

**ON THE NATURE OF DEFLECTIONS OF THE VERTICAL
DERIVED FROM DIP OBSERVATIONS
IN OCEANIC ISLANDS**

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The geoid is the equipotential surface of the earth's gravitation and rotation, and on the average coincides with the mean sea level in the open sea. It is not a regular surface as it depends on the visible irregularities of the physical surface of the earth, high mountains and deep oceans, and the invisible variable densities of masses of matter inside the earth near its surface. Had the solid earth itself resembled a perfect mathematical model, e.g. an ellipsoid of revolution with no mass anomalies inside, the geoid would have taken nearly the same form, but the irregularities in shape and density of the real earth cause the geoid to depart from a perfect mathematical form and develop into a complicated surface. The geoid has nevertheless one advantage: at sea level it is perpendicular to the direction of gravity or the vertical axis of a level theodolite, and the process of spirit levelling thus provides a straightforward means of measuring heights above the geoid. Moreover, one can also easily relate different positions on the earth's surface by referring them to a unique system of meridians and parallels derived on the geoid by means of astronomical observations. But these meridians and parallels being again irregular in form because of the complicated shape of the geoid, the system can at best be used in independent surveys within limited areas only and not adopted as a suitable basis for computing precise positions of widely separated points on the earth's surface. The possible alternative is then to choose some perfect mathematical model as the equilibrium surface of the theoretical earth which may closely approximate to the geoid, and then relate the earth's positions by means of mathematical formulas referred to a system of coordinates on the model, usually an ellipsoid of revolution called the reference surface.

The definition of the reference ellipsoid involves as many as seven constants. Two constants are involved in defining its minor axis as parallel to the earth's axis of rotation, and another two in assigning the lengths of its major and minor axis. The remaining three are needed in defining a point on the reference ellipsoid, usually an initial point or the origin of survey, by assigning its geodetic coordinates as well as height above or below the geoid. It is in this arbitrary manner that a reference surface

normally has to be defined with respect to the geoid, and as such provides the following five arbitrary quantities associated with it :

Equator's radius	a	
Flattening	$x = \frac{a - b}{a}$	
Coordinates of initial point	}	Latitude φ_g
		Longitude λ_g
Height above geoid	N	

But on adopting a particular model, e.g. the international ellipsoid as the reference surface, the quantities a and x become known and the number of arbitrary quantities reduces to three, depending on the definition or choice of datum in the geodetic survey system. Thus if any one of these quantities changes the entire geodetic system changes too. This is how we usually obtain various geodetic systems of coordinates in different parts of the world which may possibly differ quite remarkably from one another even when the same international ellipsoid is adopted as the reference surface for all.

The usual geodetic systems being thus essentially arbitrary, it is of interest to consider what is really meant by the deviation of the vertical at a point in a geodetic system. The deviation of the vertical is defined as the angle between the geoid and the reference surface (the international ellipsoid) or between the ellipsoidal normal and the vertical or direction of gravity at the point. The direction of the ellipsoidal normal at the point being again defined by its geodetic coordinates which are arbitrary, and that of the geoidal normal by its astronomical coordinates actually obtained from astronomical observations, the deflections of the vertical become as may be seen from fig. 1 :

In the meridian

$$\eta = Z_a X = \varphi_a - \varphi_g \quad (1)$$

η being small; and in the prime vertical

$$\xi = X Z_g = L_a L_g \cos \varphi_g = (\lambda_a - \lambda_g) \quad (2)$$

ξ being small,

where φ and λ denote latitude and longitude and the suffixes a and g stand for astronomical and geodetic. Thus it follows from relations (1) and (2) that the deflections of the vertical depend not only on the actual form of the geoid as brought about by the earth's irregularities of shape and density, but also on the choice or arbitrary definition of the datum in the geodetic system. Thus a deviation to the northwest may be due to some excess of mass in that direction, but may also arise from the arbitrary definition of the datum. Hence the deflections of the vertical dependent as they are on the arbitrary datum, are themselves arbitrary and can be known absolutely only when the datum is so determined. But this has not been done so far with any reasonable amount of accuracy. Therefore there is nothing at present which might properly be called the deflections of the vertical. One reasonable way of limiting the extent of our choice of datum however is by having the centre of the reference ellipsoid coincide with the centre of gravity of the actual earth. This is again of no practical use

