SPACE NAVIGATION

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Extracts from a lecture prepared for the Astronautics Seminar, Southampton, 12-19 July 1961

We restrict this study to cislunar navigation and assume that the astronaut will have sufficient power to change the course and altitude of his craft in space by making minor adjustments in speed.

We do not here consider biological and other problems outside the field of actual space navigation. Minimum equipment will be considered, since up to 1 000 pounds of fuel will be required to orbit a payload of one pound. If or when additional aids are needed, these inventions will doubtless be mothered by necessity and experience.

Since re-entry and atmospheric friction pose special problems, we restrict our study to areas outside about 500 miles from the earth's surface. Unfortunately, we do not have generally accepted units for speed, force or distance, but we do have accurate and generally accepted time units. Since the guild of air navigators will provide the principal source for space navigators, and since air navigators are required to use the nautical mile, we select it and the knot as the most suitable units for distance and speed. It is unfortunate that the natural unit of one minute of arc on the earth's surface is not exactly 6 000 feet, for convenient use with both the sexagesimal and decimal systems.

As stated by Dr. G. W. LITTLEHALES, author of the marine navigation tables, H. O. 203, " The architect does not press his own bricks, but uses the material at hand for the construction of his edifice ". In this sense we use the building blocks provided us by NEWTON, KEPLER, and the many unnamed astronomers and mathematicians who provide us with not only suitable *material,* but also remarkably efficient *tools* — even ready-to-use *lighthouses* in space.

We could contribute little to the monumental work of astronomers, but we are encouraged by history to make free use of the material they provide us. For example, when the astronomer first tabulated the declination of the sun, the early navigators could easily find, and would " run down their latitude " and accomplish practical navigation with simple equipment, and with little knowledge of longitude which involved time. When in about 1734 the marine sextant and the chronometer were invented, navigation was revolutionized and brilliant navigators soon drew handsome dividends from bank deposits of celestial data made by astronomers in past centuries.

With these lessons before us, we propose to investigate what may be done with an accurate timepiece, a marine sextant or its equivalent, and suitable star charts. We propose to make principal use of the remarkable combination " scanner-computer " sitting on the shoulders of every intelligent astronaut. The necessary base for cislunar navigation is the earth itself. Projected plans for near-earth navigation involve the use of computers, surface detection systems, and electro-magnetic communication and data links. These are very complex systems which involve either two or three coordinate transformations to obtain knowledge of position or velocity. They are not 100 % reliable.

Assuming that we have perfect equipment, what would it do ? Suppose we call for the display of the observer's geographical position in terms of latitude and longitude, and of his altitude in nautical miles above this position. If our ground-support system gives us the *planned* path of our vehicle in the form of a special ephemeris, and our equipment displays the *actual* path of the observer continuously, our sole navigation problem would be to have the displayed data follow the pre-computed data. Unfortunately, space navigation is not so simple. In the first place, it takes a lot of doing to produce such a fool-proof and failure-proof system. In any case, it would add to the cost and weight of the space vehicle. Therefore, we should first try the simplest and minimum equipment, and then add items when their need is determined.

The observer's vertical coincides with his line of sight to the earth's center. The position of the earth's center relative to star positions as viewed by the astronaut gives directly the observer's nadir which, when turned 180°, gives his geographical position in terms of sidereal hour angle and declination. For any instant of time this position may be converted quite simply to a position in terms of latitude and longitude. The distance from the observer to his geographical position may be found stadimetrically, by using the earth's diameter as a baseline. The latitude, longitude, and altitude (or distance) thus determined gives the three-dimension position required by the astronaut.

Position finding in space is only part of the space navigator's problem. He must also be able to predict future positions — to run his DR. This means that he must keep his space vehicle on its proper schedule as shown by its ephemeris. The space navigator cannot " see " his own position relative to that of the earth's center, but he can see the position of the earth's center relative to himself — which is the same thing.

To summarize this, the astronaut is provided with an ephemeris of the earth's position relative to the observer, and also with a means for finding his distance from his geographical position. He then so directs or navigates his craft as to cause the earth to follow its scheduled path. When this procedure is followed, the space navigator is doing practical navigation.

We shall now consider a specific satellite, *Echo,* which is a plastic bag 100 feet in diameter, weighing 166 pounds. The U.S. Naval Weapons Laboratory, Dahlgren, Virginia, computed the orbit of satellite Echo at five minute intervals for 5 January 1961 (see figs. 1 and 6).

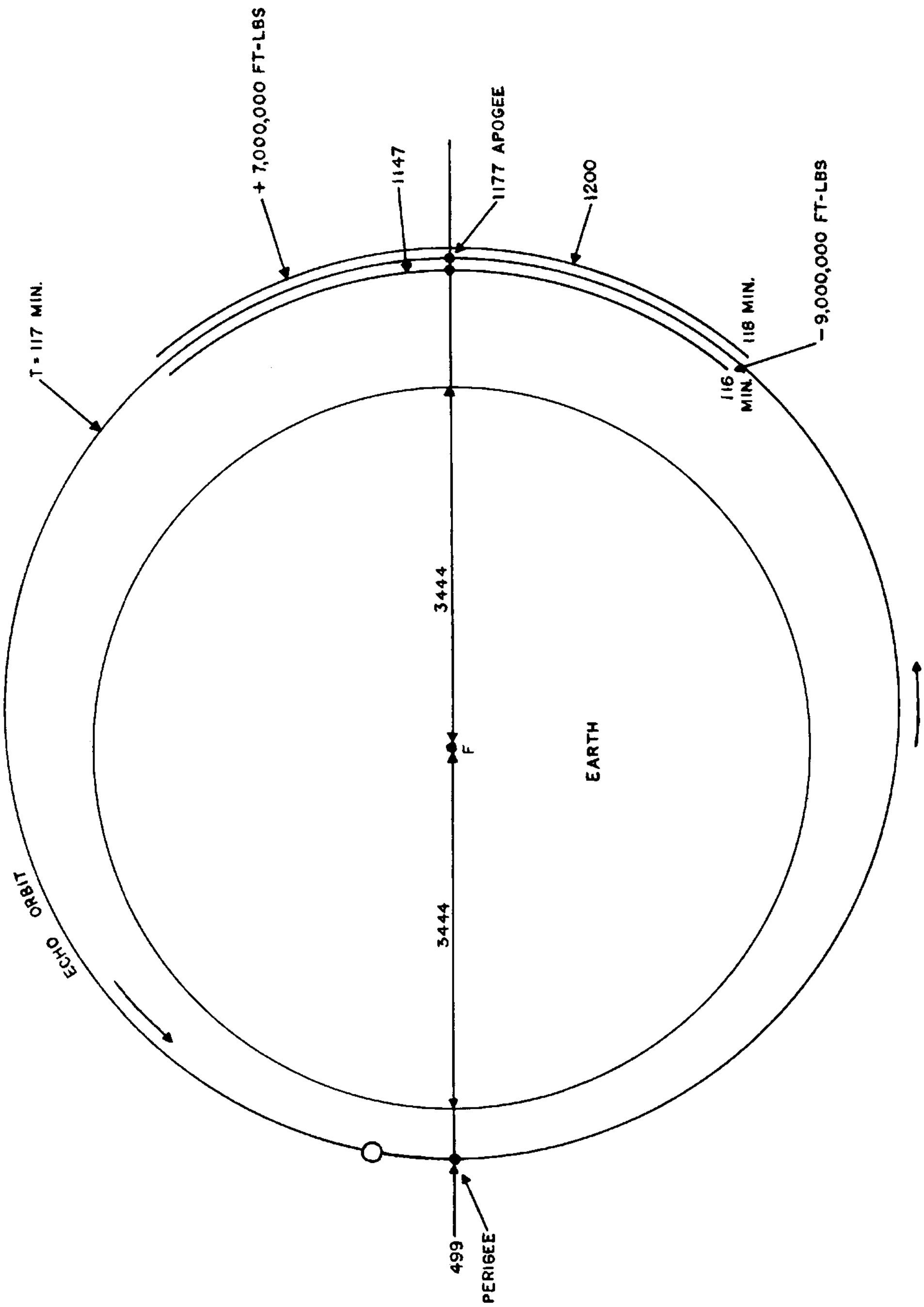

Fig. 1. — Echo satellite orbit, 0h 5 January 1961.

