TELLUROMETER MEASUREMENTS IN THE MUNICH BASE EXTENSION NETWORK

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The development of instruments and of procedures for distance measurements by means of high-frequency-modulated light or by means of electromagnetic waves has made significant progress within the past few years. Since Germany has had but little share in this work and in testing the new geodetic methods of measuring, we wanted to present for the first time practical experience with the Tellurometer measurements reported here, particularly about :

(1) fixation of the network by means of pure distance measurement;
(2) determination of the possible accuracy for trilateration lines and for the point positions;
(3) investigation of influence of meteorological conditions and of ground formation on the measurement of the transit time of electromagnetic waves.

The measurements were performed in 14 days in the autumn of 1958. All the lines of the extension network (fig. 1) and a second line of the primary triangulation were measured; the lengths were 8.2 to 56.3 km. To determine changes, if any, of the instrument, one line (baseline north-baseline south) was measured at the beginning, in the middle, and at the end of the observations. The complete measurement of the first extension triangle showed no differences between the measurements in opposite directions; hence further return observations were not made, in view of the homogeneous ground conditions in the whole network. On all stations four complete measurements were carried through. It was found that the best observation conditions, and consequently the time requirements, depend primarily on the weather and on the time of day. Overcast, mist, and rain are all favorable; with bright sunlight satisfactory measurements were possible only in the morning and in the late afternoon.

The oblique distance D′ was derived from the measured transit times and the meteorological data in the usual way, according to

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$$D' = \frac{1}{2}\,\theta\cdot c\cdot\frac{1}{n} \tag{1}$$

with

$$(n - 1)\cdot 10^6 = \frac{103{,}46}{273 + T}(B + K) \tag{2}$$

$$K = \frac{4744}{273 + T}\cdot e \tag{3}$$

where

B = air pressure, mm Hg.
e = vapor pressure of air, mm Hg.
T = air temperature, °C.

