TELLUROMETER TRILATERATION IN ARABIA

by Hans MEIER
Geodesist
Aero Service Corporation, Philadelphia

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Since the introduction of electronic distance-measuring equipment for medium-length distances (1 to 30 miles), much has been published about the principle and accuracy of the various instruments used for this purpose. The author wishes to communicate some practical experience gained with the Tellurometer (**) during a trilateration survey in the Arabian Desert. In 1958, Aero Service Corporation was given the problem of determining the coordinate position of four offshore navigation beacons near Ras Tanura in Saudi Arabia for the Arabian American Oil Company.

The existing horizontal-control points which were to be used as the base of the survey could not be recovered with certainty on the ground and were, consequently, of little value. It was, therefore, necessary to extend the survey to assure positive recovery of the previously established control points. To do the job by triangulation appeared difficult due to the adverse prevailing weather conditions in the area. Sandstorms impair the visibility, and costly tower construction would have been necessary at various stations to assure line of sight. These considerations and the fear of large lateral refraction over the desert led to the conclusion that the problem could only be solved economically by Tellurometer trilateration.

Figure 1 shows the trilateration network which was measured by one Tellurometer party of four men (two surveyors and two Arab helpers) in 30 days. 59 distances were measured varying in length from 3.5 to 58.2 km. 17 of these distances were over water — these distances being usually looked upon with suspicion. Any flat, smooth surface over which the signal travels is known to reflect the ground wave and will, therefore, interfere with the direct-wave signal resulting in a fuzzy scope presentation and usually an unreliable distance measurement.

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(**) A detailed study of this system will appear in the Supplementary Papers to SP 39.

