SOME RECENT APPROACHES TO TIDAL PROBLEMS

by W. Horn
Deutsches Hydrographisches Institut

There was a time when rivers were rather shallow, and navigation greatly subject to depths as well as currents. There followed a period of increasing dredging of rivers and more powerful engines, when ships had to worry more, if at all, about tidal streams than about tides. Now that ships are becoming bigger and bigger and of ever-increasing draughts, the question of an accurate prediction of the available depths has regained importance. This requires very accurate predictions of the astronomical tide and the best possible forecasts of additional meteorological effects, which have been forecast daily by the German Tide Service for more than 30 years already.

2. The tides, as we know, occur in different forms that depend upon the place. Fig. 1 : tides of Immingham, shows the semidiurnal type, with spring tides following the full and new moon, neap tides following the first and last quarters of the moon. Fig. 2 : tides of Do-Son, Indochina, shows the diurnal type, with spring tides following the greatest declinations to the north and south of the moon, and neap tides following her passages through the equator. With both types, a decrease of the moon's distance from the earth has an increasing effect on the range of tide, so that this range is greater on the average near the perigee than it is near the apogee of the moon. Fig. 3 : tides of Bangkok, Thailand, shows the mixed type. A smal diurnal inequality of subsequent high or low waters, indicating the influence of a small diurnal tide, is visible also in Fig. 1.

3. The harmonic analysis of the tides is usually explained as follows. It is taken for granted that the tide-generating potential P, considered as a function of the time t, can be developed in a series

$$P = \sum_n P_n \begin{matrix}\cos\\ \sin\end{matrix} s_n t,$$

where the s_n, the enumerable angular speeds correspond to certain periods which are known numbers, such as half a lunar day, half a solar day, one sidereal day, etc. It is then deduced from the hydrodynamical equations and from the equation of continuity that the height of tide ζ, referred to mean sea level, can be similarly developed in a series

$$\zeta = \sum_k \zeta_k \cos (S_k t - \alpha_k)$$

where S_k either equals certain s_n (« astronomical » constituents), or S_k is a multiple of certain s_n (« over-constituents »), or $S_k = \Sigma C_n s_n$ is a linear

(*) Condensed from a paper read at Liége on the foundation of the Belgian Centre of Oceanography and Undersea Research, 24 February, 1958.

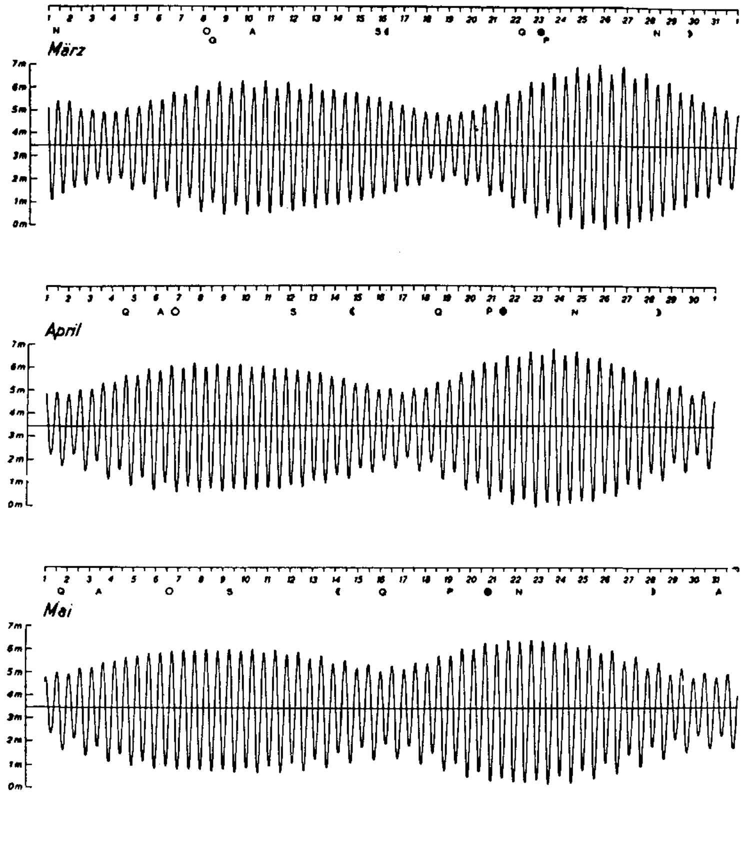

FIG. 1

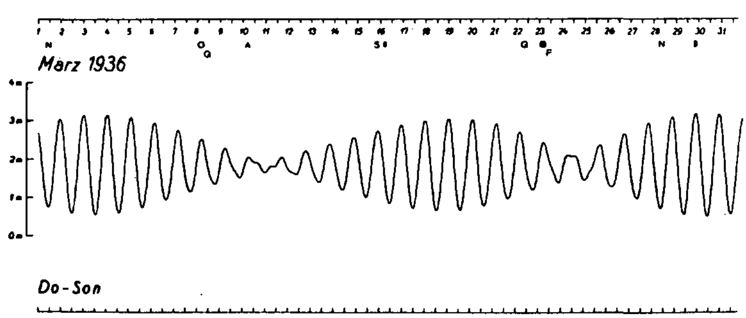

FIG. 2

combination, with integers C_n, of two, three, or more s_n (« compound » constituents). The latter two groups, originating from the non-linear terms in the equations I have referred to, bear the common name of « shallow-water constituents ». The set of constants ζ_k, α_k are called the harmonic tidal constants of the place.

It may occur that the speed of a shallow-water constituent equals that of an astronomical constituent, or comes very near to it. Also, the number

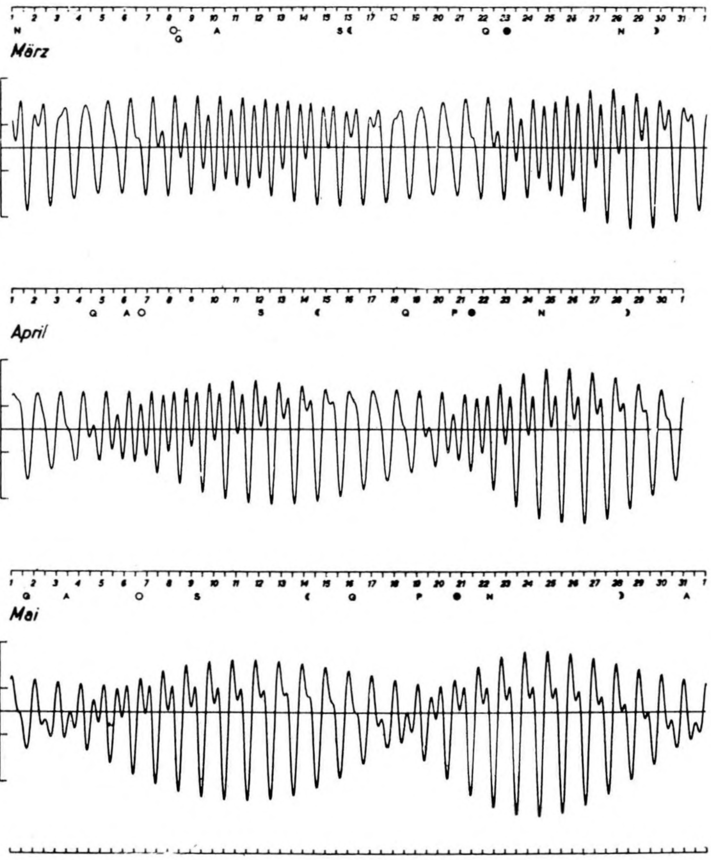

FIG. 3

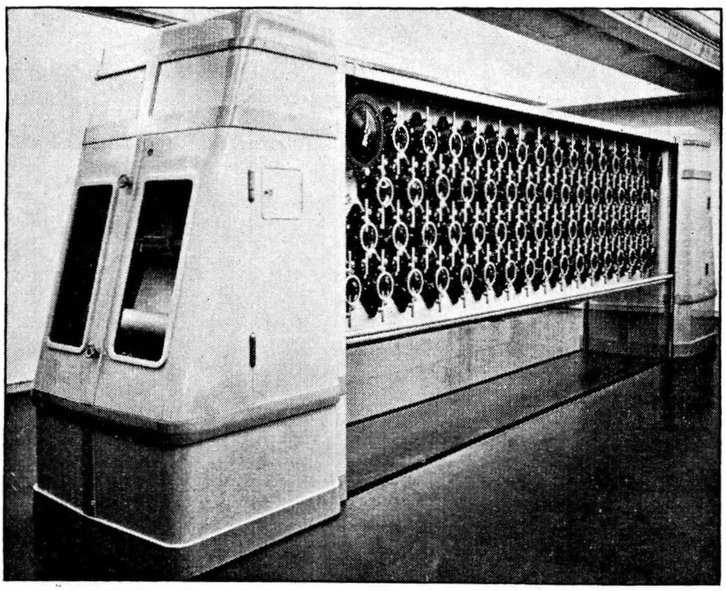

FIG. 4

of shallow-water constituents that need be taken into account greatly increases the shallower the water becomes, and the more the influence of friction makes itself felt. As can easily be imagined, a considerable confusion of constituents then arises, and it has been stated as a general rule that the whole method becomes impracticable as soon as the eighth-order constituents gain importance. On the other hand one can easily verify that in rivers, and especially in those of the German Bight, which is the innermost and shallowest part of the North Sea, shallow-water constituents of the 14th or even higher order are not negligible. For when the times of high and low waters are required, we have to differentiate with respect to time, and in the derivative the amplitudes of the constituents appear augmented proportionally to their respective order.

Fig. 4 is a photograph of one of the German tide-predicting machines. It is the biggest that exists, with installations for 62 constituents, amongst which are some eighth-diurnals. It has equally equipped front and rear sides for the simultaneous computation of two tidal functions, such as the tide and its derivative, and an automatic printer, so that no one need be present while the machine is being run. Hourly values, times and values of the maxima and minima, and/or times of zero values can be printed. Also, curves representing the two tidal functions can be drawn simultaneously. The practical navigator, however, prefers figures to curves. The machine was built in 1938 when no international exchange agreement yet existed for predictions. I still do not know of any electronic computer that would be a serious rival in the continuous predicting of tidal functions. We are therefore still very glad to have the machine, and it is used increasingly also for new kinds of tasks, such as gravity predictions. Nevertheless, in the field originally thought of, viz. the prediction of water tides, the whole method is applicable only to a limited extent, as I have said, and in fact the machine has never served to predict the tides in German ports.

4. The exposition of the harmonic analysis as I have given it here following the usual practice is not incorrect, but is incomplete to some degree and may therefore lead one astray. For instance, Paul Lévy, certainly a great mathematician, has applied to the tides the theory of almost periodic functions, very properly indeed. This theory is of an extraordinary beauty, and connects a great variety of fields of mathematical research. Lévy considers the fraction $f_k = k : S_k$, and on the assumption that f_k possesses a finite limit as k tends to infinity, deduces a number of theorems, which of course are quite correct. He concludes that theoretically about two days, and in practice not more than one month, of observations should suffice to determine empirically the harmonic constants of a place. This is a little surprising for the tidal expert, and in fact it can be shown that the tides exactly correspond to the case that Lévy deliberately leaves aside, viz. that f_k exceeds any finite limit with increasing k. In that case, he says, an infinite time interval would be required to obtain a full knowledge of the behaviour of the tides (Annales Hydrographiques, Paris 1946). Now tide gauges, the first type of automatically registering instrument, as far as I know have not existed for more than about 130 years, and one may ask what recourse anyone predicting tides would have in these circumstances.

5. Fortunately, the difficulty can be overcome by a different approach, investigating more closely the nature of the speeds S_n. For that purpose,

it will be necessary to start from the foundations of celestial mechanics. Before we do so, I wish to recall certain formulae, the first of which is Euler's equation,

$$e^{ix} = \cos x + i \sin x,$$

from which it follows that $\cos x = (e^{ix} + e^{-ix}) : 2$, $\sin x = (e^{ix} - e^{-ix}) : 2i$.

Further, let $f(x)$ denote a function of a real variable x, with period 2π, such that $f(x) = f(x + 2\pi)$. (When the period differs from 2π, a transformation of the coordinate will reduce the period to that value). Then for a very wide class of such functions the series

$$\sum_{n=-\infty}^{n=+\infty} f_n e^{ixn},$$

where

$$f_n = \frac{1}{2\pi} \int_0^{2\pi} f(r)\, e^{-inr}\, dr,$$

converges to the function $f(x)$. The theorem is generally ascribed to Fourier, but in essence Clairaud and Lagrange had it (as an interpolation formula), and one may recognize its geometrical equivalent in the Ancients' theory of epicycles, as first developed apparently by Eudoxos.

Analogous theorems are valid for functions of two or more real variables; e. g. if we have

$$f(x, y) = f(x + 2\pi, y) = f(x, y + 2\pi) = f(x + 2\pi,\ y + 2\pi),$$

then

$$f(x, y) = \sum_{\mathrm{L}} f_{\mathrm{L}}\, e^{i\mathrm{L}},$$

where L denotes any linear combination $\mathrm{A}x + \mathrm{B}y$, with $-\infty < \mathrm{A},\mathrm{B} < +\infty$, the summation being extended over all combinations of the kind, and

$$f_{\mathrm{L}} = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} f(r, s)\, e^{-i\bar{\mathrm{L}}}\, dr\, ds,$$

$\bar{\mathrm{L}}$ standing for $\mathrm{A}r + \mathrm{B}s$.

The following remark has some bearing on the much discussed problem of the search for unknown periodicities. Suppose (*Fig.* 5, 6) that the functions $f(x)$ or $f(x, y)$ are known only in a part of the interval $(0, 2\pi)$, or the square of side length 2π, respectively. We may then arbitrarily either prolong the curve representing $f(x)$, or assume values of $f(x, y)$ in the area not hatched, taking care only that the continuations fit in sufficiently smoothly and make the resulting functions periodic, and in either case there exists an infinite variety of Fourier series all representing in full detail the « observations » that have been obtained in the hatched interval, or area.