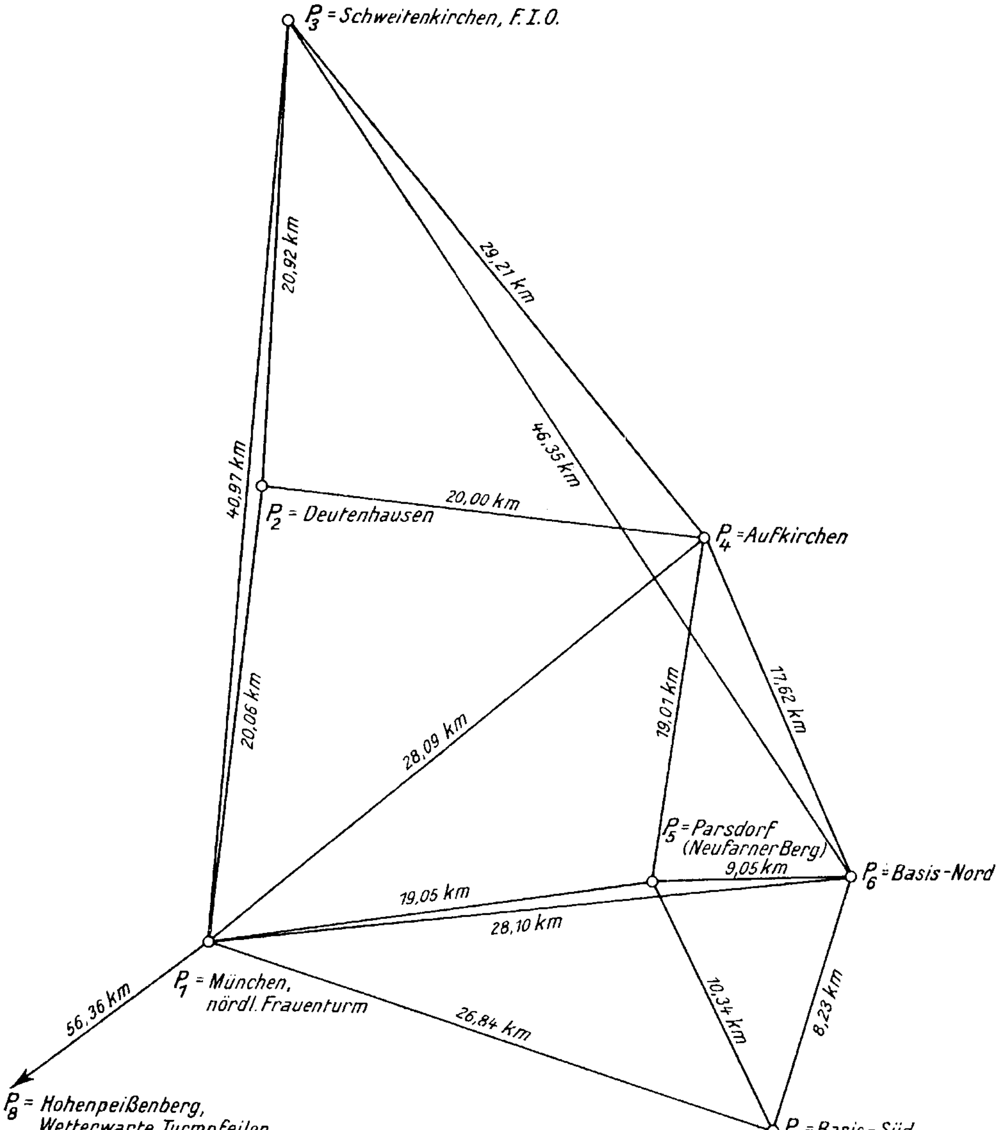

FIG. 1. — Base extension network Munich. Scale 1/250 000

The vapor pressure e was derived from

$$e = E - 0{,}48 \cdot \frac{T - t}{610 - t} \cdot B \tag{4}$$

where E is the saturated water vapor for the wet-bulb temperature t. For the light velocity c in vacuo the value 299 792.5 km/sec recommended by the International Association of Geodesy was used. As far as accuracy of the oblique distances is concerned, it made no difference whether the repeated measurements of one line were performed on the same day, on different days, in different directions, or with highly different atmospheric conditions; the results differed only by a few centimetres. A graph of the mean errors computed from the repeated measurements shows no apparent dependence of the mean errors on the distances; the maximum mean errors occur with the measurements performed on sunny days.

One line, Munich to baseline south, caused great difficulties. The precise readings oscillated highly ($\Delta_{max} = 5.10^{-9}$ sec $\cong$ 0.35 metre) and it was found that the available range of the carrier frequency was too small for the measurement (the carrier frequency of 3 000 Mc/s is varied in 12 steps over a range of $\pm$ 10 Mc/s). No explanation for this has been found; an adjacent line with similar ground contour could be measured without difficulties under the same weather conditions.

The following expression is sufficiently accurate for the reduction of the measured oblique distances D′ to the computation reference surface :

$$S = \sqrt{\frac{(D' + \Delta H)(D' - \Delta H)}{\left(1 + \frac{H_1}{R}\right)\left(1 + \frac{H_2}{R}\right)}} + \frac{(D')^3}{24\, R^2} \tag{5}$$

where

ΔH = difference of elevation.
H_1 = elevation above sea level.
R = curvature radius of the reference surface.
S = length of the geodetic line.

The numerical calculation is conveniently performed in three steps, (5) being used in the form

$$S = D' + K_1 + K_2 + K_3 \tag{6}$$

where

$$K_1 = -\frac{(\Delta H)^2}{2D'}, \quad K_2 = -D \cdot \frac{H_m}{R}, \quad K_3 = \frac{(D')^3}{24 R^2} \tag{7}$$

This neglects only the small difference between chord and arc of the ray, which amounts to only a few millimetres for measurable triangle sides, even under extreme atmospheric conditions.

The accuracy with which the quantities ΔH, H_m and R are to be introduced into the reductions, if the partial errors caused by errors $d\Delta H$, dH_m, and dR are to become smaller than 5 mm in all reductions, is important.

There results, for instance, the condition for K_1 :