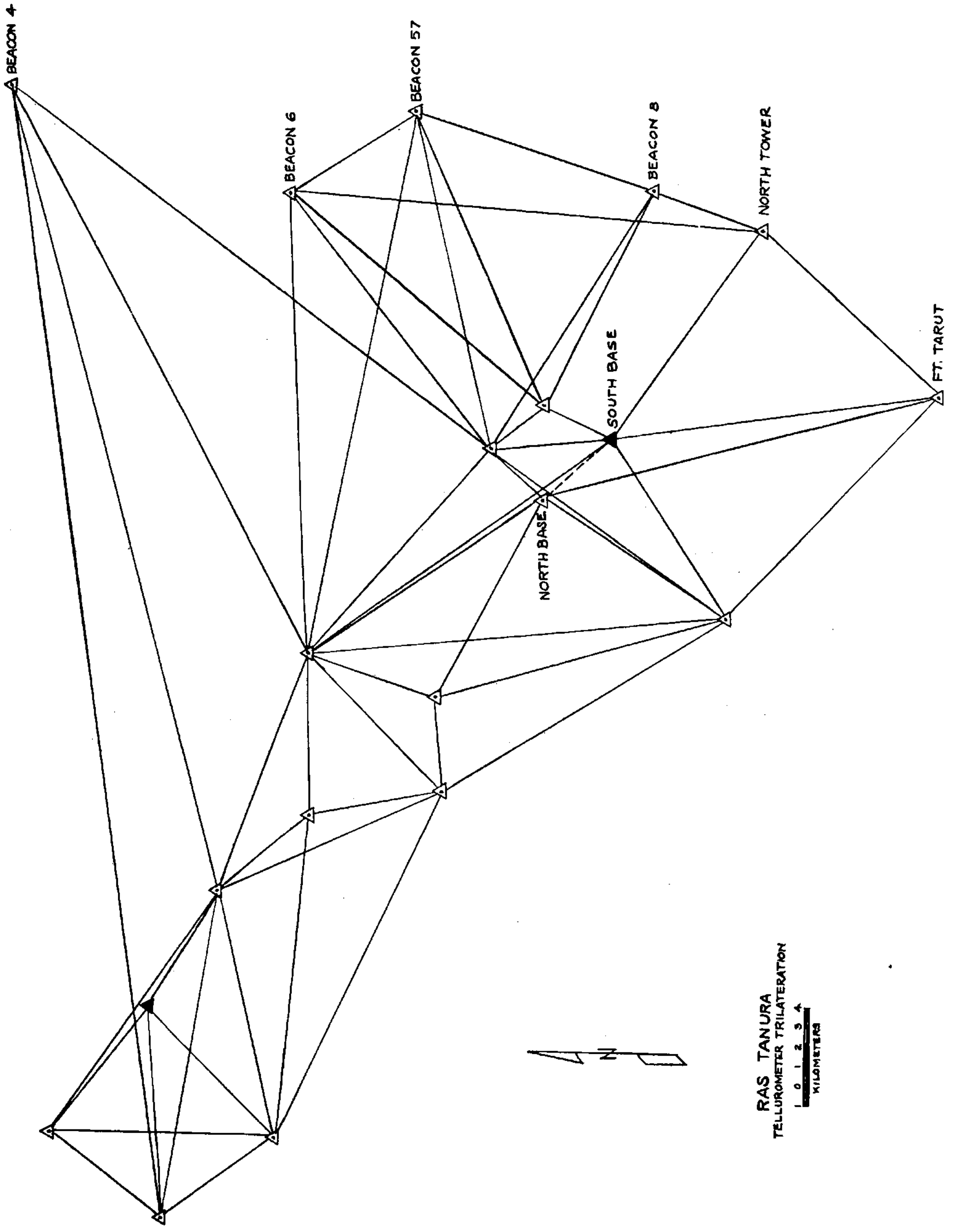

FIG. 1. — ▲ Position held in adjustment
.... Azimuth held in adjustment

None of these phenomena, however, was experienced with the *long* overwater measurements which were made at night by virtue of logistic convenience rather than on purpose. However, the short overwater distances, measured during the daytime, showed slightly fuzzy scope presentation and erratic distance readings.

All 4 distance measurements to Beacon No. 4 deserve special mention, because the remote station at the beacon site could not be seen from the master-station locations on shore. The line of sight actually was the chord distance, cutting 14 to 69 feet *below* the water surface. No trouble was experienced measuring these lines. In contrast to this rather unusual phenomenon, it was impossible to measure the much shorter distance from Beacon No. 6 to Beacon No. 4. It was not even possible to get voice communication over the beam, and a messenger had to be sent by boat to relay messages. The reason for this strange occurrence is a mystery and would certainly be worth investigating.

During the whole operation, the survey party was plagued by equipment failures. Cable contacts broke too easily, and sand penetrated into the instrument. The sandstorms blowing over the desert made the vertical-angle measurements difficult and slowed the operation, since the visibility was very poor. Trying to solve this survey problem by triangulation would have made the undertaking an uneconomical affair. Triangulation as well as traverse surveys, both depending upon horizontal-angle measurements, are not suited for desert areas, because of the unfavorable influence of the prevailing weather conditions upon the measurements. Trilateration measurements are not restrained by these conditions and can be made under otherwise impossible circumstances. The Tellurometer party experienced the handicap of angular measurements quite vividly, when considerable time was wasted waiting for the atmosphere to clear in order to make the vertical-angle readings, necessary for the slope correction of the Tellurometer measurement.

Everybody who has had the frustrating experience of sighting on a damaged or destroyed triangulation signal, can readily appreciate the big advantage of trilateration measurements. The remote station is attended and can, therefore, not be destroyed by animals or local inhabitants while in operation.

It is our contention that the triangulation method would have been quite impossible for solving this survey problem, both from a technical and from an economical point of view.

Fig. 2 shows the magnitude of the probable point errors. In the adjustment, two stations and one azimuth were held fixed. The average probable error of all points is ± 0.33 metre. The average probable position error of the 4 beacons is ± 0.43 metre (± 1.3 feet) which shows clearly that the overwater distances are not of the same quality as the overland measurements. The trilateration network was planned to satisfy specifications which allowed a probable position error of ± 0.76 metre (± 2.5 feet) for the beacon sites. This requirement was fulfilled as shown by above error figures. Table I gives a summary of the probable position error of each station. Table II shows the comparison between measured and adjusted distances.

