparent gravity slightly different both in direction and intensity from the true gravity obtained from a gravimeter at rest.

Another correction is due to the *Eotvos* effect. This effect occurs when the velocity of the ship has an east-west component, resulting in a change of the centrifugal force, which is one of the g components. According to the particular case, the outcome is a gravity increase or decrease, and to the observed shipboard value must be added a correction expressed as to magnitude and sign by

$$\Delta g_e = 7.47\ v \sin Z \cos \varphi \text{ (in milligals)}$$

an expression in which v is the speed in knots, Z the course angle, and φ the latitude. v is the speed over the bottom, and in order that g be accurate to within one milligal, v must be known to within one-eighth knot, a requisite that in the absence of data on ocean currents constitutes one of the measurement problems.

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The first trials of Graf's shipborne gravimeter were made on Lake Starnberg, located inland, in a small craft of 100 tons. They proved that in fine weather and while the craft was under way it was possible to attain an accuracy of 0.3 milligal.

Measurements were later made in the Adriatic aboard a 5 000-ton motorship, the *Messapia,* during passage from Trieste to Venice. The results on this occasion were compared with measurements in a submarine, and agreement was ascertained to average ± 2 milligals.

During the autumn of 1956, Dr. Worzel of the Lamont Geological Observatory carried out a series of simultaneous measurements with Vening Meinesz's pendulum apparatus and Graf's shipborne gravimeter, which were both installed in the same submarine. A total of 59 measurements were made in the Mediterranean, Atlantic and English Channel. Agreement averaged ± 1.2 milligals in the Channel, ± 2.3 milligals in the Mediterranean and ± 3.8 milligals in the Atlantic.

The longest cruise has been that of the U.S.S. *Compass Island* (18 000 tons) from late March to late April 1958. An Atlantic crossing from New York to Naples was first made, and after measuring profiles in the Mediterranean previously investigated in a submarine with the Vening Meinesz instrument, the vessel recrossed the Atlantic. Successful records were obtained in heavy seas down to a trough depth of 4 metres and at speeds of 18 knots. Comparable measurements at points previously observed with the Vening Meinesz instrument from a submarine supplied equivalent results to within ± 3 milligals.

It should finally be mentioned that in the U.S.A. another gravimeter, the La Coste and Romberg instrument, has been used at sea either in surface or underwater craft.

76 submarine observations were carried out during 1957 and the results compared with 16 previous and 17 simultaneous measurements obtained with the Vening Meinesz apparatus.

On a surface vessel, the converted minesweeper *Hidalgo*, measurements were carried out off the south coast of California along a 300-mile route and along profiles 10 miles apart in the Gulf of California. In this latter series, simultaneous observations were made at 27 stations with an underwater gravimeter, the mean difference between the two series being 2.7 milligals, and the amount of scatter around this mean difference only 1.3 milligals.