6. We now come to celestial mechanics. Consider n mass points, moving in Euclidean absolute space under the influence of gravitation. It is very natural for us to think of mutual attraction or forces as causes of the accelerations that the individual mass points undergo. However, in theoretical mechanics, which are concerned with the mathematical description of motions, and nothing more, the notion of force is but an abbreviation for the acceleration vector multiplied by the mass, so that it can be dispensed with; what is really important is coordinates and energy. We shall speak therefore of a kind of motion in which the accelerations depend upon the configuration of the mass points. The positions of the masses we may

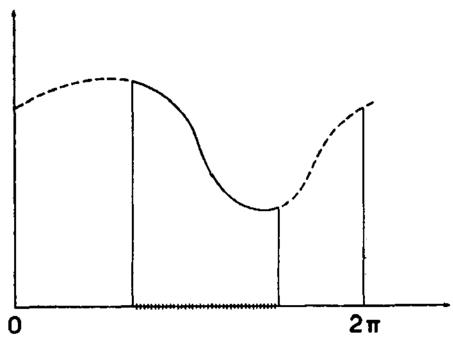

FIG. 5

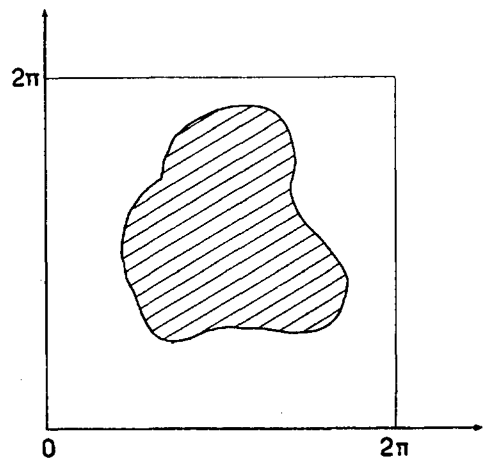

FIG. 6

describe by any kind of generalized coordinates, rectangular, polar, or other. Let K and U denote the kinetic and potential energies of the system, and $H = K + U$. We introduce the generalized velocities $\frac{dq_i}{dt} = q_i$, and the generalized momenta $p_i = \frac{\delta K}{\delta q_i}$. Then according to Hamilton and Jacobi the following « canonical equations » hold :

$$\frac{dq_i}{dt} = \frac{\delta H}{\delta p_i} \qquad \frac{dp_i}{dt} = - \frac{\delta H}{\delta q_i} .$$

It is easy to prove that the equations are true for absolue motion, but remarkably enough a system of the same form is valid also for the motion relative to any of the accelerated mass points, and in that case, i. e. after the elimination of the centre of mass and of angular momenta, the remaining number of degrees of freedom is $3n - 5$ (see e.g. A. Wintner, The Analytical Foundations of Celestial Mechanics, Princeton 1947). So, when we consider the general three-body problem, four generalized coordinates will suffice to describe the motions relative to one of the mass points. We shall call these mass points the earth, sun, and moon, and shall be concerned in what follows with motions relative to the earth.

A coordinate q_i is called ignorable, or cyclic, if $\frac{\delta H}{\delta q_i} = 0$, and consequently the corresponding momentum p_i is constant. It is not too difficult to prove that there exists in general at least an instantaneously valid system of coordinates that are all ignorable. Then all coordinates will be linear functions of the time, $q_i = q_i t + q_{io}$. The difficulty is to actually construct such a system of coordinates. The case of $n = 2$ presents no obstacles; it is treated, in the way I have indicated, in advanced courses of analytical mechanics. An ingenious solution of the three-body problem, where the main difficulty consists in the possibility of collisions, has been found, on a different basis, by Sundman in 1907, but it cannot be generalized to $n > 3$, and is useless for the practical astronomer because about 10^5 terms of the expansions would be required to obtain one coordinate of a planet with an accuracy of perhaps one degree.

Practical astronomy uses instead, for the description of the apparent motions of the sun and moon relative to the earth, a set of coordinates that has essentially already been invented intuitively in antiquity, and since then refined by the advancement of science, viz. the mean longitudes, measured in the ecliptic from the First Point of Aries, of the moon, the sun, the perigee of the moon's orbit, and the ascending node of the moon's orbit, respectively. We shall denote them by s, h, p, N, in that order, and replace N by $N' = -N$, because all but N are increasing. We may consider s, h, p, N' as coming very near at present to a system of ignorable coordinates.

7. Next we replace the mass point representing the earth by a sphere partly covered with a thin skin of water, let the sphere rotate around an axis chosen in the nearest possible conformity with reality, and assume that the resulting motions of the water masses do not practically affect the rotation of the earth or the motions of the moon and sun. We measure the orientation in space of the earth by θ, the angle between the meridian of Greenwich and the First Point of Aries, or Greenwich sidereal time, and may consider it as a fifth coordinate in which the amplified system is periodic.

Now Thomson and Tait, in their treatise on Natural Philosophy, extended the application of the theory of ignorable coordinates not only to the motions of rigid bodies, in particular to the theory of engines, but also to certain kinds of fluid motion that are governed by the motion of rigid bodies. They promised to give further examples in a second volume, which unfortunately never appeared. I am inclined to believe that they would perhaps have included the tides (height of tide, and tidal streams freed from turbulence) as a free-surface motion of the oceans that is governed by the positions relative to the earth of the moon and sun, i. e. as a function that is periodic in each of the five variables θ, s, h, p, N'.

From that definition it would immediately follow that e. g. the height of tide, and similarly the north and east components of the tidal stream, can at any place be developed in a series of the form

$$\zeta = \sum_L \zeta_L \, e^{iL}$$

where

$$L = A\theta + Bs + Ch + Dp + EN',$$

A, B, C, D, E denoting integers varying independently of each other between $-\infty$ and $+\infty$,

$$\zeta_L = \frac{1}{(2\pi)^5} \int_0^{2\pi} \ldots \int_0^{2\pi} \zeta(r_1, r_2, r_3, r_4, r_5)\; e^{-i\bar{L}}\, dr_1\, dr_2\, dr_3\, dr_4\, dr_5$$

and $\bar{L} = Ar_1 + Br_2 + Cr_3 + Dr_4 + Er_5$.

To carry through in all rigour the deliberations I have outlined would certainly be a prohibitive task, for it would include the solution, though in the restricted form of the lunar theory, of the three-body problem. Yet these deliberations may perhaps serve at least as a guide through the jungle or arithmetic that is so characteristic of most of the papers dealing with the harmonic analysis of the tides.

Hamilton, Jacobi, Thomson, Tait, Routh, Helmholtz are long dead, but not the slightest reference to what I have sketched is still to be found in textbooks of oceanography, even of tides. I hope this does not mean that I am promoting wrong ideas. The situation appears to me to be typical of present-day oceanography. On the one hand, oceanography, which not too long ago was merely a chapter of geography and cannot but continue to give full weight to a number of descriptive branches, presents entirely new problems to the more advanced sciences, and on the other hand it has still to learn a good deal from them. So there still remains, and probably for a long while yet, a lot to do for go-betweens.

The model I have constructed does not fully correspond to reality. For instance, we have ignored the flattening of the earth, from which precession and nutation originate. We shall consider as negligible both nutation and polar variations, but shall refer our five variables to what is called the mean equinox, thus introducing small accelerations in them. The influence of the other planets acts to the same effect, and brings in a sixth variable q, the mean longitude of the perigee of the sun's orbit. Also, the obliquity of the ecliptic and the eccentricity of the sun's orbit do not remain constant. The complications arising from these circumstances can be partly met by introducing more variables such as q, partly by expanding in power series the coefficients in the Fourier series, but we have already reached the limitations of the complete procedure, and fortunately we may ignore most of these effects in practice.

If we draw energy from the tides artificially, this diminishes the total energy of the mechanical system by transforming a portion of it into heat or electricity, and as a consequence the mean motion of the moon will be retarded, and its distance from the earth increased. The rotation of the earth will also be retarded. Tidal friction works to the same effect.

8. We may also obtain our result in a more pedestrian way. The earth and moon monthly revolve around their common centre of mass, which annually describes an elliptic orbit, with slowly turning axis, around the sun. In this motion, the distances of the earth and moon from their centre of mass are almost inversely proportional to their respective masses, and the centre of mass always remains within the earth's surface, but does of course never coincide with the earth's centre. The moon and earth orbits are inclined by about 5° to the ecliptic. Then the true longitudes λ, true latitudes β, and the ratios of the true to the « mean » distances $r : c$ of the moon and sun, relative to the earth's centre, can, when we let

$$X = Q(s - p) + R(h - q) + V(s - h) + W(s - h),$$
$$Y = Q(s - p) + R(h - q) + (2V + 1)(s - N) + W(s - h),$$
$$Q, R, V, W = 0, \pm 1, \pm 2, \pm 3, \ldots,$$

be expanded in the form

$$\lambda = \text{mean longitude } s \text{ or } h + \sum_x C_x \sin X$$
$$r/c = \sum_x C_x' \sin X, \qquad \beta = \sum_y C_y \sin Y.$$

In the case of the moon, a few hundred terms per equation are required to obtain the necessary accuracy. In the case of the sun, very few suffice, and in the expansions of its longitude and distance the terms with arguments $R(h - q)$, representing the inequality of elliptic motion, predominate. These formulae seem, with the exception of a paper in the German Hydrographic Review (1948) in which I quoted them, to have appeared in print for the last time about half a century ago, in E. W. Brown's famous treatise on his new lunar theory. They were first given by Delaunay in 1860.

From them it can easily be deduced that the tide-generating potential P of the moon and sun is periodic in θ, s, h, p, N', q, so that it is developable in a six-dimensional Fourier series,

$$P = \sum_L P_L e^{iL},$$

and that actually all arguments of the form

$$L = A\theta + Bs + Ch + Dp + EN' + Fq$$

occur in this expansion.

Consequently, the differential equations of the tides, linear as well as non-linear, can be satisfied by expanding in similar series the height of tide, and the north and east components of the tidal stream. In the linear case we have term-to-term correspondence with the tide-generating potential. In the non-linear case, any term in the development of the tides directly corresponds as well to the term of equal speed in the tide-generating potential, as it indirectly originates in an infinite number of ways from mutual interference : in fact, the set of e^{iL} is complete, the derivatives with respect to time and space are of the same form, and the product of any pair of such exponentials also is. Whether direct correspondence with the tide-generating potential and/or certain numberless shallow-water combinations prevail, depends upon the place. As a general rule, the higher the order of a term, the more likely does it originate from shallow-water effects.

It is impossible in principle to separate by observation lunar from solar contributions to either the tide-generating potential or tides. It is only in their order of magnitude that corresponding terms of these contributions usually differ. But there are exceptions, such as the lunar and solar constituents K_1.

9. The derivatives with respect to time of the six variables θ (= mean sidereal time), s, h, p, N', q are of course known only to a finite, and in fact rather small, number of decimals. Yet, if we consider them as constants, we have to assume them as incommensurable with each other, simply

because this probability is infinitely greater than the alternative one. Then, if we write

$$\zeta(t) = \sum_{L} \zeta_L \, e^{iL},$$

an infinite time interval of observations will be required to determine the coefficients

$$\zeta_L = \frac{1}{(2\pi)^6} \int_0^{2\pi} \cdots \int_0^{2\pi} \zeta(r_1, r_2, r_3, r_4, r_5, r_6) \, e^{-iL} \, dr_1 \, dr_2 \, dr_3 \, dr_4 \, dr_5 \, dr_6$$

in complete conformity with Lévy's statement : the set of speeds $S_k = \frac{dL}{dt}$ is enumerable, and the fraction $f_k = k : S_k$ exceeds any finite limit as k^5. Consequently, when only one year of observations is available, and if the full set of functions e^{iL} can be made use of, there exists an infinite variety of sets of constants ζ_L that all have the property of leading to a representation in full detail (including swell and waves) of the observations, but that will generally have no relation to the tides beyond the interval of observation. This is both more and less than we require, and it is only by confining ourselves to an approximate expression, using but a finite selection of constituents of minor order, that the task of the harmonic analysis and prediction of the tides can be reduced to a reasonable form.

Now, the periods during which the variables θ, s, h, p, N', q increase by 2π (or 360°) are 1 sidereal day, 1 tropical month, 1 tropical year, about 8.6 years, about 18.6 years, and about 21 000 years, respectively. The latter three are too long to be significantly felt within one year's observations, yet at least the first two cannot be ignored when predictions are to be made on the basis of an analysis carried out years before. It is then convenient to write

$$\zeta(t_k) = \Sigma \, j_n \, \zeta_n \cos(V_{no} + v_n + S_n \, t_k - g_n),$$

where ζ_n, g_n denote the harmonic constants of the place, V_{no} the value of the astronomical argument L on the meridian of Greenwich at 00.00 hours G.M.T. on January 1st of the year in question, t_k the time reckoned in mean solar hours from the beginning of the year, and where further V_{no} depends only on θ, s, h, occasionally p or q, S_n depending correspondingly on the derivatives of these three or four variables, while j_n, v_n are annual corrections expressing the influence of in general p, N', and q. Tables of the j_n, v_n, and $V_{no} + v_n$ can be computed for any astronomical constituent, and for any individual shallow-water constituent, when a numerical development of the tide-generating potential is available. Such a development has been given by Doodson in 1921 (Proceedings of the Royal Society, Series A, London). Tables of the three sets of values for the years 1900 to 1999 for about 80 constituents were presented by the German Hydrographic Institute at the International Hydrographic Conference in May 1957, and have been accepted for international use. They will be printed shortly.

As I have said before, there are astronomical and shallow-water constituents of identical speeds S_n. It is possible to separate them from a long series of annual analyses owing to their long-period behaviour as expressed by the respective j_n, v_n differing. So it is usual to determine in the course of more elaborate analyses a constituent that has so far been considered as astronomical and called L_2 (it represents part of the influence of the moon's motion in an elliptic orbit). But we have, by a more refined and unorthodox analysis of 19 years of observations, found that the constituent of that speed must in German waters be interpreted mainly as the shallow-water constituent $2MN_2$, a constituent that amounts to no less than 15 percent of the total mean range, whilst L_2 is almost negligible.