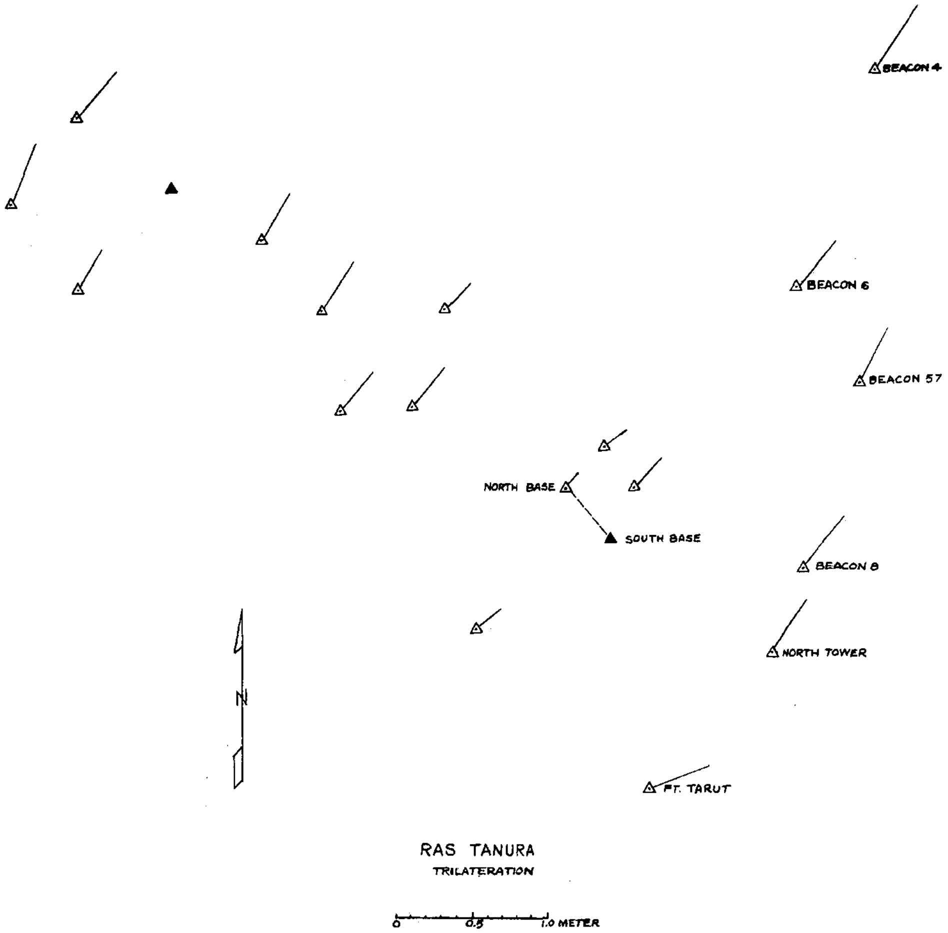

FIG. 2. — Probable point errors in meters

TABLE I

Probable error of a single observation is ± .207 metres
Probable error of each position

Station	Latitude (seconds)	Longitude (seconds)
222	± 0.013	± 0.006
013	0.005	0.007
023	0.009	0.009
024	0.009	0.006
028	0.009	0.006
029	0.008	0.008
030	0.008	0.008
031	0.010	0.008
033	0.006	0.007
035	0.004	0.006
037	0.006	0.006
N. Base	0.003	0.003
Ft. Tarut	0.005	0.015
N. Tower	0.010	0.008
Beacon 4	0.013	0.010
Beacon 6	0.010	0.009
Beacon 8	0.010	0.010
Beacon 57	0.012	0.007

TABLE II

From Station	To Station	Measured Tellurometer Distance (metres)	Final Adjusted Inverse Lengths (metres)	Difference (metres)
222	023	7 136.30	7 136.43	+ 0.13
	024	7 144.60	7 144.64	+ 0.04
	025	10 711.12	10 710.70	— 0.42
	028	16 962.68	16 962.94	+ 0.26
	Beacon 4	58 206.36	58 206.39	+ 0.03
013	029	16 942.73	16 942.85	+ 0.12
	030	15 354.18	15 354.09	— 0.09
	033	21 167.12	21 166.97	— 0.15
	035	14 774.79	14 775.08	+ 0.29
	N. Base	11 194.46	11 194.40	— 0.06
	S. Base	10 870.55	10 870.37	— 0.18
	Ft. Tarut	15 421.02	15 420.96	— 0.06
	N. Tower	19 812.63	19 812.66	+ 0.03
023	024	11 304.81	11 304.70	— 0.11
	025	7 841.52	7 840.90	— 0.62
	028	14 786.51	14 787.21	+ 0.70
024	025	9 154.45	9 154.58	+ 0.13
	028	12 866.89	12 866.82	— 0.07
	029	19 440.17	19 440.21	+ 0.04
	031	16 513.14	16 513.12	— 0.02
025	028	6 982.56	6 981.95	— 0.61