For any known or scheduled satellite path, ground support facilities can supply the astronaut with the *earth's apparent path* relative to the satellite, by shifting the latter's celestial coordinates, sidereal hour angle and declination 180°. This concept assumes the observer to be at the center of the universe — a reasonable assumption so long as the astronaut remains in the solar system, as is demonstrated in fig. 2. Since the solar system itself may be considered to be a point in the universe, our assumed restriction in range to cislunar navigation should cause no appreciable errors of position *relative to stars*. At the same time the relatively short distance from a moving satellite to the earth causes the latter's *apparent* position to change rapidly, so that the satellite's position, relative to the earth, is sharply defined in the star field.

While we have considered and illustrated in fig. 6 the position of the earth relative to a satellite as the preferred technique, it is possible to use the direct tabulation of the satellite's position *relative* to the earth's center, as we now do with the moon, sun, planets and stars, as shown in the Air Almanac (originally designed by the author in 1933). To use the satellite ephemeris direct, it is necessary for the observer to fix the position of the satellite relative to zenithal stars. Figure 3 illustrates how this may be done. In fact, this method has the advantage that some reference stars are not obscured by the earth, but it requires the use of a two-way telescope, with its added weight.

Heretofore, to judge by published material on the subject, it was generally assumed that the astronaut must either have the ability to work intricate problems of celestial mechanics, or else have substitute electronic equipment to solve these problems automatically. Isn't this about the same as asking the airman to compute the sun's ephemeris ?

Why not have ground support facilities double for "space observatories" and supply precomputed ephemerides of the earth and of the moon relative to any desired satellite paths ? If the astronaut is supplied with such precomputed data, and this data proves to be correct, his duties as navigator will consist largely of verifying the earth's position as it seems to move through the heavens like a large Sputnik, while in fact the observer on the satellite is moving in his own self-ordained path in space.

In practice, the satellite will not indefinitely follow its precomputed path. Errors of computation will be experienced, especially when extrapolated far into the future. On occasions the astronaut will change his intended destination. Unless he has the ability to make such changes, he

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FIG. 2. — Relative distances in space.

(1) *The Universe.* — Observable distance from our galaxy in any direction about 4 billion light years.
(2) *Our galaxy* (the Milky Way System). — Estimated as 80 000 light years in diameter; Sun is approximately 26 000 light years from center.
(3) *Solar system.* — Sun moving in space at approximately 420 000 knots. Orbit of Pluto about 11 *light hours* in diameter. Distance from Sun to Earth about 8 *light minutes*. Earth moving around Sun at 58 000 knots.

Note : Earth and Moon shown at double size in proportion to distance.

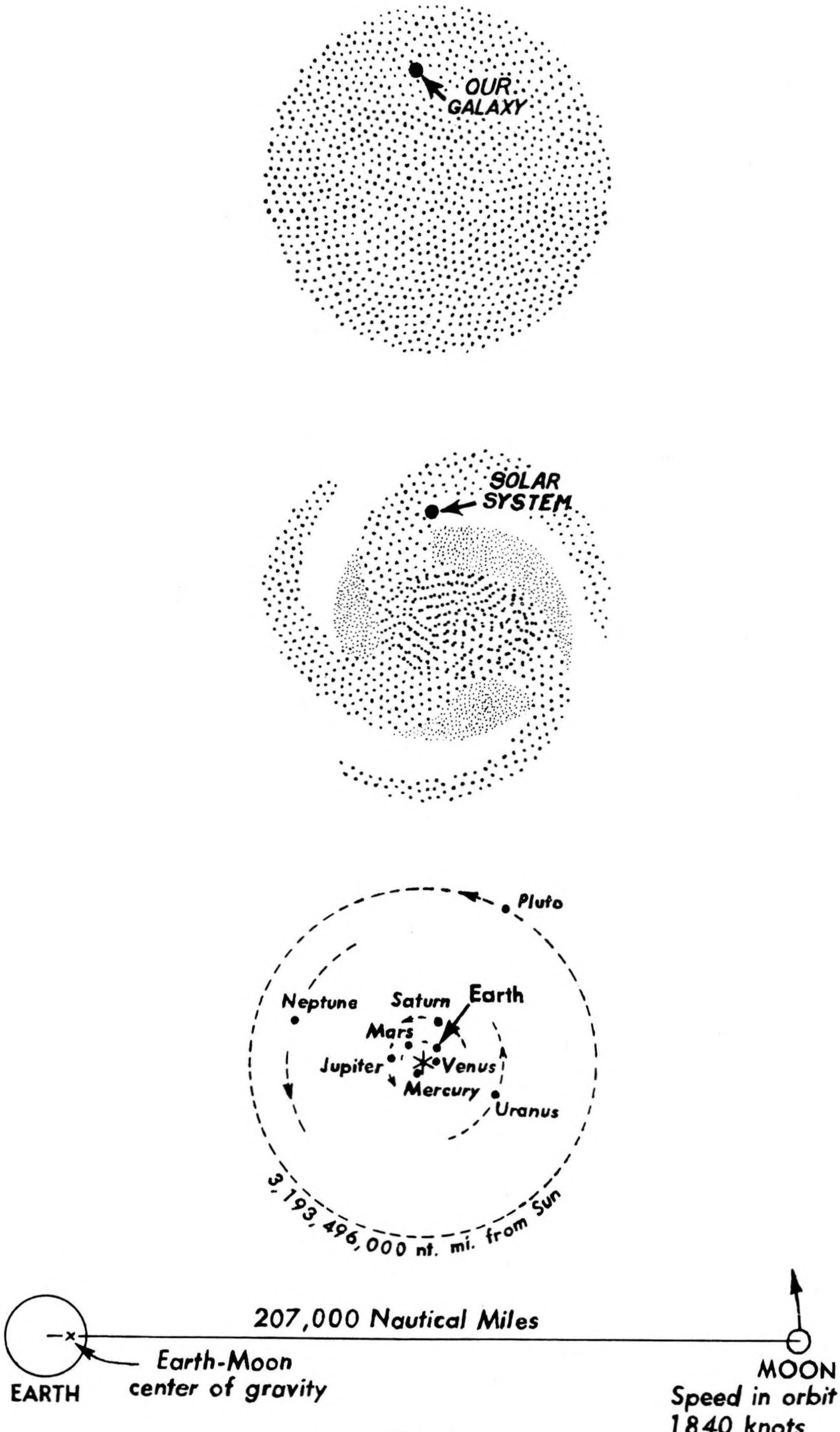
OUR GALAXY
SOLAR SYSTEM
Pluto
Neptune
Saturn
Earth
Mars
Jupiter
Venus
Mercury
Uranus
3,193,496,000 nt. mi. from Sun
207,000 Nautical Miles
Earth-Moon center of gravity
EARTH
MOON
Speed in orbit
1840 knots