Since Legendre and Gauss the most natural method of analysis would be the application of the least-squares method. In fact, it is only by prescribing a minimum condition, such as

$$\sum_k [\zeta_k - \zeta(t_k)]^2 = \text{Min.},$$

where ζ_k denotes the observed values, that the task of harmonic analysis, understood as an approximation by a finite expression, can be rendered a mathematically determinate one. This would lead, in the case of 369 days of hourly observations, and of 64 constituents plus the height of mean sea level, to 8 857 equations of error with 129 unknowns, to be reduced to 129 normal equations, and to their solution. Such a task has so far been considered as impracticable, and since the time of Thomson and Darwin a number of less rigorous methods have been devised, some of which are very ingenious and fairly effective. They can, however, be judged only as approximations to the least-squares method, and it is easy to invent a more or less probable scatter, that makes them fail in this or that respect with a finite series of observations.

We have resolved, after careful consideration, to introduce the least-squares method in full rigour, using our punched-card machines. If series of observations of the same length, viz. of 8 857 consecutive hourly heights, are always analysed, one may solve the normal equations indeterminately by computing the inverse of their matrix, and to read the observations then takes much more time than to carry through an analysis by means of the machines. Also, the computations for a number of analyses can be performed simultaneously. It is convenient to transform for practical computations the formula expressing $\zeta(t_k)$ into

$$\zeta(t_k) = \sum_{n=0}^{n=64} j_n A_n \cos(V_{no} + v_n + S_n t_k) + \sum_{n=1}^{n=64} j_n B_n \sin(V_{no} + v_n + S_n t_k)$$

$V_{00} = S_0 = 0$, and to introduce a central time origin. Then the matrix of the normal equations splits into two, one for cosine and one for sine. Table 1 gives the identification numbers, symbols, coefficients in the arguments V_{no}, and speeds in degrees per mean solar hour (1950), of the 64 constituents we use. Table 2*a* gives the coefficients in the (symmetric) normal equations for cosine ande sine, Table 2*b* the respective inverse matrices. About 60 analyses of annual observations have already been carried out by this method. Some matrices for shorter periods of observation are also available, and these have been inverted in a way that consecutively supplies the solutions for 1, 2,. . . ., *n* unknown constituents.

10. I now shall consider the particular type of semidiurnal tides, i. e. the case when the constituent with argument $2\tau = 2(\theta - s)$ predominates, such that the number of all maxima and minima of that individual term

equals the number of high and low waters of the total tide. By simply rearranging the variables we may then introduce instead of

$$L = A\theta + Bs + Ch + Dp + EN' + Fq$$

the arguments

$$L_\tau = A(\theta - s) + (B - A)s + Ch + Dp + EN' + Fq,$$

τ being the mean lunar time, which indicates the orientation of the earth relative to the mean moon. Again, the set of the functions e^{iL_τ} is complete, and for the tides at any place there holds an expansion

$$\zeta = \sum_{L_\tau} \zeta_{L_\tau} e^{iL_\tau}$$

If we attribute to τ a constant value τ_0, $\zeta(\tau_0)$ reduces to a periodic function of s, h, p, N' q only, such that expansions of the form

$$\zeta(\tau_0) = \sum_{L'} \zeta_{\tau_0, L'} e^{iL'},$$

with $L' = as + bh + cp + dN' + eq$, are valid for every τ_0, and the sequence of isolated lunar daily values $\zeta(\tau_0)$ will fluctuate less than the total tide curve that we may, but need not, imagine to pass through them. Diminished fluctuation however means that a much smaller number of terms is required to sufficiently represent the respective values of $\zeta(\tau_0)$, at the price, it is true, of having to develop separately the value of ζ for every τ_0. Yet a step-by-step computation of ζ will generally meet the practical requirements, and this explains the disadvantage of the classical harmonic method : it aims at a continuous representation of the tide, and consequently cannot avoid introduction of the variable θ, which is the root of the trouble.

In the case of mixed tides, with alternating periods of semidiurnal and diurnal character, no periodic approximation to the tides except mean sea level exists, and the classical method cannot be dispensed with, at least for the purpose of a first approximation.

That the classical method is inadequate becomes particularly evident when a table of high and low waters must be computed for a shallow-water port where the tides are of a semidiurnal character : one would a priori regard as uneconomical the prediction of a continuous infinity of points just to find the coordinates of two of them. Taking advantage of the obviously near-periodic character of these tides, we shall therefore try to compute the times and heights of the high and low waters directly. In fact, when we denote by $L_{\tau,0}$ the value of L_τ at the mean moon's meridian passage, by S_τ the increment of $L_{\tau,0}$ per half mean lunar day, and by Δt_1 and Δt_2 the time intervals between the mean moon's meridian passage, upper or lower, and the following high and low water, Δt_1 and Δt_2 must be solutions of the equation

$$\frac{d\zeta}{d\Delta t} = i \sum_{L_{\tau,0}} \zeta_{\tau,0}\, S_\tau\, e^{i(L_{\tau,0} + S_\tau \Delta t)} = 0,$$

and since the value of τ in $L_{\tau,0}$ is a constant, by the definition of $L_{\tau,0}$, Δt_1 and Δt_2 must be periodic functions of the values s_0, h_0, p_0, N'_0, q_0, the latter being taken at the mean moon's meridian passage. Consequently separate expansions of the form

$$\Delta t = \sum_{L'_0} \Delta_{L'_0} e^{iL'_0},$$

with $L'_0 = as_0 + bh_0 + cp_0 + dN'_0 + eq_0$, hold for Δt_1 and Δt_2, and similarly for the high and low water heights.

The time difference between the true and mean moon's transits is a function of s_0, h_0, p_0, N'_0, q_0, expressible in the same form. We may therefore interpret Δt_1 and Δt_2 also as the lunitidal high and low water intervals with respect to the true moon's transits, which diminishes their amounts, while the arguments L'_0 are still to be taken at equal time intervals of half a mean lunar day. If finally we discriminate between the expansions with even and odd values of A in $L_{\tau,0}$, there follow separate developments of the high and low water intervals and heights corresponding to the upper and lower transits of the true moon, in total eight series per port. The step-by-step computation of the values that these series assume at equal time intervals of one mean lunar day can be performed simultaneously by punched-card machines, and it is in this way that the official tide tables for German ports have been predicted since the issue for 1954.

The method of analysis again is the least-squares method, applied to 19 years of observations, more accurately : to eight series of each 6 689 consecutive daily high and low water intervals or heights, with central time origin. Table 3 gives the identification numbers, coefficients in the arguments L'_0, and argument increments per mean lunar day, of the 45 terms we have found to guarantee a sufficient representation of the high and low water in German ports. N'_0 and p_0 are left in the arguments, so that there is no need for annual corrections similar to the j_n, v_n; q_0 has been assumed as constant. About 60 analyses also of this kind have been carried out already. When preparing the observed lunitidal high and low water intervals for the analysis, we plot them and correct meteorologically disturbed values as indicated by Fig. 7. Without such corrections, which are

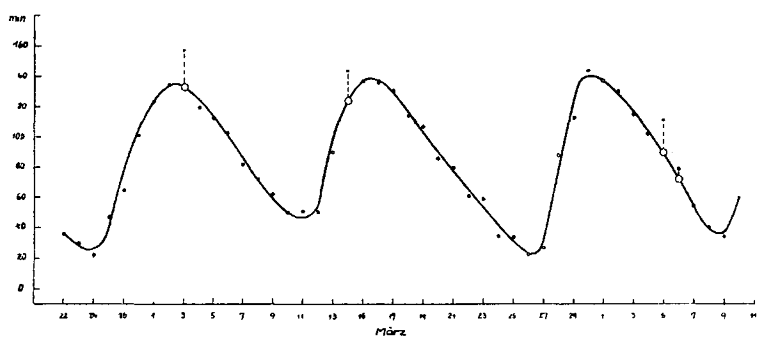

FIG. 7

applied only to reduce very obvious disturbances, the standard error would be insignificantly augmented. We have tested the method by applying it also to the tides in the Dutch port of Flushing (Vlissingen), and further to the Indian port of Bombay, where the tides show a great diurnal inequality, and in every case the results were very satisfactory. In the Tables 4*a* and 4*b* there are given the results for Bombay, with the kind permission of the Survey of India, Geodetic and Research Branch. Tables 5*a* and 5*b* give the coefficients in the normal equations for cosine and sine, and the inverse matrices.

The preceding method of predicting high and low waters directly is in essence a translation into analytical form of the method of J. W. Lubbock, who in 1832 systematized what a number of known and unknown predecessors had already been practising for a long while. Lubbock, like his predecessors, considered the lunitidal intervals and heights as depending upon the positions relative to the earth of the moon and sun at the moments of the moon's meridian passages at Greenwich. Only he expressed these positions by the equatorial coordinates of the heavenly bodies, in which it is impossible to satisfy the differential equations of the tides, except for the equilibrium tide, as first studied in detail by D. Bernouilli. Lubbock consequently resorted to statistics. But it is impossible to reconstruct from statistical tables the particularities of the individual case. When, in about 1868, W. Thomson and W. Ferrel independently of each other introduced the classical harmonic method, the possibility of which had been clearly envisaged already by Laplace, this was naturally considered as fundamentally superior to Lubbock's method, for it was of analytical character, and made it possible for the first time to predict tides of the mixed type.

Yet to insist on this point of view would not do full justice to Lubbock's method. In fact, it is still in use for a number of European ports, because the classical harmonic method practically fails, and the principle of Lubbock's method, though not the original technique, is profoundly sound, as I have tried to show. We have simply combined what is best in both methods, viz. the analytical procedure of the harmonic method, and the principle of computing isolated values directly, which characterizes Lubbock's method.

I am grateful that circumstances have allowed carrying out the work I have described from 1948 onward (it was devised much earlier). In particular I am thankful to Dr. Böhnecke who gave me freedom as far as the nature of an official service permits, and to the excellent team without whose skilful and almost sportive cooperation the undertaking, always running parallel to a good deal of routine work, could never have been completed. I wish to mention by name at least W. Habich and Dr. K. Munkelt.

To say more about the way we use our I B M machines would be of little value for non-experts, for this largely depends upon the types of machines that are available.

11. I have also little to say about the problem of the distribution in space of tides. The most important theoretical contribution of the last decades, at least in my opinion, is due to J. Proudman, now retired, who in 1916 commenced with some weighty papers dealing with the foundations of the differential equations of the tides, and thence, among other things, built up, theorem by theorem, a general theory of these equations in analogy to the theory of elliptic equations. This work, which was necessary because of gyroscopic effects, especially of the unorthodox form of the boundary condition, has rarely been adequately appreciated, and unfortunately never been presented as a whole. Even from Proudman's book on Dynamical Oceanography no one could guess to what extent its author has been active in the field I have indicated.

More recently, the application of the method of finite differences has become fashionable, and in fact there can be little doubt that the study of the tides in natural rivers and basins leads to arithmetic. Yet, if we leave aside tidal hydraulics, or the theory of river tides, in which the influence of the earth's rotation can be ignored, and the resulting equations of the hyperbolic type do not present too serious obstacles to treatment by the method of finite differences, the situation is still anything but satisfactory, and Professor Proudman has justly given his survey, which he read as presidential adress at the IAPO meeting at Rome in 1954, the title of : *The Unknown Tides of the Oceans.*

Actually, the boundary condition that the tidal streams flow parallel to the coasts is extremely difficult to handle in practice, and if one assumes as known the coastal values of the harmonic constants of the tides, the construction of cotidal lines across the oceans is still faced with the obstacle presented by the edge of the shelves, and with the difficulty that unknown values are to be assumed along the open boundaries. The method of finite differences always supplies a « solution ». What is necessary is to prove that it is true, and this can be done only by constructing manifolds of solutions for a variety of networks, as well as of the parameters that enter the equations. We have studied the distribution in space of the semidiurnal constituent M_2 in the rather difficult case of the Gulf of Mexico, and, leaving the values at a number of boundary points indeterminate we found, for networks of 9, 49, and 117 interior points, solutions that must be considered as increasingly unlikely. This does of course not mean that the solution resulting for 9 points can be attributed the highest credit, or that a progressive refinement of the network would not ultimately lead to the correct solution. But in the latter case it might well prove inevitable to introduce the third equation of motion and/or higher-order differences, which would not only tremendously increase the computational work but also lead to a break of the cotidal lines where the wave passes from deep water onto the shelves (a possibility already envisaged by Proudman). At least the question is still open as to what extent the tides observed on the coasts are representative for the tides in the deep oceanic basins.

The problem is not quite so difficult for the seas that cover the shelves. But there the method of finite differences so far has not led far beyond what one would dare construct without computation, viz. by simply drawing lines such as to satisfy the observations. On the contrary, what makes these computations interesting is that they provide an opportunity for testing the assumed values of the parameters, such as the law of friction.

Yet a number of problems remain. For instance, the question as to what extent the coastal observations are representative again arises when the waters before the coasts are very shallow. Also, the over-constituents in the eastern part of the English Channel, in contrast to those of the western part, are the greatest that occur on the European shelf, so that the complex tide in that channel much more resembles a standing oscillation than the principal constituent M_2 does. Consequently, something must be wrong energetically with all constructions of cotidal lines for the Channel in which it is assumed that the tides are sinusoidal everywhere. I should think that the number of over-simplified charts we have already by now suffices, and that an attempt should be made to approach reality more closely. Otherwise I should personally prefer as more stimulating to the imagination papers dealing with geometric basins, such as G. I. Taylor's on the reflection of Kelvin waves, or Proudman's on the expansion in terms of Poincaré waves of the tides in a straight channel.

The alternative I have to propose at present is very sober, viz. to observe the tides and tidal streams on the shelves at so great a number of stations that the application of the method of finite differences leads to a largely over-determinate system of equations that has to be smoothed out according to the rule of least squares. Such « hydrodynamic interpolation », as I should like to call it, could of course be applied only to mean tides, or to mean spring tides, or mean neap tides. The true tides never repeat themselves; any complete picture of them can be valid only for a limited time interval, just as tide tables are. As a summary, if we have come to resort to arithmetic as the means of constructing cotidal charts, we must not be too disappointed to find that this way leads to a kind of surveying. To what degree the boundary values fit in remains to be tested.