From Station	To Station	Measured Tellurometer Distance (metres)	Final Adjusted Inverse Lengths (metres)	Difference (metres)
028	029	12 260.59	12 260.76	+ 0.17
	031	6 008.65	6 008.50	— 0.15
	033	12 954.27	12 954.54	+ 0.27
	Beacon 4	42 343.39	42 343.38	— 0.01
029	030	4 849.27	4 849.29	+ 0.02
	031	6 693.56	6 693.44	— 0.12
	033	9 780.38	9 780.42	+ 0.04
030	033	6 700.26	6 700.18	— 0.08
	N. Base	11 716.80	11 716.88	+ 0.08
031	033	8 275.28	8 275.17	— 0.11
033	035	13 819.12	13 819.04	— 0.08
	N. Base	14 279.85	14 280.03	+ 0.18
	S. Base	18 710.05	18 710.19	+ 0.14
	Beacon 4	32 548.96	32 548.93	— 0.03
	Beacon 6	23 528.58	23 528.52	— 0.06
	Beacon 57	28 060.63	28 060.71	+ 0.08
035	037	3 498.15	3 498.09	— 0.06
	N. Base	3 602.48	3 602.28	— 0.20
	S. Base	6 123.86	6 123.77	— 0.09
	Beacon 8	30 385.66	30 385.68	+ 0.02
	Beacon 6	16 494.85	16 494.79	— 0.06
	Beacon 8	15 448.91	15 448.95	+ 0.04
	Beacon 57	17 508.05	17 508.05	0.00
037		4 531.80	4 531.76	— 0.04
		3 735.62	3 735.60	— 0.02
	Beacon 6	16 816.69	16 816.78	+ 0.09
	Beacon 8	12 219.78	12 219.75	— 0.03
	Beacon 57	16 275.30	16 275.13	— 0.17
N. Base	S. Base	4 449.52	4 449.49	— 0.03
	Ft. Tarut	20 457.77	20 458.02	+ 0.25
S. Base	Ft. Tarut	16 584.04	16 583.74	— 0.30
	N. Tower	12 949.53	12 949.48	— 0.05
Ft. Tarut	N. Tower	12 417.32	12 417.33	+ 0.01
N. Tower	Beacon 6	23 608.34	23 608.29	— 0.05
	Beacon 8	5 704.66	5 704.80	+ 0.14
	Beacon 57	18 177.06	18 177.01	— 0.05
Beacon 6	Beacon 57	7 526.31	7 526.28	— 0.03
Beacon 8	Beacon 57	12 473.70	12 473.80	+ 0.10

This survey proved the advantage of the trilateration method over triangulation in an area with poor visibility conditions. Using a master and two remote stations would have speeded the operation considerably, since most of the unproductive time was due to traveling from point to point.

Another Tellurometer trilateration is in progress at the present time in the same area. Its basic concept is a direct outgrowth of the experience gained during the Ras Tanura survey which showed that overwater distances can be measured without line-of-sight connection between the two Tellurometer stations.

Aero Service is engaged in a sea-boundary survey between Saudi Arabia and Bahrein. Fig. 3 shows the trilateration network and the boundary points. The boundary position will be determined by halving the distance between opposite shore markers. No tower construction is anticipated. This Tellurometer party is now equipped with one master and two remote stations in order to work more efficiently. As of today, half of the project is completed.

The difficulties encountered so far are not of a technical nature but are due to vehicle trouble. More and more we realize that adequate means of transportation and logistic support are the deciding factors on a Tellurometer trilateration project, since the time required to make a distance measurement is negligible compared with the time spent to get to the station site in difficult terrain.

It is well known that all Tellurometer measurements are subject to the ambient meteorological condition which changes the velocity of the emitted signals inversely to the index of refraction of the transversed medium. Pressure, dry-bulb and wet-bulb temperature measurements are usually made at both stations, and the average index of refraction is used to reduce the Tellurometer readings. This procedure gives satisfactory results. Discrete measurements along the signal path, however, would increase the distance-measuring accuracy since a change in the 6th decimal place of the index of refraction affects the centimetres, that is the inherent measuring accuracy of the instrument.

In order to facilitate the index of refraction computations, Aero Service uses the following method which was developed by Capt. Carl Aslakson.

The basic formula for the index of refraction is a function of pressure, dry- and wet-bulb temperature. It can be separated into 3 parts which are separately functions of wet- and dry-bulb temperatures and pressure. 2 nomograms were constructed which are entered with the 3 measured parameters, extracting 2 values N_1 and N_2 which are added to obtain the index of refraction.

Figure 4 shows the two nomograms used for this purpose.

In order to use these curves it is necessary to make the pressure measurements with an altimeter calibrated according to the NACA atmosphere. Instruments calibrated according to other standard atmospheres can only be used when the readings are converted to NACA values before entering the nomograms.

Overwater measurements without line of sight between the two stations are considered as geodetic lines, and no slope or sea-level corrections are applied. The least-squares adjustment of the Tellurometer trilateration network is done by the variation of coordinate methods similar to Shoran trilateration adjustments.