Fig. 2

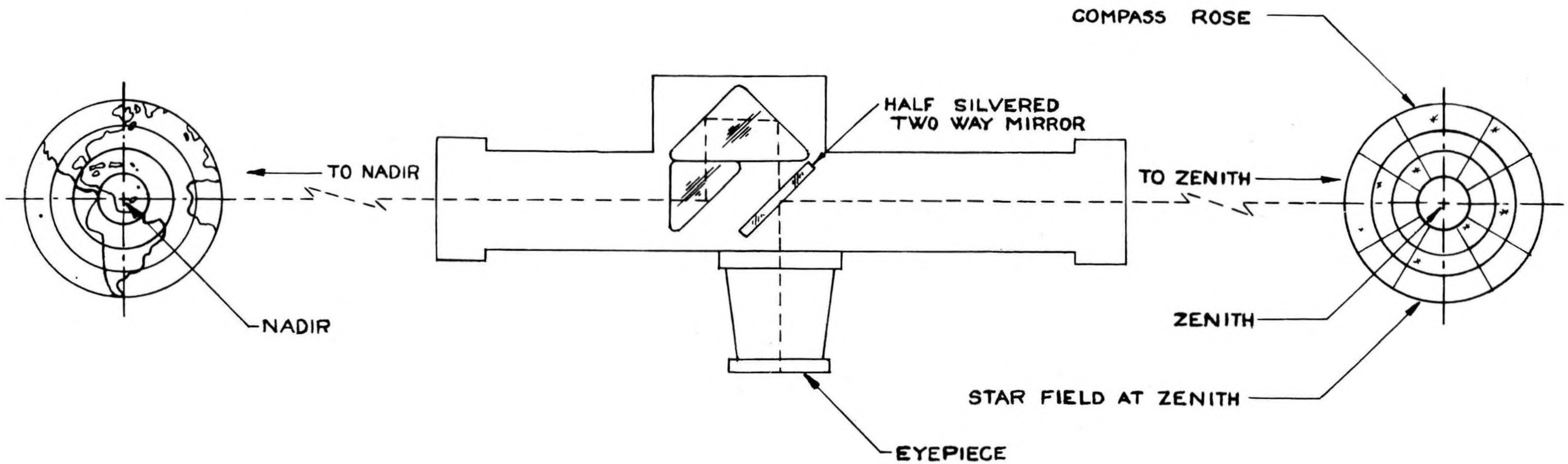

Fig. 3. — Zenith viewing telescope.

is not navigating in the full sense of being able to *direct* his craft. The ability to make considerable changes in the orbital plane, or in the elliptical orbit within its plane, will for the present be severely restricted. Present space power limitations might be compared with a battleship powered with an outboard motor. However, the prediction is made that power engineers will keep abreast of progress made in the fields of biology and navigation.

TABLE S-1

Earth Diameter	Distance Nautical Miles	Moon Diameter
121°39.2′	500	81°25.3′
101 35.0	1 000	57 53.2
88 17.2	1 500	45 14.9
78 27.8	2 000	37 13.8
70 47.6	2 500	31 39.6
64 35.4	3 000	27 33.3
59 26.7	3 500	24 23.9
55 05.7	4 000	21 53.8
51 21.8	4 500	19 51.7
48 07.2	5 000	18 10.4
42 45.3	6 000	15 32.2
35 00.7	8 000	12 02.7
29 40.3	10 000	9 50.2
25 45.4	12 000	8 18.8
22 45.6	14 000	7 11.9
18 55.1	17 500	5 49.8
16 53.1	20 000	5 08.0
13 54.0	25 000	4 08.6
11 48.8	30 000	3 28.4
9 05.3	40 000	2 37.5
7 23.1	50 000	2 06.6
6 13.2	60 000	1 45.8
5 22.3	70 000	1 30.9
4 43.7	80 000	1 19.7
4 13.3	90 000	1 10.9
3 48.8	100 000	1 03.9
3 28.6	110 000	0 58.1
3 11.7	120 000	0 53.3
2 57.3	130 000	0 49.2
2 45.0	140 000	0 45.7
2 34.2	150 000	0 42.7
2 24.8	160 000	0 40.1
2 16.4	170 000	0 37.7
2 09.0	180 000	0 35.6
2 02.3	190 000	0 33.8
1 56.3	200 000	0 32.1
1 50.9	210 000	0 30.6
1 45.9	220 000	0 29.2
1 41.4	230 000	0 27.9

FIG. 4. — Table S-1, stadimetric distances from the Sun and from the Moon.

Figure 4 shows the angle subtended by the earth's diameter, and by the moon's diameter for various distances in the earth-moon system. Values from this table are tabulated as the last column of fig. 5. A light, accurate marine sextant, with perhaps special features added, should provide a suitable means of finding the stadimetric distances, although several other methods are available, such as a variable iris telescope or a series of concentric circles on a transparent sheet with the eye at the proper focal point.

DATE	TIME Hr Min	SHA Deg Min	DECL. N/S Deg Min	RANGE MILES
5 January 1961	00 00	190 43	0 08 45	0771
	00 05	178 53	0 20 20	0871
	00 10	165 50	0 30 34	0976
	00 15	150 43	0 38 55	1077
	00 20	132 59	0 44 40	1170
	00 25	113 17	0 47 11	1247
	00 30	093 32	0 46 13	1306
	00 35	075 48	0 42 12	1343
	00 40	060 53	0 35 52	1356
	00 45	048 25	0 28 01	1346
	00 50	037 40	0 19 06	1312
	00 55	027 52	0 09 28	1255
	01 00	018 24	1 00 41	1180
	01 05	008 34	1 11 08	1089
	01 10	357 34	1 21 38	0988
	01 15	344 19	1 31 40	0884
	01 20	327 22	1 40 21	0783
	01 25	305 33	1 46 04	0694
	01 30	280 10	1 46 55	0626
	01 35	255 55	1 42 11	0585
	01 40	236 08	1 33 06	0576
	01 45	220 32	1 21 34	0599
	01 50	207 35	1 09 03	0653
	01 55	195 51	1 03 34	0731
	02 00	184 16	0 15 33	0826
	02 05	171 50	0 26 26	0930
	02 10	157 40	0 35 40	1034
	02 15	141 03	0 42 36	1131
	02 20	121 59	0 46 33	1215

FIG. 5. — Sample page of second Echo Ephemeris.

To summarize :

1) the ephemeris gives the earth's scheduled positions;
2) the observer *sees* his geographical position and measures his altitude, or distance from geographical position;
3) the astronaut *navigates* by comparing 1) and 2).

PROPOSED SPACE ALMANAC

Prior to 1929, the Nautical Almanac tabulated the positions of heavenly bodies, except the sun, in terms of right ascension and declination. For the sun, the right ascension of mean sun, equation of time, and declination were tabulated. The Air Almanac gives the Greenwich hour angle and declination for all bodies except the stars, and also the Greenwich hour angle of the first point of Aries, and the sidereal hour angle and declination for selected navigation stars.

Since the astronaut will be primarily interested in the earth's apparent position *relative* to *himself*, and since this may be determined by noting visually the earth's position relative to known stars, it is proposed that the Space Almanac give, for selected satellite orbits, the positions of the earth's center in terms of sidereal hour angle and declination, and also give altitude, or distance in nautical miles from the observer's geographical position, or sub-satellite position. For space navigation near the earth, and for re-entry, the Greenwich hour angle of Aries will be needed, since it may be combined with sidereal hour angle of any point for any selected instant to find the longitude of the observer's geographical position.