And there still remains the task of verifying arithmetically that the tides as they exist throughout the oceans are produced by gravitation.

However, I am not in a position to present a theory of the distribution in space of the constants that occur in the expansions of the high and low water intervals. This task appears almost hopeless.

TABLE 1

Harmonic analysis of hourly observations : identification numbers, symbols, coefficients in the arguments, and speeds (1950) in degrees per mean solar hour, of the constituents that are used

No.		τ	s	h	p	N′	q	
00	A_0	0	0	0	0	0	0	0.00000 00000
01	Sa	0	0	1	0	0	−1	0.04106 66776
02	Ssa	0	0	2	0	0	0	0.08213 72786
03	MSm	0	1	−2	1	0	0	0.47152 10669
04	Mm	0	1	0	−1	0	0	0.54437 46958
05	MSf	0	2	−2	0	0	0	1.01589 57627
06	Mf	0	2	0	0	0	0	1.09803 30413
07	$2Q_1$	1	−3	0	2	0	0	12.85428 62065
08	σ_1	1	−3	2	0	0	0	12.92713 98353
09	Q_1	1	−2	0	1	0	0	13.39866 09022
10	ρ_1	1	−2	2	−1	0	0	13.47151 45311
11	O_1	1	−1	0	0	0	0	13.94303 55980
12	τ_1	1	−1	2	0	0	0	14.02517 28766
13	NO_1	1	0	0	1	0	0	14.49669 39436
14	π_1	1	1	−3	0	0	1	14.91786 46831
15	P_1	1	1	−2	0	0	0	14.95893 13607
16	S_1	1	1	−1	0	0	1	15.00000 19617
17	K_1	1	1	0	0	0	0	15.04106 86393
18	φ_1	1	1	2	0	0	0	15.12320 59180
19	J_1	1	2	0	−1	0	0	15.58544 33351
20	SO_1	1	3	−2	0	0	0	16.05696 44020
21	OO_1	1	3	0	0	0	0	16.13910 16806
22	ε_2	2	−3	2	1	0	0	27.42383 37789
23	$2N_2$	2	−2	0	2	0	0	27.89535 48458
24	μ_2	2	−2	2	0	0	0	27.96820 84746
25	N_2	2	−1	0	1	0	0	28.43972 95415
26	ν_2	2	−1	2	−1	0	0	28.51258 31704
27		2	0	−2	0	0	0	28.90196 69587
28		2	0	−1	0	0	0	28.94303 55980
29	M_2	2	0	0	0	0	0	28.98410 42373
30		2	0	1	0	0	0	29.02517 28766
31		2	0	2	0	0	0	29.06624 15160
32	λ_2	2	1	−2	1	0	0	29.45562 53042
33	L_2	2	1	0	−1	0	0	29.52847 89331
34	T_2	2	2	−3	0	0	1	29.95893 33224
35	S_2	2	2	−2	0	0	0	30.00000 00000
36	K_2	2	2	0	0	0	0	30.08213 72786
37	ζ_2	2	3	−2	1	0	0	30.55365 83456
38	η_2	2	3	0	−1	0	0	30.62651 19744
39	$2SM_2$	2	4	−4	0	0	0	31.01589 57627
40	MO_3	3	−1	0	0	0	0	42.92713 98353
41	M_3	3	0	0	0	0	0	43.47615 63560
42	SO_3	3	1	−2	0	0	0	43.94303 55980
43	MK_3	3	1	0	0	0	0	44.02517 28766
44	SK_3	3	3	−2	0	0	0	45.04106 86393
45	MN_4	4	−1	0	1	0	0	57.42383 37789
46	M_4	4	0	0	0	0	0	57.96820 84746
47	SN_4	4	1	−2	1	0	0	58.43972 95415
48	MS_4	4	2	−2	0	0	0	58.98410 42373
49	MK_4	4	2	0	0	0	0	59.06624 15160

No.		τ	s	h	p	N′	q	
50	S_4	4	4	—4	0	0	0	60.00000 00000
51	SK_4	4	4	—2	0	0	0	60.08213 72786
52	$2MN_6$	6	—1	0	1	0	0	86.40793 80162
53	M_6	6	0	0	0	0	0	86.95231 27120
54	MSN_6	6	1	—2	1	0	0	87.42383 37789
55	$2MS_6$	6	2	—2	0	0	0	87.96820 84746
56	$2MK_6$	6	2	0	0	0	0	88.05034 57533
57	$2SM_6$	6	4	—4	0	0	0	88.98410 42373
58	MSK_6	6	4	—2	0	0	0	89.06624 15160
59	$3MN_8$	8	—1	0	1	0	0	115.39204 22535
60	M_8	8	0	0	0	0	0	115.93641 69493
61	$2MSN_8$	8	1	—2	1	0	0	116.40793 80162
62	$3MS_8$	8	2	—2	0	0	0	116.95231 27120
63	$2(MS)_8$	8	4	—4	0	0	0	117.96820 84746
64	$2MSK_8$	8	4	—2	0	0	0	118.05034 57533

TABLE 3

Harmonic representation of high and low water luntitidal intervals or heights : identification numbers, coefficients in the arguments, and speeds (1950) in degrees per mean lunar day, of the terms that are used.

No.	s_0	h_0	p_0	N'_0	
0	0	0	0	0	0.0000 0000 0
1	0	0	0	1	0.0548 0990 4
2	0	1	0	0	1.0201 9438 2
3	0	2	—2	0	1.8097 7174 1
4	0	2	0	0	2.0403 8876 4
5	1	—2	0	0	11.5978 4115 2
6	1	—2	1	0	11.7131 5026 3
7	1	0	—1	—1	13.4681 1210 0
8	1	0	—1	0	13.5229 2200 4
9	1	0	0	—1	13.5834 2061 2
10	1	0	0	0	13.6382 3051 6
11	1	0	0	1	13.6930 4041 9
12	1	2	—1	0	15.5633 1076 8
13	2	—4	2	0	23.4263 0052 6
14	2	—3	0	0	24.2158 7788 5
15	2	—2	0	—1	25.1812 6236 4
16	2	—2	0	0	25.2360 7226 7
17	2	—2	0	1	25.2908 8217 1
18	2	—2	1	0	25.3513 8077 9
19	2	0	—2	0	27.0458 4400 8
20	2	0	—1	0	27.1611 5251 9
21	2	0	0	—1	27.2216 5112 8
22	2	0	0	0	27.2764 6103 1
23	2	0	0	1	27.3312 1093 5
24	3	—4	1	0	36.9492 2253 0
25	3	—3	—1	0	37.7387 9988 9
26	3	—2	—1	—1	38.7041 8436 7
27	3	—2	—1	0	38.7589 9427 1
28	3	—2	—1	1	38.8138 0417 4
29	3	—2	0	0	38.8743 0278 3
30	3	—2	1	0	38.9896 1129 4
31	3	0	—3	0	40.5687 6601 2
32	3	0	—1	0	40.7993 8303 5
33	4	—5	0	0	49.4519 5015 2
34	4	—4	0	0	50.4721 4453 4

No.	s_0 h_0 p_0 N'_0	
35	4—2—2 0	52.2819 1627 5
36	4—2 0 0	52.5125 3329 8
37	4 0 0 0	54.5529 2206 2
38	5—6 1 0	62.1852 9479 7
39	5—4—1 0	63.9950 6653 8
40	5—4 0 0	64.1103 7505 0
41	5—2—1 0	66.0354 5530 2
42	6—6 0 0	75.7082 1680 1
43	6—4 0 0	77.7486 0556 5
44	8—8 0 0	100.9442 8906 8

TABLE 4*a*

Harmonic representation of the high and low water lunitidal intervals of Bombay : identification numbers, amplitudes in minutes, and phase lags in degrees, of the terms that are used.

No.	High water intervals				Low water intervals			
0	78^m44		73^m68		442^m47		444^m79	
1	1.62	222.2	2.18	200.9	2.28	186.6	2.16	206.8
2	1.52	252.7	1.64	258.5	4.64	220.4	4.55	221.4
3	.90	7.2	.72	357.0	.67	322.0	.97	292.0
4	1.54	335.9	1.73	340.0	2.03	302.7	2.52	301.7
5	6.56	40.3	7.40	213.6	4.84	186.2	4.19	5.6
6	9.60	292.8	10.26	289.1	12.03	283.1	13.19	276.5
7	.62	93.6	.42	104.5	.80	78.3	.50	138.0
8	6.81	108.8	6.84	103.1	10.56	91.8	11.45	92.6
9	.79	33.5	.47	231.1	.92	16.0	.76	182.6
10	27.74	86.0	29.79	261.3	24.20	138.7	28.56	312.5
11	4.67	194.9	5.06	6.7	3.80	220.2	4.20	43.1
12	.86	271.9	.51	279.2	.68	277.4	.60	237.1
13	.18	135.9	.48	79.6	.97	148.5	1.41	135.6
14	2.82	4.4	3.12	352.3	3.10	17.7	3.12	1.1
15	1.51	65.8	1.31	62.0	1.35	46.6	1.62	39.7
16	44.59	168.2	45.48	158.3	46.76	159.7	50.05	145.1
17	.95	203.4	.90	190.8	.20	186.6	.27	67.8
18	4.04	269.0	3.34	85.7	2.24	314.9	2.91	167.6
19	.84	310.9	.63	290.3	1.18	276.7	1.41	272.0
20	2.34	112.6	2.15	304.6	.60	266.6	.99	31.9
21	.30	234.2	.69	45.9	.52	34.4	.29	65.0
22	2.24	291.5	2.31	262.8	8.56	250.9	9.12	241.8
23	1.36	316.2	1.34	297.0	3.34	312.6	2.98	306.4
24	3.86	12.0	4.26	348.5	4.96	348.8	5.46	325.9
25	.46	58.9	.30	67.3	.10	83.2	.12	308.2
26	.27	129.2	.36	144.8	.45	155.4	.04	53.4
27	2.64	206.1	2.51	196.2	1.97	220.5	2.41	189.2
28	.76	55.9	.42	251.0	.17	88.7	.45	236.8
29	8.65	151.1	8.90	319.0	6.23	195.4	7.75	4.9
30	1.92	155.6	1.83	157.8	2.35	127.1	2.25	103.1
31	.10	103.5	.23	124.7	.43	41.8	.21	106.3
32	1.08	324.5	.92	293.5	1.45	309.1	1.16	286.1
33	1.69	83.4	1.33	46.7	.81	49.9	.64	15.6
34	7.79	243.9	8.21	223.4	8.46	210.6	9.56	182.8
35	.16	279.3	.25	277.9	.14	176.7	.21	284.8
36	4.55	53.1	4.57	27.2	4.13	353.9	4.99	325.9
37	.20	195.8	.25	204.0	.76	91.4	.86	41.8
38	1.37	69.6	1.55	38.9	1.74	28.8	2.00	355.2
39	.90	297.6	.87	261.0	.45	237.6	.78	200.7

No.	High water intervals				Low water intervals			
40	2.98	245.9	2.94	38.9	2.11	228.7	2.36	36.7
41	.80	99.7	.69	67.1	.78	23.7	.94	326.9
42	2.03	308.0	2.28	279.8	2.13	250.5	2.29	212.9
43	1.79	132.7	1.83	91.5	1.75	38.2	2.30	0.0
44	.63	13.4	.71	320.8	.80	278.3	.77	231.2

TABLE 4*b*

Harmonic representation of the high and low water heights of Bombay : identification numbers, amplitudes in feet, and phase lags in degrees, of the terms that are used.

No.	High water heights				Low water heights			
	ft		ft		ft		ft	
0	14.762		14.752		6.173		6.243	
1	.075	287.3	.093	287.1	.119	95.8	.122	103.6
2	.125	238.0	.127	237.4	.011	282.1	.010	299.3
3	.016	184.0	.019	198.8	.025	9.7	.023	1.0
4	.095	38.7	.095	37.8	.201	345.4	.203	346.3
5	.368	154.5	.336	327.0	.394	101.9	.392	276.0
6	.023	45.8	.032	64.8	.026	194.8	.032	169.8
7	.002	140.4	.008	105.5	.020	344.9	.027	323.6
8	.864	55.3	.865	47.9	.860	231.7	.831	225.9
9	.010	36.2	.019	352.4	.039	148.5	.024	72.8
10	1.099	130.8	1.029	305.5	1.877	85.4	1.862	259.9
11	.143	223.0	.131	27.6	.278	189.1	.274	2.4
12	.012	359.7	.008	340.6	.026	219.1	.019	201.8
13	.009	265.5	.007	209.0	.018	354.2	.017	342.9
14	.110	79.3	.111	69.1	.124	254.0	.124	244.7
15	.018	133.2	.019	129.4	.038	318.8	.030	301.5
16	1.682	252.8	1.684	242.5	1.648	66.3	1.640	52.8
17	.027	165.5	.027	166.7	.043	348.9	.038	343.2
18	.070	359.1	.063	155.4	.063	256.8	.085	63.9
19	.077	152.9	.074	137.0	.086	315.5	.085	304.3
20	.102	178.8	.090	348.7	.278	125.2	.284	294.2
21	.021	264.1	.004	124.4	.048	226.4	.051	33.3
22	.431	54.3	.441	39.9	.343	214.8	.361	201.2
23	.116	147.5	.115	133.3	.097	319.1	.106	306.7
24	.076	85.3	.084	65.6	.098	250.5	.095	229.8
25	.026	124.4	.024	123.2	.013	312.9	.016	312.3
26	.007	226.5	.008	207.7	.003	36.3	.003	354.7
27	.230	310.8	.236	292.4	.246	116.3	.250	96.3
28	.006	97.0	.011	286.4	.014	36.9	.011	61.1
29	.204	245.6	.199	46.0	.056	37.8	.070	224.9
30	.040	248.4	.016	172.2	.034	27.3	.040	13.5
31	.006	213.8	.007	244.1	.008	124.3	.008	152.5
32	.097	107.8	.093	88.2	.083	258.6	.090	245.2
33	.020	152.7	.020	122.5	.014	291.6	.012	269.1
34	.143	311.4	.149	290.7	.122	105.1	.129	80.1
35	.026	8.4	.024	7.4	.020	211.0	.020	182.0
36	.084	139.4	.084	115.7	.066	258.8	.071	234.6
37	.007	315.5	.009	293.9	.005	21.7	.006	298.9
38	.019	167.0	.018	121.7	.026	305.3	.027	268.0
39	.030	31.8	.034	8.6	.036	170.7	.040	131.7
40	.050	323.4	.054	125.8	.019	139.1	.017	293.9
41	.031	214.2	.031	184.4	.023	313.3	.021	286.5
42	.026	15.0	.027	344.0	.021	153.2	.022	130.9
43	.022	226.7	.021	183.1	.018	313.4	.018	274.7
44	.004	61.5	.008	45.0	.005	170.2	.004	168.4