When planning trilateration networks, one has to keep in mind that more lines must be measured than in a triangulation scheme if the same strength of figure shall be maintained. Trilateration networks are, therefore, more elaborate and not as symmetrical as triangulation chains of quadrilaterals. In order to detect any possible constant measuring error K, it is advisable to insert " sliver triangles ", which are long thin triangles,

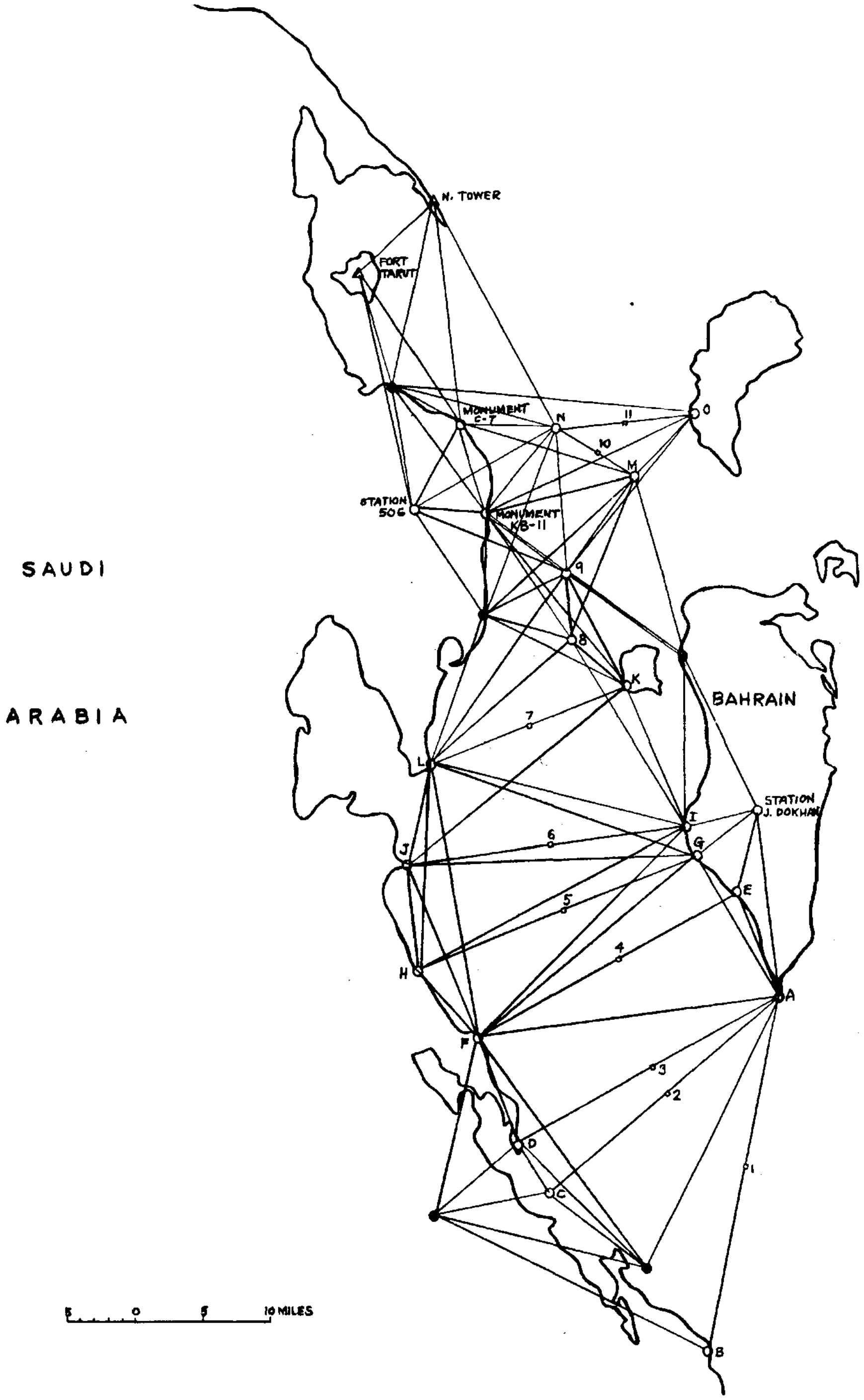

FIG. 3. — △ Triangulation Station
○ Marked and Observed Points
● Auxiliary Points
o Boundary Points

in the trilateration net. This configuration is foreign to triangulation networks. If there are a sufficient number of sliver triangles in a network and if a constant measuring error is actually present in all distance measurements, the value of K can be determined by inserting K as an unknown in the observation equations.

I hope that with this description of Aero's experience with Tellurometer trilateration in Arabia, I was able to demonstrate the practical significance of electronic trilateration surveys.