The first faltering space penetration will be near the earth. The astronaut should therefore, at this early stage, be supplied with tabulated selected low orbits calculated to fit his needs. At low orbits satellite velocities and kinetic energy are relatively high. The total energy of a satellite is the sum of its kinetic and potential energy. The higher the orbit, the greater the energy required to boost the satellite to that orbit.

Heretofore, it has been customary to compute by celestial mechanics the selected satellite orbits. This is possible for un-manned satellites, since the intricate calculation may be done at leisure on the ground, and with considerable assistance from elaborate electronic computers. It will not be feasible, and fortunately it is not necessary, for the space navigator to do complicated problems in celestial mechanics. To require the astronaut to compute his position in space would be comparable to having the mariner navigate without the Nautical Almanac, or the aeronaut navigate without the Air Almanac. We must supply the astronaut with a *Space Almanac.*

With the support of the U.S. Naval Weapons Laboratory at Dahlgren, Virginia, and of the Department of Science at the U.S. Naval Academy, we have attempted to provide a space navigator for Echo satellite with usable data which we hope will grow up to be a Space Almanac.

After a somewhat involved first attempt, our second try is shown in fig. 5 in more readable form; it gives, for 5 January 1961 at five minute intervals, the earth's sidereal hour angle and declination as seen from Echo, together with the distance from Echo to its geographical position. Declination has the numerical value of the latitude, and sidereal hour angle may be combined with the Greenwich hour angle of Aries to find longitude. Normally the space navigator would not require the longitude but would use the sidereal hour angle of the earth's center.

When, in 1929, we shifted from right ascension to Greenwich hour angle to simplify celestial navigation, we stressed the fact that in reality we projected the celestial triangle to the earth and used the terrestrial triangle — a convenience to the earth-bound navigator. However, when the astronaut goes into space he is more interested in celestial positions and will ignore the earth's rotation which determines longitude. Therefore, we might say that the *Space Almanac* will shift terrestrial coordinates back to celestial coordinates. We might even go further in this line of reasoning and say that once the astronaut is clear of the earth's atmosphere, and not involved in re-entry problems, his navigation will be 100 % celestial, in the sense that he will not have such mundane aids to navigation as lighthouses, radio beams, or railroads.

FROM ECHO I 5 JANUARY 1961

		Earth			Moon			
Log	GMT	SHA	Dec	Range	SHA	Dec	Range	V
	h m	° ′	° ′					
	0 00	190 43	N08 45	669				
	05	178 53	N20 20	756				
	10	165 50	N30 34	847				
	15	150 43	N38 55	935				
	20	132 59	N44 40	1015				
	25	113 17	N47 11	1082				
	30	93 32	N46 13	1134				
	35	75 48	N42 12	1166				
	40	60 53	N35 52	1177				
	45	48 25	N28 01	1168				
	50	37 40	N19 06	1139				
	55	27 52	N09 28	1089				
	1 00	18 24	S00 41	1024				
	05	8 34	S11 08	945				
	10	357 34	S21 38	857				
	15	344 19	S31 40	767				
	20	327 22	S40 21	679				
	25	305 33	S46 04	602				
	30	280 10	S46 55	543				
	35	255 55	S42 11	507				
	40	236 08	S33 06	500				
	45	220 32	S21 34	520				
	50	207 35	S09 03	567				
	55	195 51	S03 34	634				
	2 00	184 16	N15 33	717				

a 4283.5 miles i 47.273°
T 117.18 min e 0.07923

This is the proposed layout for the Space Almanac; with two hours of the Earth ephemeris included. The moon ephemeris and velocity (V) to be inserted. Perigee, apogee and other needed data to be put in the *Space Log* column.

Fig. 6. — Proposed page layout for Space Almanac.

The first Space Almanac will find more use by earth-bound people than by astronauts in orbit. We therefore propose that our first published Space Almanac cater for the needs of teachers and students. One way to help them will be to supplement the ephemeris illustrated in fig. 5 with sufficient data to give the orbital parameters such as velocity, eccentricity, inclination, perigee, apogee, and period.

This layout, while prepared for the ultimate use of the space navigator, caters for the needs of the instructor and of students one or two generations ahead of the astronaut himself. This is based on the assumption that several years will be required to put any considerable numbers of space men in orbit. The plan is to assemble needed data such as accepted constants and to give for selected orbits sufficient data for use in the classroom as well as in space.

A proposed page layout for the Space Almanac is shown in fig. 6. The added data in each line are in reality answers to problems which may be solved by the basic tabulated data and constants. To stress this use of the proposed almanac, we shall include here some computations which are summarized in fig. 7.

ECHO SATELLITE
5 January 1961

Assumed period T	Semi-major axis a	Radius vector Min/Max	Total energy U	Change in energy	Perigee	Apogee	Velocity at perigee	Velocity at apogee	Area swept per unit time
116	4 255	3 919/4 591	-1.405×10^{9}	-9×10^{6}	475	1 147	15 000 kt	12 800 kt	8 100
117	4 282	3 943/4 621	-1.396×10^{9}	0	499	1 177	14 900 kt	12 700 kt	8 160
118	4 304	3 964/4 644	-1.389×10^{9}	$+7 \times 10^{6}$	520	1 200	14 800 kt	12 600 kt	8 150

In the above table, all distances are in nautical miles, speeds in knots, periods in minutes, other times are in seconds, and energies in foot-pounds. The following constant values were used :

M_e Mass of Earth $= 5.976 \times 10^{27}$ gm
m Satellite mass $= 166$ lbs $= 7.54 \times 10^{4}$
G Gravitational Constant $= 6.670 \times 10^{-8}$ cm³/gm sec²

The following orbital parameters were the same for all assumed periods :

e Eccentricity $= 0.0792$
i Inclination $= 47°16'$

FIG. 7. — Echo satellite tabulated data, 5 January 1961.

Commander Frank ANDREWS, Head of the Science Department, U.S. Naval Academy, undertook to work some elementary problems based on the Echo satellite ephemeris. The following remarks are based largely on the results of this work.

PROBLEM I

To define all the parameters of a satellite orbit, given that the satellite has a mass, M = 166 pounds, and an orbital period P = 117 minutes. Other constants which will be used are :

Mass of earth .... $M_e = 5.98 \times 10^{24}$ kilograms
Universal G constant ...... $\gamma = 6.67 \times 10^{-8}$ cm³ gm⁻¹ sec⁻².

1. Calculate semi-major axis a for P = 117

$$P = 2\pi a \sqrt{\frac{a}{\gamma M_e}}.$$

Solving for a, we get : $a = 4\,282$ statute miles.

2. Calculate a for P = 118, $p = 116$

$$\frac{117}{118} = \left(\frac{4282}{a_{118}}\right)^{3/2} \qquad a_{118} = \left(\frac{118}{117}\right)^{2/3} \times 4282 = 4303 \text{ miles}$$

$$\frac{117}{116} = \left(\frac{4282}{a_{116}}\right)^{3/2} \qquad a_{116} = \left(\frac{116}{117}\right)^{2/3} \times 4282 = 4255 \text{ miles}$$

3. Calculate e of Echo given that perigee = 499 miles, apogee = 1 177 miles. See fig. 7.

$$\begin{array}{r} 1177 \\ 3444 \\ \hline + 4621 \end{array} = R_{max} \qquad \begin{array}{r} 499 \\ 3444 \\ \hline + 3943 \end{array} = R_{min}$$

$$a = \frac{R_{max} + R_{min}}{2} = 4282$$

$$c = \begin{array}{r} 4621 \\ -4282 \\ \hline 339 \end{array}$$

$$e = \frac{c}{a} = \frac{339}{4282} = 0.0792 .$$

4. Calculate total energy of each orbit

$$U = -\frac{\gamma m M}{2a}$$

$U = -13.96 \times 10^8$ ft-lbs of energy

$U_{116} \cdot a_{116} = U_{117} \cdot a_{117} = U_{118} \cdot a_{118}$

$4255\ U_{116} = (4282)\ (1396) = 4304\ U_{118}$

$U_{118} = -1.389 \times 10^9$ ft-lbs

$U_{116} = 1.405 \times 10^9$ ft-lbs.