Table 2a. Coefficients in the normal equations for the harmonic analysis of 8,857 consecutive observed hourly tidal values. 64 constituents + height of mean sea level (see Table 1). Central time origin.

cos

	00	01	02	03	04	05	06	07	08	09
	8857.0000+	90.7526-	91.1224+	230.9591-	198.7488-	2.1765+	4.8057-	6.3396+	1.2115+	7.7064-
		4473.8523+	91.2460-	232.8171+	199.6095+	2.3274-	4.6829+	6.3356-	1.2100-	7.7028+
			4473.9639+	238.5655-	202.2452-	2.7856+	4.3103-	6.3237+	1.2054+	7.6918-
01		4383.1477+		4392.5635+	476.2306-	62.5412-	53.9968-	2.1943+	0.6817+	2.2760-
02		0.3436-	4383.0361+		4461.2413+	81.0470-	62.6015-	1.6414-	0.0508-	2.6868+
03		22.7365+	46.4966-	4464.4365+		4427.4119+	46.2867+	6.3878-	1.2322-	7.7439+
04		12.8024+	25.9854-	478.4071-	4395.7587+		4430.9004+	6.3545-	1.1845-	7.7644+
05		3.7620-	7.5914+	136.2076-	149.9121-	4429.5881+		4430.7474+	475.5279-	100.0848-
06		3.2155-	6.4868+	122.2630-	129.4673-	44.8357+	4426.0996+		4429.1040+	117.3039-
07		0.1845+	0.3709-	6.0617+	5.8650+	0.6242-	0.2507-	4426.2526+		4426.8011+
08		0.2824+	0.5672-	8.3881+	8.3068+	0.2662-	0.3051+	479.1099-	4427.8960+	
09		0.1456+	0.2921-	3.4876+	3.6528+	0.5121+	0.8063+	98.6640-	113.6552-	4430.1989+
10		0.0295-	0.0594+	1.6939-	1.4825-	0.6635+	0.6038+	49.2697-	97.1889-	476.7775-
11		0.2633-	0.5288+	7.7948-	7.7246-	0.2291+	0.3030-	34.4403+	1.6297+	101.1546-
12		0.2576-	0.5174+	7.6785-	7.5970-	0.2641+	0.2565-	29.6542+	1.7286-	89.8267-
13		0.1430-	0.2870+	3.5404-	3.6740-	0.4137-	0.7025-	32.8434+	32.4364+	4.0933-
14		0.2466-	0.4993+	7.3172-	7.2610-	0.1634+	0.3187-	19.9497+	2.4633+	36.6931-

[illegible]

sin

Table 2b. Inverse matrices for the harmonic analysis of 8,857 consecutive observed hourly tidal values. See Table 2a. Unit .00000 00000 1.

cos

	00	01	02	03	04	05	06	07	08	09
	11323259+	167878+	167661-	638475+	558040+	13323+	27213+	15190-	3283-	21481+
		22479919+	335176+	1287266-	1121631-	26119-	54004-	30366+	6560+	42947-
			22484145+	1319173+	1138671+	24532+	52737+	30324-	6543-	42903+
01		22815548+		23234399+	2637241+	371110+	319652+	13864-	4576-	16825+
02		36-	22818958+		22836640+	450126+	353084+	4765+	811-	9976-
03		123973-	253485+	22716479+		22602874+	224651-	31008+	6548+	44097-
04		79167-	160927+	2522105+	23076605+		22580661+	30660+	6259+	43976-
05		12777+	25648-	776291+	850721+	22630031+		22859410+	2481629+	616387+
06		10707+	21476-	692989+	735808+	182860-	22635819+		22880853+	732077+
07		858-	1715+	42218-	41889-	565+	1532-	22886577+		22926050+
08		1182-	2361+	52667-	53011-	1848-	4682-	2501719+	22888288+	
09		537-	1072+	17125-	18279-	4125-	5346-	602665+	708599+	22909037+
10		87+	173-	10508+	9557+	3079-	2807-	361611+	605280+	2525926+
11		998+	1995-	44481+	44771+	1566+	3957+	158744-	634-	580343+

Table 5a. Coefficients in the normal equations for the harmonic analysis of 6,689 consecutive observed high and low water lunitidal intervals or heights. 44 periodic terms + constant term (see Table 3). Central time origin.