$$0{,}005 \text{ m} > -\frac{\Delta H\, d\Delta H}{D'} \tag{8}$$

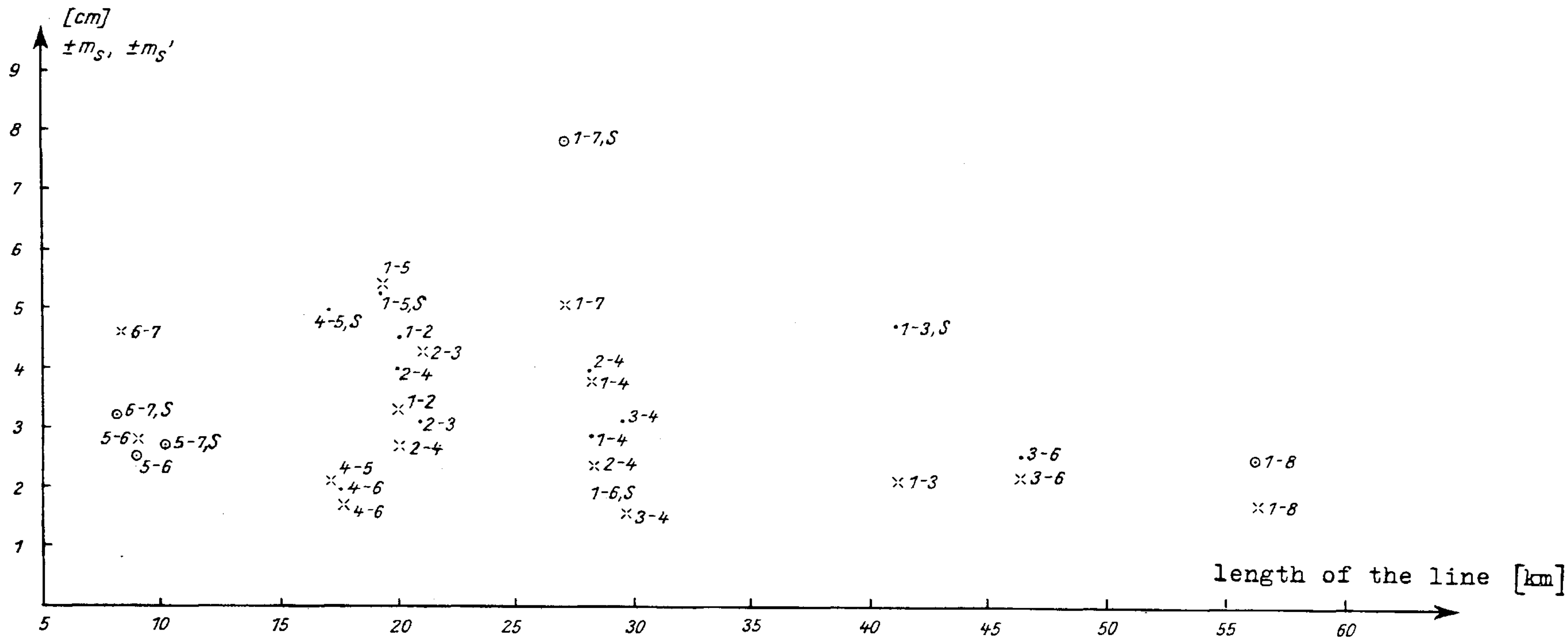

FIG. 2. — Graph of the mean errors m_s and m'_s

m_s	=	mean error computed from various measurements
m'_s	=	mean error computed from travel times measured at various frequencies
. *i.k*	=	mean error m_s for line *i.k.* computed from observations on same day
⊙ *i.k*	=	mean error m_s for line *i.k.* computed from observations on different days
× *i.k*	=	mean error m'_s for line *i.k.*
S	=	observations under sunshine

Assuming $d\Delta H = 0.5$ metre results in the values shown as follows for maximum admissible differences of elevation ΔH for various lengths :

Length	1	5	10	20	30	40	50	km
ΔH admissible	10	50	100	200	300	400	500	m

The accuracy of H_m necessary for K_2 can be found in the same way :

$$0.005 \text{ m} > -\frac{D}{R} dH_m \tag{9}$$

Admissible errors dH_m calculated for various lengths are shown as follows :

Length	1	10	20	30	40	50	km
dH_m admissible	31.85	3.18	1.59	1.06	0.80	0.64	m

Hence it can be seen that the use of barometrically determined elevations will be satisfactory only in a few rare cases, even these being measured very carefully.

For the necessary accuracy of R there results

$$\frac{1}{R^2} \cdot H_m \cdot D \cdot dR < 0.005 \text{ m} \tag{10}$$

from which the following can be computed :

D \ H_m	0.5 km	0.6 km	0.7 km	0.8 km	0.9 km	1.0 km	1.1 km
10 km	40.7 km	33.9 km	29.1 km	25.4 km	22.6 km	20.3 km	18.5 km
20 km	20.3 km	17.0 km	14.5 km	12.7 km	11.3 km	10.2 km	
30 km	13.6 km	11.3 km					
40 km	10.2 km			dR admissible $<$ 10 km			
50 km							

Generally it will be necessary to determine the curvature radius R_α for the mean latitude φ_m and the azimuth α of the side. The use of a Gauss curvature radius $R = \sqrt{MN}$ is possible only for short sides, because $(R_\alpha - \sqrt{MN})$ attains on the average 10 km for the mean latitudes 45° to 50°.

Comparison of two sides with the Bavarian primary triangulation was interesting. The result of a base extension (measured by Clauss in 1920) for the side Munich-Schweitenkirchen is shown in Table 1.

