A NEW METHOD OF CONSTRUCTING HYPERBOLIC GRID SHEETS

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IIB Note. -- Andrew C. CAMPBELL was born December 28, 1923 in Suffield, Connecticut. He enlisted in the U. S. Navy in 1941 and served for five years. The last four years of his enlistment were spent in the Pacific Theater and at the conclusion of hostilities there he was honorably discharged. He attended the University of Connecticut from 1948 to 1952, receiving a B.A degree in mathematics. He started working in the field of mapping six years ago. The first two years were spent on topographic mapping in photogrammetry. The last four years he has been employed at the U. S. Navy Hydrographic Office where he has been engaged in geodetic field work and geodetic office computations.

The increasing dependence of the hydrographer on electronic positioning systems reflects the remarkable advantages of these systems in efficiency and convenience. However, there is one disadvantage to the hyperbolic systems that continues to plague the type of fast-breaking survey occasionally conducted by the U. S. Navy Hydrographic Office. When a field party must itself position the system transmitters and make prompt use of them, the customary delay for home-office computation and plotting of the hyperbolic lines of position becomes troublesome. This paper describes a field procedure for computing and plotting small-scale survey sheets to be used with continuous-wave phase-comparison radiolocation systems within a single day from the establishment of station positions.

The customary office method used by the Hydrographic Office to compute points on the hyperbolas is to solve for the points trigonometrically, expressing them first on a local system and then in terms of X and Y Universal Transverse Mercator grid coordinates. The solution of the X and Y coordinates is best accomplished with an electronic computer as the number of points needed would often require a prohibitively large number of man-hours by desk calculator.

X' and Y' are designated as coordinates on a local system, and they are computed from the following formulas :

$$X' = Ka \cosh B$$
$$Y' = Kb \sinh B$$

where K is one half the length of a hyperbolic baseline

where $a = 1 - L(\Delta a)$

where $b = \pm \sqrt{1 - a^2}$

where L is the lane value

where $\Delta a = \dfrac{2}{\text{number of lanes in baseline}}$

where cosh B is the hyperbolic cosine of sectorial angle B

where sinh B is the hyperbolic sine of sectorial angle B

The formulas for the UTM coordinates of a point are :

$$X = X' \cos \psi + Y' \sin \psi + h$$

$$Y = - X' \sin \psi + Y' \cos \psi + k$$

where X' and Y' are coordinates on the local system

    ψ is the angle of rotation of the baseline

    h is the UTM easting of midpoint of baseline

    k is the UTM northing of midpoint of baseline

    X, Y are UTM grid coordinates of the point

In these formulas the X' and Y' are computed first and are coordinates on a local system with respect to the system's baseline. The X and Y formulas rotate and translate the values of X' and Y' into UTM grid coordinates. The office method, as briefly described here, is too time consuming for field construction of hyperbolic grid sheets from coordinates computed by desk calculators normally available aboard ship.

The new method presented here will eliminate this inconvenient waiting period. The method is precise and it will enable a field crew to construct a hyperbolic grid sheet in one day. The sole limitation of this method is that the plotted positions of the three transmitting stations must fall within the limits of the plotting area used. This can always be accomplished by reducing the scale, of course, except where large-scale plotting is mandatory.

The hyperbola is usually defined by :

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1.$$

It may also be defined by :

$$D = R + c - L(2GLW)$$

where c is a baseline

where L is the lane value of the hyperbola passing through P

where R is the distance from centre station to P

where D is the distance from end station to P

where 2 GLW is 2 times the grid lane width. (See figure 1).

The latter definition is used exclusively in the method outlined here.

Method

    I. Convert the geodetic positions of the centre and two end stations to UTM grid coordinates.

   II. Plot the three stations on a rectangular grid of largest convenient size.

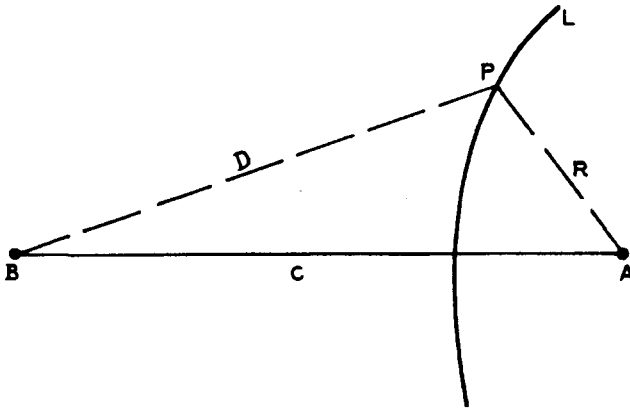 III. Compute the mean UTM grid scale factor (k) for each baseline.

Fig. 1. — A : Centre
B : End

IV. Determine the grid lane width (GLW) for each baseline with the following formula :

$$GLW = \frac{(\text{rate of propagation of radio waves) } k}{2 \text{ times mean or comparison frequency}}$$

where the rate of propagation of radio waves is 299 690 kilometres per second, unless otherwise stated, k is the scale factor, and the mean frequency or comparison frequency for each system is given.