cos

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
	6689.0000+	120.7767-	15.5547+	58.3811-	15.4055-	9.8957-	8.9085-	5.9348+	6.2534-	7.9333+	8.0568-	8.1524+	3.8582-	3.7323-	0.8276-	1.6640-	1.4113+	1.1549-	0.6101-	4.2614+	3.6704+	2.5685-	2.3640+	2.1524-	3.1343+	1.8787-	1.3297-	1.4820+	1.6289-	2.4648+	2.9576+	1.7664-	0.6715+	1.1206+	1.3758-	2.2104-	1.7772-	2.0196-	1.8990-	0.3763-	1.1378-	0.1414+	1.3162+	1.4952+	1.2318-	00
		3404.7875+	15.9199-	58.3800+	15.4747+	9.8794+	8.8949+	5.9236-	6.2417+	7.9195-	8.0429+	8.1385-	3.8505+	3.7257+	0.8268+	1.6618+	1.4095-	1.1535+	0.6085+	4.2543-	3.6645-	2.5646+	2.3604-	2.1492+	3.1291-	1.8754+	1.3273+	1.4793-	1.6260+	2.4605-	2.9526-	1.7636+	0.6702-	1.1189-	1.3736+	2.2067+	1.7743+	2.0163+	1.8958+	0.3755+	1.1359+	0.1413-	1.3140-	1.4927-	1.2298+	01
			3336.7973+	81.9434+	15.3567+	9.8788+	8.8405+	5.9758-	6.2886+	7.9318-	8.0494+	8.1393-	3.8945+	3.7225+	0.7940+	1.6270+	1.3759-	1.1213+	0.6299+	4.2244-	3.6291-	2.5300+	2.3265-	2.1161+	3.1051-	1.8708+	1.3273+	1.4779-	1.6232+	2.4487-	2.9327-	1.7429+	0.6747-	1.1047-	1.3581+	2.1912+	1.7572+	1.9988+	1.8819+	0.3764+	1.1301+	0.1366-	1.3022-	1.4802-	1.2196+	02
01		3284.2125+		3333.1955+	179.1133+	3.9531-	2.9397-	3.1094+	3.1843-	3.4916+	3.4797-	3.4567+	2.1947-	1.6810-	0.0018-	0.3675-	0.2645+	0.1612-	0.5313-	1.6356+	1.2967+	0.7944-	0.7071+	0.6178-	1.2005+	0.8340-	0.6286-	0.6840+	0.7369-	1.0275+	1.1675+	0.5953-	0.3700+	0.3676+	0.4746-	0.8722	0.6477	0.7595-	0.7451-	0.1889-	0.4759-	0.0138+	0.4919+	0.5693+	0.4723-	03
02		7.2816-	3352.2027+		3337.0915+	9.8284-	8.6339-	6.1002+	6.3956-	7.9275+	8.0272-	8.0996+	4.0039-	3.6927-	0.6938-	1.5166-	1.2704+	1.0210-	0.6892-	4.1138+	3.5056+	2.4151-	2.2147+	2.0077-	3.0178+	1.8471-	1.3200-	1.4656+	1.6059-	2.4005+	2.8585+	1.6727-	0.6841+	1.0570+	1.3054-	2.1339-	1.6977-	1.9369-	1.8311-	0.3766-	1.1070-	0.1224+	1.2605+	1.4355+	1.1834-	04
03		3.1843+	42.7945+	3355.8045+		3344.5884+	214.0939+	19.9352+	18.1635-	8.6732+	6.9971-	5.3920+	10.3516-	1.9943-	2.8980+	2.5001+	2.5317-	2.5545+	2.5606-	0.2784-	1.1881-	1.7874+	1.8470-	1.8998+	0.1851-	0.7676-	0.8331-	0.8018+	0.7679-	0.4941+	0.0937+	0.7153+	0.8142+	0.5130-	0.4572+	0.0708-	0.3350+	0.2077+	0.0311-	0.3034-	0.2305-	0.2972-	0.1523-	0.0980-	0.0565+	05
04		3.5357+	0.1981+	208.6685+	3351.9085+		3342.6338+	29.9502+	28.4849-	21.7925+	20.0543-	18.3537+	13.5956-	3.9947-	2.2099+	1.3900+	1.5595-	1.7225+	2.6990-	2.0109+	0.8536+	0.2857+	0.4503-	0.6122+	1.3362+	1.6052-	1.3994-	1.4428+	1.4813-	1.6267+	1.4918+	0.1753-	1.0654+	0.0417+	0.2085-	1.0713-	0.5008-	0.7246-	0.8799-	0.4471-	0.7208-	0.2099-	0.4433+	0.5717+	0.4883-	06
05		0.0364+	0.8904+	0.7770-	1.7716-	3344.4116+		3346.6457+	62.5301-	217.0405+	164.9816-	137.8480+	4.0374+	2.0956-	3.4564-	3.8679-	3.6439+	3.4093-	1.6846+	4.2231+	4.4593+	3.9546-	3.8132+	3.6598-	2.6937+	0.8675-	0.3645-	0.5121+	0.6573-	1.5531+	2.2797+	1.9910-	0.1501-	1.2735+	1.4172-	1.6043-	1.6118-	1.6811-	1.3739-	0.0282-	0.6474-	0.3467+	1.0789+	1.1626+	0.9271-	07
06		0.2774+	0.2017+	3.3136+	0.3784-	216.0766+	3346.3662+		3346.6307+	361.8407-	216.9205+	164.8496-	5.7488-	1.7541+	3.5817+	3.9213+	3.7133-	3.4943+	1.8685-	4.0099-	4.3302-	3.9190+	3.7920-	3.6530+	2.5540-	0.7374+	0.2575+	0.3996-	0.5397+	1.4106-	2.1354-	1.9463+	0.2245+	1.2496-	1.3790+	1.5015+	1.5495+	1.6018+	1.2887+	0.0039-	0.5856+	0.3557-	1.0295-	1.1028-	0.8771+	08
07		0.3298+	1.2987-	6.0630+	2.6001+	23.3010+	31.6142+	3342.3543+		3345.8826+	61.6726-	61.4695+	15.9707+	0.6623+	3.9768-	3.8198-	3.7392+	3.6467-	2.8443+	2.1909+	3.0193+	3.2564-	3.2368+	3.2062-	1.3930+	0.1749+	0.4570+	0.3647-	0.2716+	0.3509+	0.9899+	1.4617-	0.6796-	0.9716+	0.9967-	0.6930-	0.9962-	0.9341-	0.6152-	0.2095+	0.1315-	0.3789+	0.6123+	0.6114+	0.4715-	09
08		0.3069-	1.2682+	5.7242-	2.5374-	21.3350-	29.8962-	58.2466-	3342.3693+		3345.6820+	61.4645-	17.8294-	1.0357-	4.0104+	3.7756+	3.7155-	3.6433+	2.9755-	1.8899-	2.7909-	3.1272+	3.1245-	3.1110+	1.1998-	0.3152-	0.5638-	0.4801+	0.3952-	0.1830-	0.8027-	1.3728+	0.7440+	0.9191-	0.9277+	0.5608+	0.9006+	0.8214+	0.5047+	0.2394-	0.0597+	0.3787-	0.5414-	0.5290-	0.4038+	10
09		0.1373+	0.9983-	3.1618+	1.9882+	10.3372+	21.4436+	213.1300+	358.0441-	3343.1174+		3345.4672+	19.7337+	1.4092+	4.0313-	3.7190-	3.6796+	3.6281-	3.0980+	1.5810+	2.5522+	2.9872-	3.0014+	3.0052-	1.0022+	0.4548+	0.6692+	0.5943-	0.5179+	0.0139+	0.6124+	1.2792-	0.8061-	0.8636+	0.8556-	0.4265-	0.8020-	0.7058-	0.3923-	0.2685+	0.0123+	0.3773+	0.4688+	0.4449+	0.3348-	11
10		0.1096-	0.9391+	2.7221-	1.8685-	8.4084-	19.4442-	161.1850-	213.2501+	59.1041-	3343.3180+		3346.1602+	3.7043+	3.8524+	4.4932+	4.1704-	3.8366+	1.4612-	5.5753-	5.5930-	4.7222+	4.5112-	4.2866+	3.3345-	1.3083+	0.6937+	0.8691-	1.0409+	2.0826-	2.8760-	2.2759+	0.0434-	1.4199-	1.6097+	1.9801+	1.8777+	1.9936+	1.6751+	0.1161+	0.8468+	0.3396-	1.2608-	1.3756-	1.0981+	12
11		0.0817+	0.8772-	2.2770+	1.7434+	6.5469+	17.4857+	134.1776+	161.3171-	59.1055+	59.3122-	3343.5328+		3345.7438+	61.5854+	30.2119+	28.5600-	26.9091+	18.7342-	11.9924-	14.7346-	14.8256+	14.4678-	14.0707+	3.6836-	0.1572+	0.5391-	0.3205+	0.1023-	1.3151-	2.6681-	3.1622+	1.0965+	1.8596-	1.9386+	1.5880+	1.9904+	1.9123+	1.3210+	0.2518-	0.4196+	0.6079-	1.1580-	1.1844-	0.9009+	13
12		0.3503-	1.1259+	6.0608-	2.2494-	14.0220-	15.9596-	7.9016+	9.6567-	19.8687+	21.6690-	23.5025+	3342.8398+		3344.0832+	12.2218-	8.3377+	4.8171-	15.2845-	20.7256+	15.8811+	9.5568-	8.3770+	7.2097-	5.3005+	3.5740-	2.6050-	2.8291+	3.0424-	4.2322+	4.7895+	2.3287-	1.4418+	1.2185+	1.5541-	2.8017-	2.0745-	2.3862-	2.2172-	0.5574-	1.4019-	0.0395+	1.3840+	1.5928+	1.2677-	14
13		0.1771-	0.6050+	3.0922-	1.1990-	4.2075-	4.9138-	1.0592+	1.3802-	3.4176+	3.6944-	3.9626+	0.7468+	3343.2562+		3343.7055+	59.6460-	59.5996+	162.8083-	28.6873+	19.0387+	8.6678-	6.8978+	5.1938-	5.2464+	4.1722-	3.2070-	3.4088+	3.5979-	4.6012+	4.8791+	1.8502-	1.9769+	0.9014+	1.2626-	2.7719-	1.8389-	2.2101-	2.1620-	0.6936-	1.4730-	0.1119-	1.2580+	1.4850+	1.1889-	15
14		0.2731+	0.6173-	4.3292+	1.2304+	6.0904+	5.3469+	5.1926-	5.4605+	6.6438-	6.7589+	6.8520-	4.9296+	63.1525+	3344.9168+		3343.8121+	59.7571-	214.8767+	30.2957-	20.8548-	10.4336+	8.5913-	6.8154+	5.4037-	4.0973+	3.1019+	3.3148-	3.5153+	4.5973-	4.9643-	2.0305+	1.8584-	1.0058-	1.3637+	2.8146+	1.9296+	2.2864+	2.1986+	0.6609+	1.4662+	0.0695+	1.3038-	1.5271-	1.2188+	16
15		0.2505+	0.5272-	3.9161+	1.0519+	5.4332+	4.5448+	5.0406-	5.2510+	6.0759-	6.1402+	6.1837-	5.0031+	31.3524+	10.9797-	3345.2945+		3343.9274+	359.8009-	31.8993+	22.7066+	12.2660-	10.3512+	8.5033-	5.5443+	4.0075-	2.9850-	3.2085+	3.4198-	4.5775+	5.0333+	2.2056-	1.7324+	1.1075+	1.4609-	2.8480-	2.0143-	2.3554-	2.2278-	0.6258-	1.4544-	0.0266-	1.3453+	1.5641+	1.2447-	17
16		0.2546-	0.5483+	3.9968-	1.0936-	5.5250-	4.6939-	4.9736+	5.1952-	6.1038+	6.1803-	6.2361+	4.8419-	29.8211-	7.2170+	61.1307-	3345.1879+		3344.8098+	34.8780-	30.0791-	22.1740+	20.2469-	18.3577+	5.8842-	2.9748+	1.8862+	2.1570-	2.4187+	3.9632-	4.9671-	3.1518+	0.7105-	1.6808-	1.9705+	2.8011+	2.3893+	2.5945+	2.2180+	0.3341+	1.2420+	0.2619	1.5007-	1.6698-	1.3065+	18
17		0.2578+	0.5675-	4.0639+	1.1314+	5.5979+	4.8260+	4.8913-	5.1232+	6.1114-	6.1998+	6.2676-	4.6671+	28.2865+	3.8213-	60.9754+	61.0195-	3345.0726+		3344.3157+	214.4511+	241.2161-	192.9334+	162.6619-	0.0012-	3.8301+	3.7964+	3.6937-	3.5799+	2.6404-	1.0976-	2.3725-	3.2774-	1.4210+	1.1822-	0.5925+	0.7140-	0.2814-	0.3795+	0.8947+	0.8089+	0.7900+	0.2111+	0.0428+	0.0253+	19
18		0.2596-	0.6504+	4.2017-	1.2935-	5.6949-	5.3577-	4.0491+	4.3333-	5.7731+	5.9330-	6.0744+	3.2690-	20.7642-	15.3804-	163.3583-	215.2938+	360.0839-	3344.1902+		3343.5425+	358.8557-	213.9940+	161.9912-	2.9340-	5.0339+	4.4519+	4.4874-	4.5081+	4.3835-	3.4327-	0.6980-	3.3850-	0.5485+	0.1973-	1.8190+	0.4219+	0.9104+	1.3404+	0.9762+	1.3009+	0.6141+	0.4670-	0.6964-	0.5949+	20
19		0.0292-	0.1066-	0.2243-	0.2059+	0.4791-	0.5290+	2.3254+	2.2435-	1.4248+	1.2863-	1.1414+	3.3577-	10.6167-	18.4233+	26.5083+	28.0853-	29.6650+	32.6668-	3344.6843+		3343.4473+	59.3554-	59.2777+	5.0628+	5.2634-	4.2905-	4.4434+	4.5805-	5.2036+	4.9397+	0.9089-	2.9157+	0.3152+	0.7075-	2.6037-	1.3573-	1.8142-	1.9684-	0.8873-	1.5212-	0.3577-	0.9886+	1.2341+	0.9993-	21
20		0.1320-	0.1654+	1.9068-	0.3341-	2.6701-	1.6111-	3.6932+	3.7266-	3.5292+	3.4624-	3.3826+	4.3028-	14.3175-	13.7200+	16.8007+	18.6436-	20.5296+	28.3019-	215.7194+	3345.4575+		3343.4902+	59.4050-	5.3427-	5.2517+	4.2266+	4.3958-	4.5492+	5.2796-	5.1219-	1.1431+	2.8162-	0.4419-	0.8360+	2.6928+	1.4830+	1.9297+	2.0400+	0.8636+	1.5365+	0.3149+	1.0552-	1.3002-	1.0479+	22