5. Calculate area swept through per unit time

$$H_{117} \cdot P = \pi a^2 \sqrt{1 - e^2}$$

P = period, H = rate of sweeping area, e = eccentricity. (Note : Energy of each orbit is determined by one parameter only, i.e., a, the semi-major axis.)

$$H = \frac{\pi \cdot (4282)^2 \sqrt{1 - (0.08)^2}}{117 \text{ min. } 60 \text{ seconds}}$$

$$H_{117} = 8\,160 \text{ sq. miles/sec}$$

$$\frac{H_{118} \cdot 118}{4990} = \frac{H_{117} \cdot 117}{4920}$$

$$H_{118} = 8160 \cdot \frac{117}{118} \cdot \frac{4304}{4282} = 8150$$

$$H_{116} = 8160 \cdot \frac{117}{116} \cdot \frac{4255}{4282} = 8180$$

6. Calculate perigee for 0_{118} and O_{116}, assuming $e = .079$ for all

$C_{118} = (0.079)\ (4\,304) = 340$

$$\begin{array}{rll} R_{min} = & 4304 & \\ & -\ 340 & \\ \hline & 3964 & \\ & -\ 3444 & \text{R earth} \\ \hline & 520 & \text{perigee} \end{array} \qquad \begin{array}{rll} R_{max} = & 4304 & \\ & +\ 340 & \\ \hline & 4644 & \\ & -\ 3444 & \text{R earth} \\ \hline & 1200 & \text{apogee} \end{array}$$

$C_{116} = (0.079)\ (4\,255) = 336$

$$\begin{array}{rll} R_{min} = & 4255 & \\ & -\ 336 & \\ \hline & 3919 & \\ \text{less } R_{earth} & -\ 3444 & \\ \hline & 475 & \text{perigee} \end{array} \qquad \begin{array}{rll} R_{max} = & 4255 & \\ & +\ 336 & \\ \hline & 4591 & \\ \text{less } R_{earth} & -\ 3444 & \\ \hline & 1147 & \text{apogee} \end{array}$$

7. Calculate velocity at apogee

$$H = \frac{1}{2} R^2 \frac{d\theta}{dt} = \frac{1}{2} R \,.\, V$$

$$V_{117} = \frac{2(8160)}{R_a} = \frac{2(8160)}{1177 + 3444} = \frac{16\,320}{4621} = 3.53 \text{ nautical miles/sec.}$$

These and other calculations show values that are readily supplied in the ephemeris by the computing machine. Except for instruction, and to provide the astronaut a more complete understanding of the Space Almanac, the space navigator would not find it necessary, nor would it be feasible, for him, to make these detailed calculations. We should provide the space navigator with a suitable Space Almanac which includes short cuts

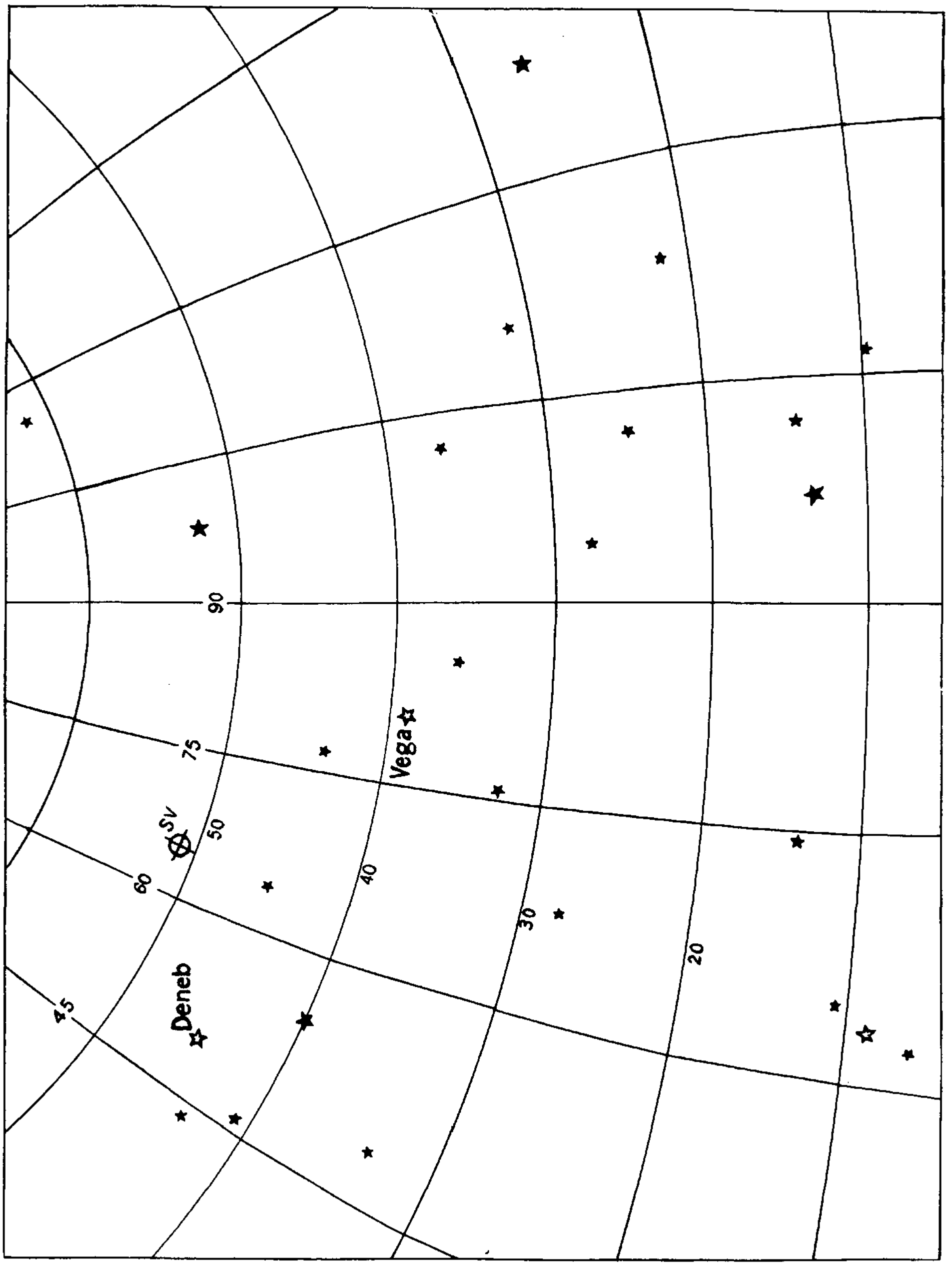

FIG. 8. — Plot of zenith of space vehicle (SV) in star field.

such as properly designed nomograms as aids for accomplishing efficient space navigation.

THREE DIMENSION SPACE FIX

Example : The space navigator of a space vehicle in orbit 12 July 1961 observes the center of the earth with the type of telescope shown in fig. 3, and at the same time estimates by eye by referring to a star chart of the area that the Zenith cross-wire is at declination 51 °N, and at a sidereal hour angle of 65°. This is in the northern part of the constellation Cygnus, in the area between Deneb and Vega. At the same time he measures the diameter of the earth to be 21° 30′.2.