V. Determine the number of lanes contained in each baseline with the following :

$$\text{number of lanes} = \frac{\text{grid length of base}}{GLW} = \frac{c}{GLW}$$

The grid length (c) of each base is the hypotenuse of a right triangle obtained by :

$$c = \sqrt{(\Delta N)^2 + (\Delta E)^2}$$

where $N\Delta$ and $E\Delta$ are the difference between northings and eastings of the end and centre stations.

CHENAUX LANES	10	20	30	40	R 50	60	70	80	90
20	D	D + 10	D + 20	D + 30	D + 40	D + 50	D + 60	D + 70	D + 80
40	D − 1[20(2GLW)]				D + 40 − 1[20(2GLW)]				
60					D + 40 − 2[20(2GLW)]				
80									
100			D + 20 − 4[20(GLW)]						
120									
140									
160					D + 40 − 7[20(GLW)]				

Fig. 2

VI. With the centre station as the centre of a system of circles, draft
pencil circles or radius (R) equal to 10 centimetres, 20 centimetres,
30 centimetres and so forth until the entire working area of the sheet
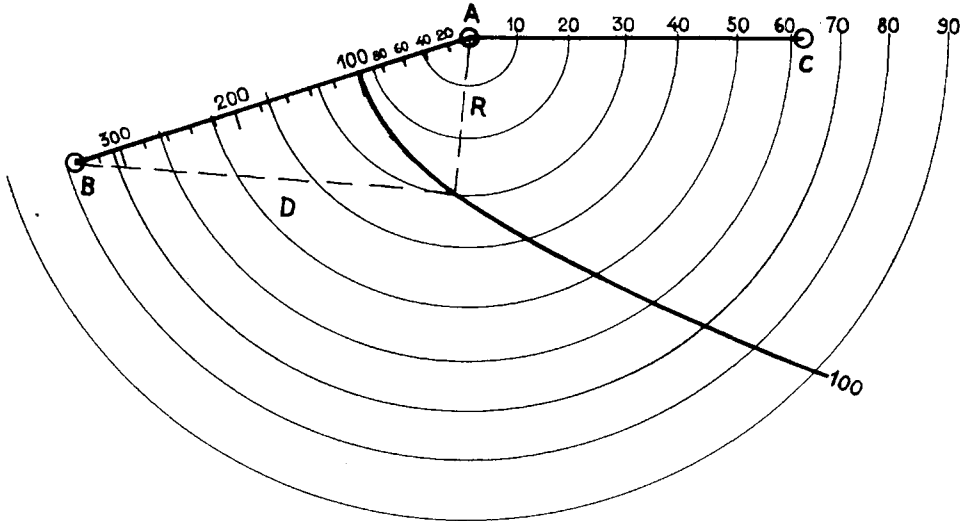is covered with arcs of circles. See figure 3.

FIG. 3. — A : Centre
B : Red End
C : Green End

VII. Select a hyperbolic lane interval that will be convenient to plot, useful
to the survey and which will be equal to between $1\frac{1}{2}$ to 2 centimetres
along the baseline. Plot the lane interval selected along the baseline
with zero at the centre station and maximum at the end station. See
centre to end station intervals on figure 3.

VIII. Selecting a sheet of data paper, list in the first column the lane
interval previously selected, and increment by that interval until a
maximum value for the baseline is reached. See figure 2, where a
lane interval of 20 is used as an example.

IX. Set a column aside for each value of R. See figure 2.

X. Reduce c and GLW from metres to centimetres at the scale selected.
Compute the value for D, when L equals 20 and R equals 10. The
expression of D here would be : $D = 10 + c - 20(2GLW)$.

XI. Values of D in any given row are increased by an increment of R.
For example : the difference between D in row lane 20, column R 10
and D in row lane 20, column R 20 is exactly 10 centimetres. There-
fore, only the first value of a row need be computed; the remaining
rows are merely incremented by 10. See figure 2.

XII. Values of D in any given column are decreased by a constant of
L(2GLW), where L is the lane interval selected. 20 was used as the
lane-interval value in figure 2. For example : the value of 20(2GLW)
is a constant. In order to obtain successive values in column R 10,
each value is decreased by 20(2GLW) which gives the value in the

next row beneath. See figure 2 where column R 10 and column R 50 are illustrated. It is apparent that only the first value of a column need be computed.

XIII. With the use of a metre bar and a beam compass with steel points, plot the intersections of each D with R. For example : in figure 3, plot D for column R 30 and lane 100. Column R 30 is a circle of 30 centimetres' radius from the centre station. The distance D is taken from figure 2 (column R 30, lane 100) and measured on the metre bar. With one point of the beam compass on the end station, the place where the other point intersects the radius R 30 is where lane 100 passes through.

XIV. With a spline and colored ink, draft in hyperbolas for one system.

XV. When Step XIV has been completed, go back to Step IV and repeat process for the other system of hyperbolas.

The method presented here has been carefully tested and proven. On a recent field trip, the author computed the intersections of D for both systems, and with the assistance of several others plotted the points and inked both systems of hyperbolas, and all in one day. During this time, two survey ships were standing by, awaiting the completion of the hyperbolic grid sheets. The rapid completion of the hyperbolic grid sheets represented a considerable saving in time to the ships, as this method is considerable faster than any other tried.