21		0.2006+	0.3758-	3.0698+	0.7511+	4.1519+	3.2145+	4.3008-	4.4289+	4.8003-	4.8060+	4.7950-	4.4501+	15.3146+	7.8664-	6.7376-	8.5769+	10.4888-	21.0622+	243.0682-	361.0291-	3345.5527+		3343.5464+	5.6075+	5.2216-	4.1474-	4.3324+	4.5017-	5.3381+	5.2884+	1.3755-	2.7060+	0.5678+	0.9624-	2.7730-	1.6043-	2.0392-	2.1048-	0.8367-	1.5466-	0.2709-	1.1185+	1.3620+	1.0931-	23
22		0.2081-	0.4023+	3.2008-	0.8034-	4.3117-	3.4079-	4.3231+	4.4635-	4.9156+	4.9323-	4.9318+	4.3973-	15.0874-	6.7816+	5.0411+	6.8141-	8.6593+	19.2515-	194.8484+	216.1765+	61.4213-	3345.5098+		3344.3079+	61.7769+	30.0970+	28.5324-	26.9690+	19.3644-	7.6663-	10.6206-	14.0300-	3.0978+	2.4953-	1.1899+	1.4281-	0.5397-	0.6529+	1.5148+	1.3679+	1.3155+	0.3327+	0.0672+	0.0370+	24
23		0.2148+	0.4272-	3.3206+	0.8528+	4.4563+	3.5886+	4.3313-	4.4834+	5.0140-	5.0414+	5.0515-	4.3309+	14.8186+	5.7142-	3.4166-	5.1235+	6.9022-	17.4817+	164.6333-	164.1754-	61.2973+	61.3717-	3345.4536+		3345.2885+	10.8771-	7.0346+	3.5598-	16.1383-	32.1485-	19.5792+	6.9658+	4.3350-	4.3006+	3.1546+	4.0643+	3.6509+	2.1450+	0.4763-	0.6106+	1.0183-	1.7060-	1.7127-	1.2028+	25
24		0.0257-	0.0319-	0.2815-	0.0602+	0.4249-	0.0751+	1.2090+	1.1783-	0.8431+	0.7828-	0.7192+	1.5573-	2.5697-	3.3383+	3.3710+	3.5048-	3.6281+	4.0115-	0.3751+	1.7961-	3.4409+	3.6696-	3.8888+	3344.6921+		3345.1326+	61.0477-	60.9715+	163.8722-	164.4103-	27.7557+	5.4702+	4.3472-	4.4296+	3.9574+	4.3715+	4.0751+	2.5782+	0.1864-	1.0055+	0.8425-	1.8284-	1.8941-	1.3392+	26
25		0.1392-	0.3888+	2.3031-	0.7700-	2.6961-	2.7258-	1.4212+	1.5650-	2.3813+	2.4763-	2.5636+	0.7717-	1.8050-	2.6793-	3.6393-	3.4595+	3.2670-	1.6207+	5.2121+	5.7353+	5.2560-	5.1368+	4.9998-	62.9610+	3343.7115+		3345.1840+	61.0946-	215.8793+	194.6385+	29.3037-	7.1698-	4.5023+	4.5343-	3.7992-	4.3940-	4.0365-	2.4821-	0.2861+	0.8919-	0.9149+	1.8129+	1.8593+	1.3084-	27
26		0.1547-	0.4088+	2.5262-	0.8101-	2.9634-	2.8841-	1.7758+	1.9215-	2.7145+	2.8019-	2.8804+	1.1584-	2.4145-	2.0721-	3.0463-	2.8329+	2.6088-	0.8362+	5.3760+	5.4639+	4.6425-	4.4737+	4.2888-	31.4673+	12.3244-	3343.8674+		3345.2261+	360.7391-	242.8722-	30.8503+	8.9405+	4.6443-	4.6252+	3.6271+	4.4023+	3.9844+	2.3772+	0.3855-	0.7748+	0.9847-	1.7914-	1.8183-	1.2732+	28
27		0.1496+	0.4008-	2.4504+	0.7941+	2.8686+	2.8186+	1.6670-	1.8108+	2.6028-	2.6913+	2.7714-	1.0459+	2.2194+	2.1913+	3.1398+	2.9386-	2.7264+	1.0192-	5.2148-	5.4083-	4.6904+	4.5372-	4.3679+	29.8486-	8 5201+	59.7289-	3343.8160+		3345.2476+	215.6394+	33.5392-	19.0223-	5.0571+	4.7141-	2.1085-	3.9885-	3.2297-	1.4586-	0.9988+	0.0672+	1.3473+	1.4838+	1.3813+	0.9284-	29
28		0.1440-	0.3915+	2.3666-	0.7754-	2.7646-	2.7438-	1.5531+	1.6946-	2.4828+	2.5723-	2.6536+	0.9305-	2.0184-	2.3020-	3.2217-	3.0332+	2.8336-	1.1970+	5.0375+	5.3350+	4.7219-	4.5846+	4.4313-	28.2267+	5.0785-	59.6035+	59.6821-	3343.7739+		3344.7520+	31.3487-	28.4131-	4.6232+	3.9655-	0.0265+	2.7891-	1.7747-	0.1385-	1.5214+	1.0215+	1.5184+	0.8613+	0.6293+	0.3647-	30
29		0.0960+	0.2998-	1.6312+	0.5927+	1.8699+	2.0437+	0.6850-	0.8006+	1.5058-	1.5942+	1.6780-	0.0999+	0.5576+	2.8781+	3.5512+	3.4595-	3.3558+	2.2901-	3.5614-	4.5250-	4.6394+	4.6161-	4.5776+	20.1340-	14.5266-	162.2945-	214.2912+	359.1458-	3343.7524+		3343.7556+	193.5762+	0.4556-	0.4856-	4.8896-	1.8572-	2.7059-	2.8061-	1.5865-	2.3598-	0.8281-	1.0480+	1.3870+	1.0559-	31
30		0.0238+	0.1404-	0.4960+	0.2759+	0.5107+	0.8721+	0.4230+	0.3582-	0.1218-	0.1927+	0.2635-	0.8563-	1.1901-	2.9878+	3.2572+	3.2912-	3.3146+	3.0896-	1.2391-	2.7691-	3.6778+	3.7865-	3.8840+	7.7392-	30.7282-	162.8849-	193.1433+	241.4121-	214.5311+	3344.2480+		3344.8483+	4.5667+	4.7724-	5.1553-	4.8803-	4.6763-	3.0843-	0.1787-	1.4792-	0.6070+	1.9196+	2.0508+	1.4448-	32
31		0.1340+	0.2738-	2.0773+	0.5450+	2.4883+	2.0351+	2.2341-	2.3150+	2.6256-	2.6412+	2.6477-	2.0438+	3.5384+	0.9468-	0.2705-	0.5094+	0.7480-	2.2309+	3.8809-	2.4781-	0.8227+	0.5575-	0.2885+	11.9885-	19.5697+	27.4400+	29.0774-	30.7140+	34.0056-	32.4145-	3345.2444+		3343.9551+	8.3637+	20.0603+	8.2126+	7.8236+	4.7876+	2.3408+	3.6507+	1.0678+	1.5197-	1.9175-	1.3074+	33
32		0.1603+	0.4024-	2.5876+	0.7979+	3.0254+	2.8426+	2.0022-	2.1395+	2.8529-	2.9264+	2.9904-	1.4547+	2.7696+	1.3269+	2.2240+	1.9998-	1.7678+	0.0470-	4.9781-	4.6718-	3.6329+	3.4372-	3.2288+	15.5252-	8.1928+	6.4687+	8.2356-	10.0699+	20.4761-	29.9679-	194.2056+	3344.1517+		3343.8841+	29.6100-	8.3412-	8.0208-	4.8625-	2.8809-	4.0412-	1.5485-	1.2663+	1.7170+	1.1793-	34
33		0.1232-	0.2711+	1.9347-	0.5385-	2.2355-	1.9205-	1.8064+	1.8874-	2.2466+	2.2733-	2.2923+	1.5348-	2.4714-	0.1927+	0.3477-	0.1782+	0.0076-	1.1107-	2.9065+	2.1602+	1.1476-	0.9785+	0.8054-	4.4590+	4.5735-	4.3018-	4.5400+	4.7648-	5.7121+	5.7653+	1.7444-	4.2153+	3345.0449+		3344.7797+	193.6238+	14.5095+	0.4327+	3.8658-	2.5635-	3.5725-	1.6488-	1.1788-	0.5854+	35
34		0.1111+	0.2374-	1.7358+	0.4718+	2.0076+	1.6905+	1.6862-	1.7553+	2.0467-	2.0656+	2.0774-	1.4683+	2.3227+	0.3701-	0.1077+	0.0475+	0.2032-	1.2009+	2.5501-	1.7854-	0.8179+	0.6592-	0.4977+	3.7581-	4.3383+	4.1879+	4.3742-	4.5471+	5.1817-	4.9675-	0.8595+	4.2399-	7.1911+	3345.1159+		3343.8877+	8.2434-	4.7747-	4.0860-	4.7989-	2.6430-	0.6234+	1.1749+	0.8444-	36
35		0.0276-	0.1112+	0.5035-	0.2189-	0.5329-	0.6951-	0.0246-	0.0196-	0.3190+	0.3601-	0.4004+	0.2795+	0.2718+	1.3162-	1.4531-	1.4467+	1.4356-	1.2129+	0.8188+	1.3525+	1.6052-	1.6270+	1.6436-	1.3500+	1.8658+	2.5912+	2.4542-	2.3077+	1.0676-	0.5590+	4.0189-	3.7777-	19.0886+	28.7710-	3344.2203+		3344.0698+	4.2613-	5.4848-	5.5041-	3.9185-	0.1843-	0.4574+	0.4219-	37
36		0.0824+	0.1617-	1.2673+	0.3219+	1.4728+	1.1723+	1.3647-	1.4081+	1.5603-	1.5641+	1.5625-	1.2567+	1.9175+	0.6542-	0.3135	0.4344+	0.5542-	1.2811+	1.7797-	1.0319-	0.2025+	0.0718-	0.0597-	2.4301-	3.7130+	3.7649+	3.8615-	3.9456+	4.0683-	3.4643-	0.4795-	4.0282-	6.9460+	7.0643-	194.1580+	3345.1123+		3344.7169+	28.6298-	19.2449-	14.2425-	2.9015-	1.8465-	0.6803+	38
37		0.0496+	0.0799-	0.7385+	0.1597+	0.8712+	0.6108+	0.9639-	0.9808+	0.9963-	0.9864+	0.9730-	0.9640+	1.4082+	0.8461-	0.6503-	0.7317+	0.8108-	1.2451+	0.9780-	0.3060-	0.3381-	0.4342+	0.5292-	1.2148-	2.9505+	3.1616+	3.1844-	3.1964+	2.9290	2.0835-	1.4002-	3.5791-	6.5609+	6.7962-	14.7069+	7.1620-	3344.9302+		3344.7174+	215.5027+	7.6347-	4.8132-	4.5779-	2.0054+	39
38		0.0204-	0.0802+	0.3688-	0.1580-	0.3859-	0.4958-	0.0035-	0.0275-	0.2364+	0.2649-	0.2927+	0.1798+	0.1610+	0.8560-	0.9415-	0.9358+	0.9270-	0.7752+	0.5396+	0.8728+	1.0259-	1.0380+	1.0469-	0.7584+	0.9724+	1.3246+	1.2503-	1.1713+	0.5240-	0.3153+	1.9671-	1.8074-	3.8513+	4.0460-	0.0588-	4.2376-	4.0379-	3344.2831+		3345.0350+	19.4511-	5.1010-	4.3137-	1.7835+	40
39		0.1065-	0.2615+	1.7100-	0.5181-	1.9079-	1.7633	1.2906+	1.3718-	1.7872+	1.8275-	1.8618+	0.9497-	1.5146-	0.5197-	0.9353-	0.8210+	0.7039-	0.1335-	2.2397+	2.0171+	1.4939-	1.3961+	1.2934-	2.7466+	1.4480-	0.9685-	1.1251+	1.2786-	2.1726+	2.7983+	2.1458-	0.3555+	2.6796+	3.3724-	5.0427-	4.8470-	6.4520-	29.7512-	3344.2826+		3344.4159+	4.9563-	4.9604-	2.0278+	41
40		0.0859-	0.2240+	1.3968-	0.4435-	1.5467-	1.4904-	0.9303+	1.0026-	1.3939+	1.4355-	1.4724+	0.6070-	1.0232-	0.7469-	1.0835-	0.9986+	0.9102-	0.2400+	1.8498+	1.8334+	1.5221-	1.4567+	1.3864-	2.3025+	0.6310-	0.1374-	0.2819+	0.4259-	1.3441+	2.1499+	2.3733-	0.5034-	3.4596+	4.0156-	3.6383-	5.0968-	6.0959-	20.2535-	214.6678+	3343.9650+		3344.0086+	8.1303-	1.1119+	42
41		0.1058-	0.2515+	1.6869-	0.4986-	1.8853-	1.7051-	1.3452+	1.4215-	1.7982+	1.8325-	1.8607+	1.0361-	1.6098-	0.3118-	0.7185-	0.6020+	0.4833-	0.3417-	2.1676+	1.8506+	1.2712-	1.1671+	1.0589-	2.5924+	1.7105-	1.3099-	1.4491+	1.5838-	2.3231+	2.7430+	1.6656-	0.8043+	1.7009+	2.3096-	4.6830-	3.6103-	5.0145-	15.3126-	7.7707-	20.0473-	3344.5841+		3344.1682+	0.3989+	43
42		0.0563-	0.1181+	0.8760-	0.2347-	0.9856-	0.8195-	0.8372+	0.8694-	1.0017+	1.0090-	1.0129+	0.7283-	1.0524-	0.2114+	0.0044+	0.0720-	0.1394+	0.5661-	1.0501+	0.7068+	0.2896-	0.2217+	0.1529-	1.1492+	1.3672-	1.2750-	1.3214+	1.3634-	1.5095+	1.3984+	0.1289-	1.1585+	0.4046-	0.1449+	2.0835-	0.4467-	1.1228-	3.3518-	4.0953-	4.7947-	4.0559-	3344.9914+		3344.1982+	44
43		0.0327-	0.0599+	0.4964-	0.1193-	0.5656-	0.4303-	0.5560+	0.5703-	0.6104+	0.6088-	0.6051+	0.5234-	0.7306-	0.3261+	0.2069+	0.2501-	0.2927+	0.5414-	0.5769+	0.2794+	0.0283+	0.0756-	0.1227+	0.6043+	1.0796-	1.0823-	1.0982+	1.1105-	1.0834+	0.8527+	0.2765+	1.0835+	0.8113-	0.6469+	1.3148-	0.2363+	0.2807-	2.0117-	3.6775-	3.7430-	3.9480-	7.2751-	3344.8318+		
44		0.0238+	0.0470-	0.3667+	0.0936+	0.4111+	0.3282+	0.3737-	0.3856+	0.4284-	0.4295+	0.4290-	0.3370+	0.4670+	0.1526-	0.0688-	0.0974+	0.1256-	0.2980+	0.4094-	0.2400-	0.0518+	0.0221-	0.0077-	0.4133-	0.5989+	0.5770+	0.5906-	0.6022+	0.6221-	0.5300-	0.0580-	0.5443-	0.2881+	0.1964-	0.6758+	0.0108+	0.2416+	0.8017+	1.1290+	1.2097+	1.0502+	0.2994+	0.2221-	3344.8018+	
		01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	