Required : The space vehicle's position in terms of the latitude and longitude of its geographic position, and of the distance vertically above it.

Explanation : From the Air Almanac the star positions are found to be :

Deneb. — Declination 45° 09′ N, Sidereal Hour Angle 49° 58′.
Vega. — Declination 38° 45′ N, Sidereal Hour Angle 81° 05′.

Plot these positions and the relative position of the space vehicle's zenith on a chart to suitable scale (fig. 8). This gives the geographical position of the observer to be at latitude 51° N, and at sidereal hour angle of 65°. Had the Greenwich Mean Time of observation been 0h 26m 55s on 12 July 1961, the Greenwich Hour Angle of Aries would have been, for that instant, 06° 45′ W, which, added to the sidereal hour of the space vehicle (65° 00′) would place the space vehicle in longitude 1° 24′ W.

The Air Almanac gives for each day the Greenwich Hour Angle of Aries (GHA γ) at ten minute intervals. Since the annual three-volume Air Almanac totals about 900 pages and since GHA γ and a few pages of star positions from the almanac will fill the principle needs of the astronaut, a separate annual sheet may be used to find GHA γ and replace the Air Almanac. This condensation is possible due to the fact that the GHA γ may be given for 0h each day and the correction for elapsed time may be carried through 24 hours. Figures 9 and 10 give the GHA γ, or Greenwich Sidereal Time, for the entire year.

From table S1, for 21° 30′.2, the distance to the geographical position from the space vehicle is found to be 15 000 nautical miles. Therefore, the space vehicle's position is : 15 000 nautical miles vertically above latitude 51° N, longitude 1° 24′ W.

SOLUTION

GMT 0h 26m 55s on 12 July 1961

GHA for 0 h	289° 39′	taken from table shown in fig. 10
Corr for 0 h 26 m ...	6° 31′	taken from table shown in fig. 11
Corr for 55 s	0° 14′	

GREENWICH HOUR ANGLE OF ARIES OR
GREENWICH SIDEREAL TIME (EXPRESSED IN ARC) OF O HRS G. M. T.

CALENDAR YEAR 1961

Day of Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
1	100 24	130 58	158 34	189 07	218 41	249 14	278 48	309 22	339 55	9 29	40 02	69 37
2	101 23	131 57	159 33	190 06	219 40	250 13	279 48	310 21	340 54	10 28	41 02	70 36
3	102 23	132 56	160 32	191 05	220 39	251 13	280 47	311 20	341 53	11 27	42 01	71 35
4	103 22	133 55	161 31	192 04	221 38	252 12	281 46	312 19	342 52	12 27	42 00	72 34
5	104 21	134 54	162 30	193 03	222 38	253 11	282 45	313 18	343 52	13 26	43 59	73 33
6	105 20	135 53	163 29	194 03	223 37	254 10	283 44	314 17	344 51	14 25	44 58	74 32
7	106 19	136 52	164 28	195 02	224 36	255 09	284 43	315 17	345 50	15 24	45 57	75 31
8	107 18	137 52	165 28	196 01	225 35	256 08	285 42	316 16	346 49	16 23	46 56	76 31
9	108 17	138 51	166 27	197 00	226 34	257 07	286 42	317 15	347 48	17 22	47 56	77 30
10	109 17	139 50	167 26	197 59	227 33	258 07	287 41	318 14	348 47	18 21	48 55	78 29
11	110 16	140 49	168 25	198 58	228 32	259 06	288 40	319 13	349 46	19 21	49 54	79 28
12	111 15	141 48	169 24	199 57	229 31	260 05	289 39	320 12	350 46	20 20	50 53	80 27
13	112 14	142 47	170 23	200 56	230 31	261 04	290 38	321 11	351 45	21 19	51 52	81 26
14	113 13	143 46	171 22	201 56	231 30	262 03	291 37	322 11	352 44	22 18	52 51	82 25
15	114 12	144 46	172 21	202 55	232 29	263 02	292 36	323 10	353 43	23 17	53 50	83 25
16	115 11	145 45	173 21	203 54	233 28	264 01	293 36	324 09	354 42	24 16	54 50	84 24
17	116 11	146 44	174 20	204 53	234 27	265 01	294 35	325 08	355 41	25 15	55 49	85 23
18	117 10	147 43	175 19	205 52	235 26	266 00	295 34	326 07	356 40	26 15	56 48	86 22
19	118 09	148 42	176 18	206 51	236 25	266 59	296 33	327 06	357 40	27 14	57 47	87 21
20	119 08	149 41	177 17	207 50	237 25	267 58	297 32	328 05	358 39	28 13	58 46	88 20
21	120 07	150 40	178 16	208 50	238 24	268 57	298 31	329 05	359 38	29 12	59 45	89 19
22	121 06	151 40	179 15	209 49	239 23	269 56	299 30	330 04	0 37	30 11	60 44	90 19
23	122 05	152 39	180 15	210 48	240 22	270 55	300 30	331 03	1 36	31 10	61 44	91 18
24	123 05	153 38	181 14	211 47	241 21	271 54	301 29	332 02	2 35	32 09	62 43	92 17
25	124 04	154 37	182 13	212 46	242 20	272 54	302 28	333 01	3 34	33 08	63 42	93 16
26	125 03	155 36	183 12	213 45	243 19	273 53	303 27	334 00	4 33	34 08	64 41	94 15
27	126 02	156 35	184 11	214 44	244 19	274 52	304 26	334 59	5 33	35 07	65 40	95 14
28	127 01	157 34	185 10	215 44	245 18	275 51	305 25	335 58	6 32	36 06	66 39	96 13
29	128 00		186 09	216 43	246 17	276 50	306 24	336 58	7 31	37 05	67 38	97 13
30	128 59		187 09	217 42	247 16	277 49	307 23	337 57	8 30	38 04	68 38	98 12
31	129 59		188 08		248 15		308 23	338 56		39 03		99 11

FIG. 9. — Greenwich hour angle of Aries for 0h each day, 1961.