Whereas the Tellurometer measurement and the base extension by Clauss are in good agreement, a scale difference of about 5×10^{-6} arises relative to the primary triangulation. However, it was decided not to derive a correction for the primary triangulation, or a correction coefficient for the quartz frequency of the Tellurometer, because of the low number of comparison values. For the following adjustment there were used the directly measured side lengths and those equalized to the primary triangulation in accordance with

$$v_{mm} = -24 - 4{,}81 \times S_{km} \tag{11}$$

TABLE 1

Side	Telluro-meter Measu-rement (1)	Bavarian Primary Triangula-tion (2)	Base Extension Network (3)	Differences	
				(2)—(1)	(3)—(1)
Munich-Schweitenkirchen	40974.775	40974.554	40974.764	— 0.221	— 0.011
Munich-Hohenpeissenbert	56364.225	56363.93	—	— 0.295	—

For the adjustment of the trilateration network it was at first necessary to select a proper computation reference surface. The network was designed in a plane mapping (Gauss-Krüger projection) because it became possible to compute sufficiently precise coordinates for all the points of the network and hence rigorous side reductions Δs_{ik}. This implies essential simplifications against the otherwise necessary computation on the spheroid or on a Gauss sphere. Thus the measured side lengths mean the planed values s of the spheroidal side lengths S.

$$s_{ik} = S_{ik} + \Delta s_{ik} \qquad (12)$$

The following considerations were decisive for the selection of the adjustment method : Corresponding to the

$$r = s - (2p - 3) = 4$$

surplus sides there result four normal equations with adjustment by variation of coordinates. The time factor was not decisive for the selection of the method, because a program-controlled relay computing machine Z11 was available for the solution of the normal equations. Thus the adjustment by variation of coordinates was preferred for the following reasons :

(*a*) the procedure is easier to survey, and the error equations are simpler to establish than the condition equations;

(*b*) the result is given directly in terms of the coordinates of the new points, whereas otherwise they can only be computed from the adjusted side lengths.

The error equation in terms of plane coordinates has the form

$$\begin{aligned} v_{ik} = {} & \Delta y_i \sin \bar{t}_{ki} + \Delta x_i \cos \bar{t}_{ki} + \Delta y_k \sin \bar{t}_{ik} \\ & + \Delta x_k \cos \bar{t}_{ik} + \bar{s}_{ik} - s_{ik} \end{aligned} \qquad (13)$$

where $\bar{t}$ and $\bar{s}$ are the values for direction angle and side length derived from the approximate (adopted) coordinates. All the measured sides were taken with the same weights ($p_{ik} = 1$).

The adjustment was performed in four different ways :

I. Measured side lengths II. Side lengths equalized to the primary triangulation according to equation 11	Form of the base extension network without station Deutenhausen
III. Measured side lengths IV. Equalized side lengths	Complete network

Only the side lengths and the corrections ensuing from the adjustments are given as results, because they indicate the intrinsic accuracy of the Tellurometer measurement :

Side	Plane Side Lengths, m		Corrections for Adjustment, m			
	Measured	Equalized according to Equation 11	I	II	III	IV
1.2	20 056.975	056.855	—	—	— 0.054	— 0.049
1.3	40 975.237	975.016	— 0.012	— 0.003	+ 0.042	+ 0.036
1.4	28 090.262	090.033	+ 0.012	+ 0.024	+ 0.012	+ 0.024
1.5	19 053.527	053.411	— 0.038	— 0.038	— 0.038	— 0.038
1.6	28 100.454	100.295	+ 0.029	+ 0.015	+ 0.029	+ 0.015
1.7	26 838.807	838.654	— 0.008	— 0.001	— 0.008	— 0.002
2.3	20 918.424	918.299	—	—	— 0.005	— 0.043
2.4	20 003.804	003.584	—	—	+ 0.001	+ 0.005
3.4	29 206.183	206.018	— 0.052	— 0.059	— 0.058	— 0.066
3.6	46 348.713	348.466	+ 0.070	+ 0.066	+ 0.074	+ 0.067
4.5	17 009.573	009.467	— 0.002	+ 0.006	— 0.002	+ 0.007
4.6	17 623.069	622.960	— 0.070	— 0.068	— 0.090	— 0.068
5.6	9 047.662	047.594	— 0.044	— 0.034	— 0.044	— 0.035
5.7	10 337.583	337.509	+ 0.010	— 0.002	+ 0.010	— 0.002
6.7	8 231.927	231.863	— 0.009	+ 0.003	— 0.008	+ 0.004

On the average the corrections for the measured sides amount to ± 3.5 cm. Thus the geometrical network conditions are well fulfilled; gross systematical errors do not occur. Furthermore, the correctness of the distribution of weights is proved; large and small corrections occur with the same frequency for the long and short sides. Thus accuracy is not a function of length. Such further factors as ground or unfavorable atmospheric conditions which are reflected in the deviations of the individual measurements can evidently not be used for assignment of weights. The mean error of a measured side after adjustment is ± 0.07 metre. Comparison with the mean errors derived from repeated observations suggests an additional error affecting the measurements, which, however, cannot be explained. The average position accuracy of the new points fixed by pure distance measurements amounts to ± 0.1 metre. Evaluation of the external accuracy and eventual scale errors will be reserved for later comparison with the results of baseline measurement and triangulation.