sin

Table 5b. Inverse matrices for the harmonic analysis of 6,689 consecutive observed high and low water lunitidal intervals or heights. See Table 5a. Unit .00000 00000 1.

cos

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
	14962251+	525140+	74389-	252079+	54151+	41468+	37450+	21221-	20498+	30355-	32123+	33030-	17617+	15943+	3478+	7278+	6245-	5305+	3752+	16410-	13200-	8271+	8177-	7589+	14078-	8409+	4806+	5217-	5290+	8949-	11375-	7475+	3682-	4899-	6057+	9299+	7223+	8805+	8374+	1479+	4996+	481-	5729-	6528-	5380+	00
		29400274+	152117+	504210-	108947-	82807-	74794-	42364+	40922-	60611+	64143-	65956+	35167-	31833-	6951-	14538-	12476+	10596-	7489-	32768+	26361+	16519-	16331+	15157-	28112+	16791-	9595-	10416+	10562-	17868+	22713+	14928-	7350+	9784+	12096-	18569-	14423-	17583-	16720-	2951-	9976-	961+	11439+	13036+	10744-	01
			29989951+	736424-	100990-	86443-	77530-	44704+	43146-	63297+	66953-	68802+	37133-	33219-	6931-	14850-	12711+	10770-	7967-	33974+	27246+	16960-	16755+	15533-	29112+	17481-	10029-	10875+	11022-	18549+	23525+	15388-	7702+	10075+	12475-	19241-	14898-	18185	17320-	3091-	10356-	960+	11828+	13488+	11117-	02
01		30448910+		30119312+	1609126-	34165+	25140+	23394-	22228+	26505-	27745+	28079-	20133+	14813+	143-	3433+	2608-	1925+	5104+	13080-	9608-	4846+	4672-	4136+	11018-	7569+	4742+	5026-	5028+	7501-	8968-	5126+	3837-	3268-	4254+	7481+	5318+	6732+	6687+	1528+	4226+	15-	4355-	5056-	4198+	03
02		66490+	29836183+		30055203+	83446+	73747+	44005-	42398+	61015-	64474+	66163-	36538+	31852+	6023+	13616+	11592-	9766+	7932+	32135-	25601-	15719+	15507-	14337+	27448-	16664+	9635+	10427-	10554+	17577-	22188-	14369+	7435-	9385-	11661+	18160+	13970+	17098+	16332+	2978+	9808+	837-	11110-	12686-	10460+	04
03		27804-	382056-	29920610+		30025056+	1918675-	154568-	136752+	40749-	34988+	24814-	86588+	17086+	25761-	20880-	20775+	20099-	17625+	1400+	8631+	15048-	15845+	16330-	2436+	5910+	6447+	6098-	5677+	2160-	1438+	6566-	6389-	4818+	4428-	15-	3513-	2496-	346-	2370+	1440+	2597+	1745+	1339+	880-	05
04		30376-	21889+	1862342-	29949819+		30047244+	231578-	209270+	149124-	147202+	137919-	119372+	35496+	19674-	10750-	11880+	12370-	20732+	17380-	7008-	4539-	5391+	6513-	12224-	14060+	10842+	10937-	10621+	11094-	10233-	1482+	9607-	356-	1837+	9039+	3791+	6234+	7709+	3599+	6107+	1934+	3821-	4945-	4235+	06
05		126-	8275-	9503+	15647+	30029488+		30128077+	206759+	1881810-	1413658+	1168416-	14672-	17642+	26377+	29144+	27158-	24909+	6608-	29397-	29199-	25482+	25893-	25224+	22840-	8288+	2496+	3351-	3757-	11669-	17808-	15591+	820-	10269-	11588+	12952+	12513+	13936+	11659+	355+	5786+	2465-	8903-	9678-	7730+	07
06		2387-	1353-	28447-	4976+	1933842-	30016062+		30425680+	3214319+	1875318-	1395096+	20357+	15350-	25379-	27662-	25837+	23745-	6871+	26965+	27116+	24008-	24420+	23830-	20979+	7314-	1999-	2795+	3185-	10566-	16333+	14529-	470+	9586+	10773-	11828-	11577-	12832-	10660-	223-	5221-	2361+	8201+	8892+	7094-	08
07		2774-	11140+	49833-	17949-	182346-	246236-	30157468+		30370071+	250352+	315023-	135263-	10738-	30938+	27047+	26378-	25123+	17706-	9550-	15880-	20490+	21303-	21536+	8496-	2644-	4718-	4183+	3713-	1473-	6227-	9894+	5120+	6881-	6925+	3659+	6423+	5982+	3566+	1845-	446+	2823-	3931-	3817-	2890+	09
08		2523+	10327-	45567+	16687+	162206+	222160+	179101+	30451687+		30095045+	395066+	153194+	13111+	32788-	28263-	27645+	26391-	19302+	8748+	15809+	21181-	22061+	22362-	7988+	3470+	5404+	4865-	4369+	893+	5786+	10003-	5744-	6991+	6956-	3240-	6339-	5758-	3219-	2072+	127-	2979+	3799+	3622+	2718-	10
09		761-	7381+	19438-	12979-	51590-	143798-	1855666-	3190150+	30384530+		30023778+	170863-	16045-	33608+	28371+	27874-	26702+	20598-	6908-	14679-	20918+	21844-	22238+	6674-	4560-	6154-	5638+	5133-	86+	4719-	9544+	6371+	6726-	6567+	2371+	5806+	5041+	2466+	2302-	387-	3038-	3353-	3088-	2274+	11
10		670+	7427-	18342+	13156+	45240+	140428+	1386622+	1850354-	236009+	30109206+		29889079+	30056-	35783-	40055-	37083+	33819-	7042+	42328+	40901+	34544-	34995+	33938-	30662+	12010-	4231-	5337+	5827-	16027+	23820+	20065-	1947+	12959+	14738-	17076-	16045-	18016-	15215-	764-	7747-	2890+	11367+	12421+	9904-	12
11		484-	7071+	15531-	12680-	33510-	129247-	1141595-	1370375+	301728-	382049+	30035569+		29908514+	554283-	260493-	243225+	222741-	117038+	90565+	108648+	114950-	117158+	115465-	35643+	3706-	4903+	3291-	2197+	12989+	25815+	28429-	6775-	17232+	18188-	14802-	17994-	18296-	13006-	1987+	4800-	5219+	11008+	11384+	8674-	13
12		3139+	11021-	54039+	17286+	119048+	139495+	45221-	49226+	164837-	183943+	201631-	29920349+		29917899+	121999+	88905-	63304+	147621+	170600-	121750-	61155+	58163-	50906+	50111-	33172+	20014+	21196-	21178+	31703-	37879-	20818+	15184-	11266-	14437+	24716+	17605+	21815+	20473+	4583+	12768+	97-	12642-	14580-	11608+	14
13		1715+	6143-	29481+	9602+	38360+	44918+	7796-	8528+	31912-	34916+	37533-	2667-	29932054+		29995120+	433036+	371442-	1386765+	223444-	134192-	36581+	30820-	20608+	45784-	36340+	24010+	24839-	24478+	31648-	34645-	14632+	18978-	7287-	10469+	22642+	13961+	18432+	18418+	5520+	12423+	1385+	10476-	12449-	9993+	15
14		2487-	6229+	39184-	8987-	53631-	47202-	39171+	37937-	52744+	56072-	57829+	46482-	568209-	29910558+		30045769+	321894+	1868601-	231098+	143858+	47622-	41971+	31553-	46452+	35375-	22928-	23825+	23540-	31374+	35030+	15807-	17947+	8009+	11131-	22685-	14485-	18812-	18489-	5216-	12249-	1049-	10707+	12632+	10107-	16
15		2102-	4867+	32579-	6864-	44211-	36604-	36063+	34717-	44223+	46799-	47943+	44353-	269052-	111026+	29981719+		30268994+	3211200+	232368-	148430-	55281+	49816-	39444+	45715-	33705+	21502+	22425-	22207+	30326-	34378-	16263+	16691-	8330-	11323+	22099+	14481+	18582+	18035+	4846+	11786+	769+	10586-	12419-	9915+	17
16		2108+	4982-	32808+	7069+	44323+	37277+	35088-	33821+	43991-	46599+	47813-	42588+	252659+	79016-	445534+	30034184+		30438239+	221182+	188252+	140237-	137646+	128744-	44521+	20398-	9076-	10449+	10919-	23529+	32634+	23978-	5497+	13349+	15440-	19543-	17016-	19361-	16314-	1642-	8804-	2183+	11206+	12382+	9648-	18
17		2054-	4927+	32059-	7022-	43152-	36718-	33370+	32199-	42568+	45127-	46358+	40067-	232562-	54495+	382924-	332478+	30259370+		30309752+	1559333-	1962676+	1586377-	1332299+	5638-	26275-	26095-	24817+	23174-	10807+	2154-	21348+	23538+	12940-	11391+	1860-	8317+	5055+	660-	6282-	4562-	6429-	3224-	2094-	1145+	19
18		1890+	5342-	30575+	7946+	39920+	38609+	22484-	22067+	36812-	39415+	41095-	22550+	132974+	146467+	1389490+	1870432-	3212315+	30444644+		30525304+	3104932+	1781724-	1313193+	20256+	35682-	29037-	28625+	27375-	23039+	16187+	6435+	24597+	4838-	2347+	11862-	1244-	5400-	8947-	6572-	8657-	4753-	2729+	4347+	3796-	20
19		20-	1483+	2210-	2707-	838-	8717-	13582-	12469+	3503-	3046+	2102-	23235+	76952+	152428-	208065-	215578+	217175-	206561+	30311102+		30395800+	220013+	286740-	45121-	40957+	28331-	28896-	28225+	32984-	33515-	10201+	22498-	4326-	7484+	20867+	11150+	15710+	16512+	6035+	11854+	2386+	8631-	10555-	8494+	21
20		860+	1127-	12178+	1215+	17175+	9380+	22332-	21126+	19457-	20182+	20051-	30882+	103881+	105722-	118847-	128815+	134030-	177989+	1565153-	30515048+		30127327+	365568+	48320+	42721-	29257-	29903+	29247-	34757+	35783+	11624-	23117+	5058+	8359-	22115-	12146-	16874-	17519-	6190-	12431-	2308-	9278+	11284+	9058-	22
21		1670-	3695+	25642-	5131-	33769-	26969-	28762+	27601-	33648+	35505-	36230+	35192-	119980-	49319+	23146+	34914-	43465+	135769-	1975246+	3118557+	30382743+		30050560+	50838-	43167+	29097+	29843-	29248+	35699-	37499-	13329+	22814-	5990-	9345+	22943+	13118+	17861+	18209+	6099+	12694+	2049+	9839-	11863-	9492+	23
22		1770+	3967-	27240+	5533+	35766+	28854+	29935-	28745+	35466-	37447+	38246-	36366+	122980+	46265-	17126-	29050+	37831-	133433+	1599753-	1795963-	233408+	30113603+		29920429+	554903-	257488-	240675+	220471-	120076+	15674+	98383+	123640+	29594-	24150+	9896-	15129+	6441+	5186-	13068-	11234-	12277-	3673-	1295-	111+	24
23		1839-	4201+	28409-	5896-	37154-	30416-	30297+	29124-	36588+	38666-	39547+	36415-	122268-	39541+	7223+	18981-	27826+	125172-	1346277+	1327672+	299969-	379096+	30037270+		29909049+	122571+	92320-	70953+	139124+	291006+	172154-	52534-	39491+	39244-	26636-	35298-	32943-	19292-	4459+	5883-	9022+	15334+	15400+	10778-	25
24		179+	411+	1677+	931-	3064+	1633-	9455-	8786+	5023-	4978+	4586-	13870+	23943+	31765-	29710-	30353+	30042-	30436+	6237-	12886+	31594-	33997+	35978-	29917833+		30042564+	374069+	295278-	1325163+	1345986+	218021-	17780-	35145+	36362-	32521-	34415-	34196-	22298-	992+	9584-	6119+	15252+	15983+	11328-	26
25		1238+	3782-	20352+	5713+	22913+	23518+	9724-	9672+	19083-	20512+	21525-	7292+	14357+	24569+	31915+	30193-	28104+	9184-	38316-	41238-	39075+	40295-	39979+	566068-	29923488+		30118387+	228935+	1793550-	1599481-	225455+	28191+	35628-	36486+	30967+	34089+	33406+	21292+	1599-	8710+	6508-	14904-	15494-	10938+	27
26		1311+	3747-	21209+	5573+	24072+	23424+	12374-	12141+	20664-	22097+	23017-	11049+	20325+	16160+	23842+	21986-	20012+	1362-	38681-	36787-	30092+	30674-	29873+	269745-	134436+	30051466+		30386931+	3116291+	1975004+	225284-	34253-	34807+	35411-	29023-	32803-	31851-	19983-	1920+	7890-	6536+	14203+	14687+	10339-	28
27		1256-	3624+	20355-	5404-	23051-	22610-	11543+	11344-	19693+	21073-	21974+	10098-	18650-	16699-	23991-	22230+	20320-	2416+	36920+	35780+	30014-	30656+	29956-	252378+	104234-	364964+	30127665+		30515876+	1566076-	227643+	133348+	35779-	32976+	11185+	26478+	21313+	8678+	7576-	1133-	9755-	9790-	8902-	5875+	29
28		1180+	3427-	19150+	5117+	21653+	21352+	10651-	10480+	18438-	19741+	20599-	9182+	17004-	16443+	23246+	21603-	19798+	2960-	34579-	33928-	28917+	29572-	28957+	231445-	82496+	286353-	219863+	30395788+		30308601+	210061+	225246+	32723-	26907+	7393-	17354+	8689+	3228-	12170-	9710-	11744-	4416-	2311-	904+	30
29		585-	2072+	9985-	3223-	10828-	12534-	2168+	2340-	8264+	9006-	9641+	372+	379-	21082-	23971-	23291+	22165-	12951+	16931+	23500+	27484-	28706+	29068-	125085+	127780+	1314807+	1783355-	3106533+	30523529+		30018524+	1731838-	4063+	4323+	41568+	13621+	23329+	24713+	12884+	20003+	7563+	9062-	12045-	9203+	31
30		53+	534+	57-	1033-	776+	2633-	6155-	5679+	2407-	2286+	1943-	9402+	15404+	22889-	21717-	22030+	21684-	20806+	1804-	10770+	22749-	24354+	25594-	14386+	281196+	1335485+	1589415-	1965538+	1561340-	30308259+		30000948+	42622-	43995+	41796+	41270+	41042+	26481+	123+	12177+	5852-	16853-	17848-	12503+	32
31		1163-	2646+	17918-	3701-	20989-	17088-	16395+	15731-	19657+	20735-	21167+	18387-	31004-	8685+	1437+	3205-	4450+	18436-	31869+	13732+	2439-	1407+	363+	110360+	172289-	214865-	222927+	223222-	231731+	219547+	30005939+		29906862+	77278-	176397-	64175-	70083-	43684-	19625-	32144-	10166-	13644+	17275+	11814-	33
32		1368-	3815+	21995-	5643-	24836-	23714-	13479+	13176-	21463+	22910-	23803+	12486-	21901-	12850-	20312-	18469+	16596-	1303-	36863+	33463+	25628-	25961+	25033-	136869+	64042-	26047-	36727+	42667-	143847+	236500+	1736327-	30008457+		29908776+	261855+	60416+	71516+	44456+	24367+	35450+	14593+	11333-	15453-	10664+	34
33		1109+	2700-	17293+	3853+	19473+	16715+	13587-	13092+	17584-	18609+	19092-	14370+	22209+	1986-	3571+	2219-	1144+	10209+	24011-	16479-	6479+	6009-	4904+	42033-	41876+	34183+	35237-	34775+	41222-	42760-	15339+	40405-	29896999+		30002377+	1736008-	132870-	5013-	31583+	19150+	30872+	14856+	10871+	5412-	35
34		1001-	2373+	15539-	3358-	17541-	14730-	12772+	12282-	15989+	16895-	17293+	13756-	21019-	3632+	1441-	221+	702+	10692-	21356+	13763+	4042-	3510+	2447-	35645+	39777-	33743-	34457+	33807-	37126+	36069+	7491-	40106+	66615-	29897532+		30006956+	82414+	44226+	32679+	40375+	22393+	6554-	11315-	8040+	36
35		264+	1093-	4712+	1742+	4726+	6204+	302+	128-	3172-	3524+	3874-	2051-	2540-	11642+	12130+	11917-	11440+	8008-	5344-	9420-	12544+	13162-	13438+	12020-	15322-	21377-	20134+	18740-	4496+	9855-	34406+	30133+	168047-	254684+	30007502+		29905480+	39581+	46507+	46982+	35497+	1502+	4280-	3967+	37
36		764-	1710+	11728-	2377-	13309-	10692-	10483+	10045-	12354+	13016-	13265+	11640-	17441-	5426+	1464+	2396-	3029+	10280-	15848+	8920+	512-	45-	952+	24305+	33047-	29548-	29792+	28995-	28379+	24488+	1950+	35206+	53168-	49196+	1740850-	29996309+		29902000+	246401+	157353+	128972+	26561+	17049+	6210-	38
37		458-	851+	6795-	1105-	7898-	5516-	7597+	7224-	7738+	8090-	8147+	9008-	13169-	7764+	5125+	5709-	5992+	9930-	9298+	3078+	3855+	4452-	5177+	12445+	26891-	26261-	26017+	25039-	20139+	13047+	11864+	32009+	58623-	60353+	134496-	72578+	29897496+		30024975+	1932228-	58638+	40654+	38885+	17038-	39
38		185+	777-	3308+	1241+	3255+	4333+	311+	183-	2140-	2384+	2630-	1553-	1871-	7870+	8101+	7963-	7647+	5421-	3369-	6093-	8205+	8609-	8794+	6574-	8671-	11658-	10976+	10208-	2593+	5063-	17319+	15253+	35132-	36991+	489-	38979+	37415+	29905973+		30021885+	171133+	43390+	36378+	14890-	40
39		902+	2430-	14378+	3558+	15554+	14427+	8958-	8710+	13372-	14232+	14727-	8504+	12564+	4261+	7829+	6903-	6027+	2449+	16632-	14026-	9551+	9549-	9019+	23929-	12791+	6638+	7510-	7794+	15652-	21398-	17571+	4859-	22590-	28664+	41245+	38632+	54731+	256013+	30028303+		29902544+	44924+	44849+	18308-	41
40		705+	2024-	11395+	3013+	12174+	11892+	6013-	5902+	10165-	10866+	11319-	5135+	7783+	6489+	9040+	8343-	7596+	770-	12942-	12381-	10261+	10441-	10160+	19329-	4938+	590-	310-	844+	9591-	16906-	19639+	3349+	30034-	34777+	27948+	41905+	51556+	165855+	1925289-	30030754+		29904758+	73043+	10140-	42
41		948+	2485-	15016+	3614+	16289+	14793+	9898-	9596+	14151-	15035+	15518-	9687+	14131+	2782+	6606+	5634-	4754+	3939+	17263-	13793-	8442+	8344-	7725+	24051-	15585+	9672+	10444-	10572+	17206-	21585-	14721+	8513-	15712-	21350+	40528+	30434+	45287+	138732+	59863+	176707+	29901365+		29903319+	3787-	43
42		510+	1191-	7889+	1678+	8666+	7192+	6373-	6120+	7846-	8281+	8463-	6822+	9559+	1994-	246+	284+	672-	4933+	8905-	5541-	1324+	1078-	619+	11003-	12502+	10274+	10422-	10172+	10772-	10080-	1225+	10808-	3565+	1154-	18525+	2979+	10092+	30727+	34533+	41229+	36937+	29895839+		29902661+	44
43		301+	624-	4549+	846+	5086+	3856+	4343-	4145+	4784-	5022+	5090-	4916+	6787+	3009-	1552-	1875+	2067-	4440+	5232-	2420-	832-	1080+	1423-	6029-	9884+	8934+	8903-	8595+	7584-	5716-	2296-	9832-	7234+	5702-	11947+	2763-	2563+	18649+	31216+	31911+	35787+	65221+	29897174+		
44		216-	476+	3303-	660-	3625-	2882-	2866+	2743-	3333+	3508-	3570+	3146-	4271-	1425+	471+	688-	831+	2503-	3555+	1933+	8+	141-	348+	3997+	5464-	4706-	4723+	4577-	4374+	3660+	477+	4991+	2586-	1739+	6041-	269+	2141-	7350-	9563-	10333-	9554-	2748-	1920+	29897163+	
		01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	

sin