CORRECTION TO BE ADDED TO TABULATED GREENWICH SIDEREAL TIME FOR ELAPSED GREENWICH MEAN TIME

Min.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 0	15 3	30 5	45 7	60 10	75 12	90 15	105 17	120 20	135 22	150 25	165 27	180 30
1	0 15	15 18	30 20	45 22	60 25	75 27	90 30	105 32	120 35	135 37	150 40	165 42	180 45
2	0 30	15 33	30 35	45 38	60 40	75 42	90 45	105 47	120 50	135 52	150 55	165 57	181 0
3	0 45	15 48	30 50	45 53	60 55	75 57	91 0	106 2	121 5	136 7	151 10	166 12	181 15
4	1 0	16 3	31 5	46 8	61 10	76 13	91 15	106 17	121 20	136 22	151 25	166 27	181 30
5	1 15	16 18	31 20	46 23	61 25	76 28	91 30	106 33	121 35	136 37	151 40	166 42	181 45
6	1 30	16 33	31 35	46 38	61 40	76 43	91 45	106 48	121 50	136 52	151 55	166 57	182 0
7	1 45	16 48	31 50	46 53	61 55	76 58	92 0	107 3	122 5	137 8	152 10	167 12	182 15
8	2 0	17 3	32 5	47 8	62 10	77 13	92 15	107 18	122 20	137 23	152 25	167 27	182 30
9	2 15	17 18	32 20	47 23	62 25	77 28	92 30	107 33	122 35	137 38	152 40	167 43	182 45
10	2 30	17 33	32 35	47 38	62 40	77 43	92 45	107 48	122 50	137 53	152 55	167 58	183 0
11	2 46	17 48	32 50	47 53	62 55	77 58	93 0	108 3	123 5	138 8	153 10	168 13	183 15
12	3 1	18 3	33 5	48 8	63 10	78 13	93 15	108 18	123 20	138 23	153 25	168 28	183 30
13	3 16	18 18	33 21	48 23	63 25	78 28	93 30	108 33	123 35	138 38	153 40	168 43	183 45
14	3 31	18 33	33 36	48 38	63 40	78 43	93 45	108 48	123 50	138 53	153 55	168 58	184 0
15	3 46	18 48	33 51	48 53	63 56	78 58	94 0	109 3	124 5	139 8	154 10	169 13	184 15
16	4 1	19 3	34 6	49 8	64 11	79 13	94 15	109 18	124 20	139 23	154 25	169 28	184 30
17	4 16	19 18	34 21	49 23	64 26	79 28	94 31	109 33	124 35	139 38	154 40	169 43	184 45
18	4 31	19 33	34 36	49 38	64 41	79 43	94 46	109 48	124 50	139 53	154 55	169 58	185 0
19	4 46	19 48	34 51	49 53	64 56	79 58	95 1	110 3	125 6	140 8	155 10	170 13	185 15
20	5 1	20 3	35 6	50 8	65 11	80 13	95 16	110 18	125 21	140 23	155 26	170 28	185 30
21	5 16	20 18	35 21	50 23	65 26	80 28	95 31	110 33	125 36	140 38	155 41	170 43	185 45
22	5 31	20 33	35 36	50 38	65 41	80 43	95 46	110 48	125 51	140 53	155 56	170 58	186 1
23	5 46	20 48	35 51	50 53	65 56	80 58	96 1	111 3	126 6	141 8	156 11	171 13	186 16
24	6 1	21 3	36 6	51 8	66 11	81 13	96 16	111 18	126 21	141 23	156 26	171 28	186 31
25	6 16	21 19	36 21	51 23	66 26	81 28	96 31	111 33	126 36	141 38	156 41	171 43	186 46
26	6 31	21 34	36 36	51 39	66 41	81 43	96 46	111 48	126 51	141 53	156 56	171 58	187 1
27	6 46	21 49	36 51	51 54	66 56	81 58	97 1	112 3	127 6	142 8	157 11	172 13	187 16
28	7 1	22 4	37 6	52 9	67 11	82 14	97 16	112 18	127 21	142 23	157 26	172 28	187 31
29	7 16	22 19	37 21	52 24	67 26	82 29	97 31	112 33	127 36	142 38	157 41	172 43	187 46
30	7 31	22 34	37 36	52 39	67 41	82 44	97 46	112 49	127 51	142 53	157 56	172 58	188 1
31	7 46	22 49	37 51	52 54	67 56	82 59	98 1	113 4	128 6	143 8	158 11	173 13	188 16
32	8 1	23 4	38 6	53 9	68 11	83 14	98 16	113 19	128 21	143 24	158 26	173 28	188 31
33	8 16	23 19	38 21	53 24	68 26	83 29	98 31	113 34	128 36	143 39	158 41	173 44	188 46
34	8 31	23 34	38 36	53 39	68 41	83 44	98 46	113 49	128 51	143 54	158 56	173 59	189 1
35	8 46	23 49	38 51	53 54	68 56	83 59	99 1	114 4	129 6	144 9	159 11	174 14	189 16
36	9 2	24 4	39 6	54 9	69 11	84 14	99 16	114 19	129 21	144 24	159 26	174 29	189 31
37	9 17	24 19	39 21	54 24	69 26	84 29	99 31	114 34	129 36	144 39	159 41	174 44	189 46
38	9 32	24 34	39 37	54 39	69 41	84 44	99 46	114 49	129 51	144 54	159 56	174 59	190 1
39	9 47	24 49	39 52	54 54	69 57	84 59	100 1	115 4	130 6	145 9	160 11	175 14	190 16
40	10 2	25 4	40 7	55 9	70 12	85 14	100 16	115 19	130 21	145 24	160 26	175 29	190 31
41	10 17	25 19	40 22	55 24	70 27	85 29	100 32	115 34	130 36	145 39	160 41	175 44	190 46
42	10 32	25 34	40 37	55 39	70 42	85 44	100 47	115 49	130 51	145 54	160 56	175 59	191 1
43	10 47	25 49	40 52	55 54	70 57	85 59	101 2	116 4	131 7	146 9	161 11	176 14	191 16
44	11 2	26 4	41 7	56 9	71 12	86 14	101 17	116 19	131 22	146 24	161 26	176 29	191 31
45	11 17	26 19	41 22	56 24	71 27	86 29	101 32	116 34	131 37	146 39	161 42	176 44	191 46
46	11 32	26 34	41 37	56 39	71 42	86 44	101 47	116 49	131 52	146 54	161 57	176 59	192 2
47	11 47	26 49	41 52	56 54	71 57	86 59	102 2	117 4	132 7	147 9	162 12	177 14	192 17
48	12 2	27 4	42 7	57 9	72 12	87 14	102 17	117 19	132 22	147 24	162 27	177 29	192 32
49	12 17	27 20	42 22	57 24	72 27	87 29	102 32	117 34	132 37	147 39	162 42	177 44	192 47
50	12 32	27 35	42 37	57 39	72 42	87 44	102 47	117 49	132 52	147 54	162 57	177 59	193 2
51	12 47	27 50	42 52	57 55	72 57	87 59	103 2	118 4	133 7	148 9	163 12	178 14	193 17
52	13 2	28 5	43 7	58 10	73 12	88 15	103 17	118 19	133 22	148 24	163 27	178 29	193 32
53	13 17	28 20	43 22	58 25	73 27	88 30	103 32	118 34	133 37	148 39	163 42	178 44	193 47
54	13 32	28 35	43 37	58 40	73 42	88 45	103 47	118 50	133 52	148 54	163 57	178 59	194 2
55	13 47	28 50	43 52	58 55	73 57	89 0	104 2	119 5	134 7	149 9	164 12	179 14	194 17
56	14 2	29 5	44 7	59 10	74 12	89 15	104 17	119 20	134 22	149 25	164 27	179 29	194 32
57	14 17	29 20	44 22	59 25	74 27	89 30	104 32	119 35	134 37	149 40	164 42	179 44	194 47
58	14 32	29 35	44 37	59 40	74 42	89 45	104 47	119 50	134 52	149 55	164 57	180 0	195 2
59	14 47	29 50	44 52	59 55	74 57	90 0	105 2	120 5	135 7	150 10	165 12	180 15	195 17
60	15 3	30 5	45 7	60 10	75 12	90 15	105 17	120 20	135 22	150 25	165 27	180 30	195 32