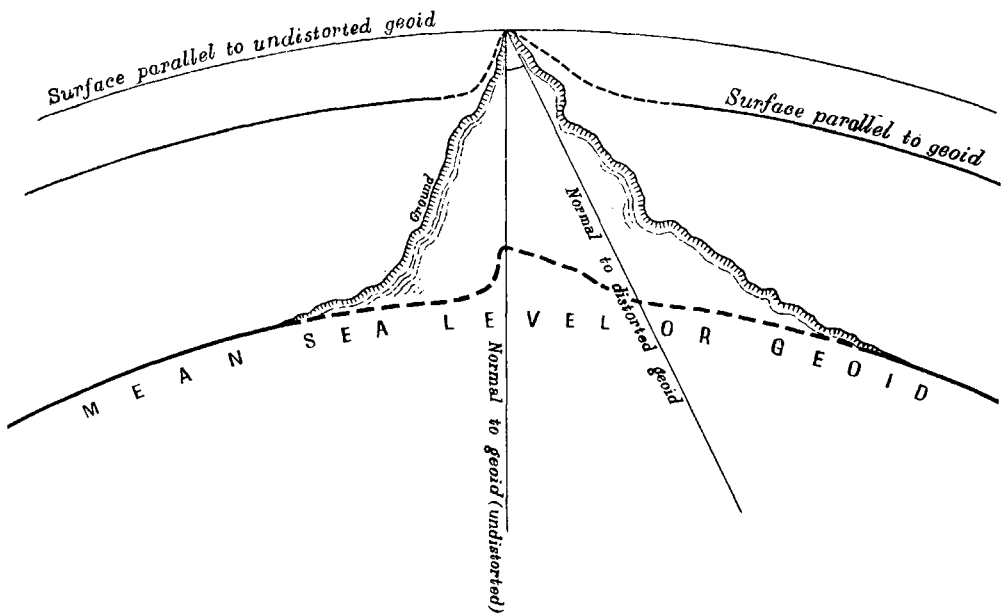


Fig. 1.

to a triangulator, who has no means to relate his measures to the earth's centre of gravity. He has therefore no other alternative than to begin the computations of geodetic points with only an arbitrary datum, which he tries to improve later in order to obtain a closer fit over the largest possible extent between the surfaces of the reference ellipsoid and the geoid. The process is known as astro-geodetic; it is very long, and it is seldom possible to connect the datums so defined readily with one another. Moreover, it cannot be applied at all on the oceans. The only hope is then the gravimetric method. As early as 1849, G. G. STOKES did some pioneer work in the field of gravimetry on the basis of the above consideration, which may eventually enable us to refer our geodetic measurements to an ellipsoid of reference having its centre coincident with the earth's centre of gravity. His formula for deriving the absolute elevation of the geoid above the reference ellipsoid and the one as modified by VENING-MEINESZ in 1928, for computing the absolute deflections of the vertical at a given point, are all expressed in terms of gravity anomalies. This again presupposes knowledge of gravity over the entire surface of the earth, which is obviously impossible of attainment now or in the distant future.

There is thus nothing much to be hoped for even in the remote future so far as the practical application of these formulas is concerned. However, a method has recently been suggested by the author for computing the absolute deflections of the vertical by making use of gravity data limited to a 100-mile radius only, but the results of its practical application are yet to be seen before it can be actually accepted as a general method. Even then we seem not yet to have made much headway in the gravimetric method. Thus what is actually possible now to obtain in the existing geodetic systems based on arbitrary datums, is only the relative deflection

of the vertical associated with them, which is subject to change whenever there is a corresponding alteration in the definition of the datums. It is thus essential that the datum of the existing geodetic systems be uniquely known. In that event we shall have what has rightly been called by HEISKANEN the World Geodetic System. "This is the ideal, the end goal of the scientific geodetic study. Then we shall have a geodetically one uniform world". Unfortunately in the present state of geodesy, we are yet quite far from this and the correct evaluation of absolute deflections of the vertical now being out of the question we must be contented with their relative values only.

However, from HIRVONEN's pioneer work (1934), TANNI's important results (1948) and HEISKANEN's recent investigations (1959) on geoid elevation (though based on rather insufficient gravity material and for that matter subject to substantial modification when additional gravity data are taken into account), we can obtain a fair idea of the absolute undulations or the ups and downs of the geoid with respect to the international ellipsoid with its centre coincident with the centre of gravity of the earth. [TANNI's Map 1: Continental Undulations of the Actual Geoid, appearing at the end of his publication (Isostatic Institute of I.A.G., No. 18) and HEISKANEN's figures 1-4: Gravimetrically Computed Geoid, given in his article *The Columbus Geoid*, pp. 844-46]. These would make it sufficiently clear among other things that even in the open sea the geoid very rarely coincides with the perfectly oriented reference ellipsoid, and that the variations may in many cases be remarkably large also. Thus, since the slope of the geoid is simply another term for the deflection of the vertical in a direction perpendicular to the contours, we may easily conclude that considerable deflections of the vertical exist even in the sea level surface. Here it may normally be very difficult for someone with only a professional interest in the subject to imagine that any kind of regional mass distribution is actually responsible for such heavy swelling of the geoid in the area in relation to the reference ellipsoid. Moreover we can also quite easily deduce the difference: $\Delta\eta$, $\Delta\xi$ between the deflections of the vertical at any desired point P (φ , λ) for any given tilt: meridional $\Delta\eta_0$ prime vertical $\Delta\xi_0$ and vertical separation ΔN_0 at the origin 0 (φ_0 , λ_0) of survey between the international ellipsoid of reference, one perfectly oriented and the other as adopted in any existing geodetic system, from the formulas given by BOMFORD in *Geodesy*, p. 129, and obtain the results as follows:

$$\Delta\eta = -(1 + f \cos^2 \varphi) \left[\frac{1}{a} \sin u \cdot \cos \omega (N_0 \cos u_0 - a \Delta\eta_0 \sin u_0) + \right. \\ \left. - \sin u \cdot \sin \omega \cdot \Delta\xi_0 - \frac{1}{a} \cos u (N_0 \sin u_0 + a \Delta\eta_0 \cos u_0) \right] \text{cosec } 1'' \text{ (second)} \quad (3)$$

$$\Delta\xi = - \left[\frac{\sin \omega}{a} (N_0 \cos u_0 - a \Delta\eta_0 \sin u_0) - \cos \omega \Delta\xi_0 \right] \text{cosec } 1'' \text{ (second)} \quad (4)$$

where $\tan u = (1 - f) \tan \varphi$ and $\omega = \lambda - \lambda_0$

Hence if at P the relative deflections of the vertical be η_R , ξ_R and the corresponding absolute deflections of the vertical be η_A , ξ_A , we obtain the following relations:

$$\eta_A = \eta_R + \Delta\eta \quad (5)$$

$$\xi_A = \xi_R + \Delta\xi \quad (6)$$

Thus the relations (5) and (6) provide us with a quantitative idea of the deflections of the vertical generally met with even at sea level in the open sea, due to any possible error involved in the arbitrary definition of the adopted reference ellipsoid or the usual tilt of the geoid brought about merely by the irregularities in mass distributions inside the earth or both. Hence it would not be quite correct to maintain that the geoid and the reference ellipsoid of an existing geodetic system are necessarily exactly identical at sea level in the open sea. This being so, the kind of deflections of the vertical obtained by the dip method can neither be termed as absolute nor even relative, since it is really not possible to assume, as stated before, that the geoid and the reference ellipsoid, whether perfectly oriented or not, would necessarily coincide at sea level in the open sea. We simply cannot obtain in this way what may be properly called absolute or relative deflections of the vertical; for otherwise there would be no real need to determine the deflections of the vertical, at least over the oceanic part, which is the most extensive feature of the earth's surface and poses the most delicate problem for geodesists; and as a result, the problem of determining the real shape of the earth, one of the main objectives of scientific geodetic study, would have been easily solved once for all. What then we actually derive from the method of dip is simply a measure of the

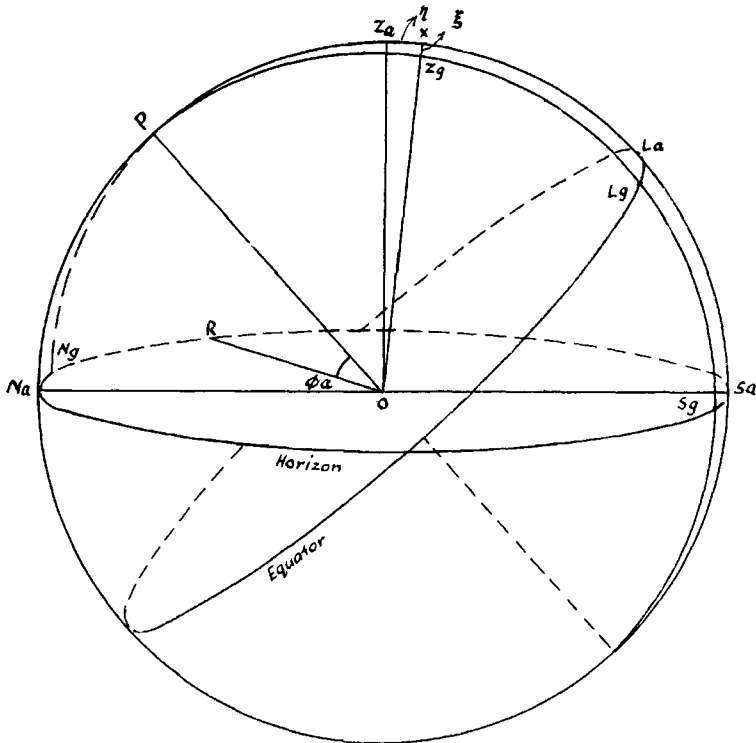


Fig. 2.

mean tilt of the distorted or swelling geoid below the observation point on the island, with the mean sea-level surface or the geoid in the open sea all round the island, and not the deflections of the vertical in the sense as used in geodesy. This has been given in a slightly exaggerated form in figure 2 in order to make the point more clear. The determination of the tilt of the distorted geoid in oceanic islands, however, has the great practical utility of correcting observed astronomical positions for the control of local triangulation and charting.

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