Min.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	195 32	210 35	225 37	240 39	255 42	270 44	285 47	300 49	315 52	330 54	345 57
1	195 47	210 50	225 52	240 55	255 57	270 59	286 2	301 4	316 7	331 9	346 12
2	196 2	211 5	226 7	241 10	256 12	271 14	286 17	301 19	316 22	331 24	346 27
3	196 17	211 20	226 22	241 25	256 27	271 30	286 32	301 34	316 37	331 39	346 42
4	196 32	211 35	226 37	241 40	256 42	271 45	286 47	301 49	316 52	331 54	346 57
5	196 47	211 50	226 52	241 55	256 57	272 0	287 2	302 5	317 7	332 9	347 12
6	197 2	212 5	227 7	242 10	257 12	272 15	287 17	302 20	317 22	332 25	347 27
7	197 17	212 20	227 22	242 25	257 27	272 30	287 32	302 35	317 37	332 40	347 42
8	197 32	212 35	227 37	242 40	257 42	272 45	287 47	302 50	317 52	332 55	347 57
9	197 47	212 50	227 52	242 55	257 57	273 0	288 2	303 5	318 7	333 10	348 12
10	198 2	213 5	228 7	243 10	258 12	273 15	288 17	303 20	318 22	333 25	348 27
11	198 18	213 20	228 22	243 25	258 27	273 30	288 32	303 35	318 37	333 40	348 42
12	198 33	213 35	228 38	243 40	258 42	273 45	288 47	303 50	318 52	333 55	348 57
13	198 48	213 50	228 53	243 55	258 57	274 0	289 2	304 5	319 7	334 10	349 12
14	199 3	214 5	229 8	244 10	259 13	274 15	289 17	304 20	319 22	334 25	349 27
15	199 18	214 20	229 23	244 25	259 28	274 30	289 32	304 35	319 37	334 40	349 42
16	199 33	214 35	229 38	244 40	259 43	274 45	289 48	304 50	319 52	334 55	349 57
17	199 48	214 50	229 53	244 55	259 58	275 0	290 3	305 5	320 7	335 10	350 12
18	200 3	215 5	230 8	245 10	260 13	275 15	290 18	305 20	320 23	335 25	350 27
19	200 18	215 20	230 23	245 25	260 28	275 30	290 33	305 35	320 38	335 40	350 43
20	200 33	215 35	230 38	245 40	260 43	275 45	290 48	305 50	320 53	335 55	350 58
21	200 48	215 50	230 53	245 55	260 58	276 0	291 3	306 5	321 8	336 10	351 13
22	201 3	216 5	231 8	246 10	261 13	276 15	291 18	306 20	321 23	336 25	351 28
23	201 18	216 20	231 23	246 25	261 28	276 30	291 33	306 35	321 38	336 40	351 43
24	201 33	216 36	231 38	246 40	261 43	276 45	291 48	306 50	321 53	336 55	351 58
25	201 48	216 51	231 53	246 55	261 58	277 0	292 3	307 5	322 8	337 10	352 13
26	202 3	217 6	232 8	247 11	262 13	277 15	292 18	307 20	322 23	337 25	352 28
27	202 18	217 21	232 23	247 26	262 28	277 31	292 33	307 35	322 38	337 40	352 43
28	202 33	217 36	232 38	247 41	262 43	277 46	292 48	307 50	322 53	337 55	352 58
29	202 48	217 51	232 53	247 56	262 58	278 1	293 3	308 6	323 8	338 10	353 13
30	203 3	218 6	233 8	248 11	263 13	278 16	293 18	308 21	323 23	338 25	353 28
31	203 18	218 21	233 23	248 26	263 28	278 31	293 33	308 36	323 38	338 41	353 43
32	203 33	218 36	233 38	248 41	263 43	278 46	293 48	308 51	323 53	338 56	353 58
33	203 48	218 51	233 53	248 56	263 58	279 1	294 3	309 6	324 8	339 11	354 13
34	204 3	219 6	234 8	249 11	264 13	279 16	294 18	309 21	324 23	339 26	354 28
35	204 19	219 21	234 23	249 26	264 28	279 31	294 33	309 36	324 38	339 41	354 43
36	204 34	219 36	234 38	249 41	264 43	279 46	294 48	309 51	324 53	339 56	354 58
37	204 49	219 51	234 54	249 56	264 58	280 1	295 3	310 6	325 8	340 11	355 13
38	205 4	220 6	235 9	250 11	265 13	280 16	295 18	310 21	325 23	340 26	355 28
39	205 19	220 21	235 24	250 26	265 29	280 31	295 33	310 36	325 38	340 41	355 43
40	205 34	220 36	235 39	250 41	265 44	280 46	295 49	310 51	325 53	340 56	355 58
41	205 49	220 51	235 54	250 56	265 59	281 1	296 4	311 6	326 8	341 11	356 13
42	206 4	221 6	236 9	251 11	266 14	281 16	296 19	311 21	326 24	341 26	356 28
43	206 19	221 21	236 24	251 26	266 29	281 31	296 34	311 36	326 39	341 41	356 43
44	206 34	221 36	236 39	251 41	266 44	281 46	296 49	311 51	326 54	341 56	356 59
45	206 49	221 51	236 54	251 56	266 59	282 1	297 4	312 6	327 9	342 11	357 14
46	207 4	222 6	237 9	252 11	267 14	282 16	297 19	312 21	327 24	342 26	357 29
47	207 19	222 21	237 24	252 26	267 29	282 31	297 34	312 36	327 39	342 41	357 44
48	207 34	222 37	237 39	252 41	267 44	282 46	297 49	312 51	327 54	342 56	357 59
49	207 49	222 52	237 54	252 56	267 59	283 1	298 4	313 6	328 9	343 11	358 14
50	208 4	223 7	238 9	253 12	268 14	283 16	298 19	313 21	328 24	343 26	358 29
51	208 19	223 22	238 24	253 27	268 29	283 31	298 34	313 36	328 39	343 41	358 44
52	208 34	223 37	238 39	253 42	268 44	283 47	298 49	313 51	328 54	343 56	358 59
53	208 49	223 52	238 54	253 57	268 59	284 2	299 4	314 7	329 9	344 11	359 14
54	209 4	224 7	239 9	254 12	269 14	284 17	299 19	314 22	329 24	344 26	359 29
55	209 19	224 22	239 24	254 27	269 29	284 32	299 34	314 37	329 39	344 42	359 44
56	209 34	224 37	239 39	254 42	269 44	284 47	299 49	314 52	329 54	344 57	359 59
57	209 49	224 52	239 54	254 57	269 59	285 2	300 4	315 7	330 9	345 12	0 14
58	210 4	225 7	240 9	255 12	270 14	285 17	300 19	315 22	330 24	345 27	0 29
59	210 20	225 22	240 24	255 27	270 29	285 32	300 34	315 37	330 39	345 42	0 44
60	210 35	225 37	240 39	255 42	270 44	285 47	300 49	315 52	330 54	345 57	0 59

Sec.	Corr.
	′
0	0
1	0
2	1
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60	15

FIG. 10

GHA for 0h 26m 55s . .	296° 24′
SHA of Zenith	65° 00′
Longitude	1° 24′ W (360° subtracted)
Latitude (declination) .	51° 00′ N

A more direct method, but using the same principle, would be for the observer to view the earth and star field through a transparent star half-globe with the eye at its center, and with a grid of meridians and parallels projected against the earth and sky. The globe is oriented by placing any two stars on the globe over the corresponding stars in the sky. In this position the observer notes the position of the nadir against the grid. Since the nadir is 180° from the zenith, the globe is marked to show the correct latitude and sidereal hour angle, from which longitude may be found as previously explained.

For greater accuracy, photograph the earth and star field with a polaroid camera and superpose the film on a star chart printed on a transparent sheet at the same scale as the photograph, then scale the declination and sidereal hour angle of the observed nadir, and change this position 180° to get the observer's geographical position. Proceed as explained heretofore to get the longitude of the observer.

The local vertical, determined by the direct viewing of the earth's center, may be used to control the attitude of the space vehicle. This attitude control could of course be made relative to the line of sight to any fixed star, but it appears more logical and more useful to fix the space vehicle's attitude relative to the earth itself.