PRECISE TELLUROMETER TRAVERSING

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Introduction

In several countries, the tellurometer traverse has already become the standard method of providing geodetic control. In other countries, the tellurometer is still suspect. All agree that it usually gives results of high accuracy; but in certain circumstances, which are not always easy to detect, accuracy falls off. The problem, therefore, is to find a field routine which not only uses the tellurometer to its best advantage, but automatically reveals a weak measurement.

Ground swing

If the difference in length between the direct and reflected radio paths is small, as occurs when there is a graze in mid-ray, the ground swing is so small that it causes no inaccuracy. Close grazes are of course undesirable for the angle measurement, and they absorb half the power transmitted between the instruments, so that on long lines it becomes difficult to read the trace. But a satisfactory compromise is nearly always possible. One needs little field experience to be able to detect unsuitable lines at a glance, and on a new traverse, it should always be possible to select lines which prove to have an average swing of less than 3 mμs, and a maximum of less than 6 mμs. When working on towers, swings are usually very small.

When reinforcing an existing triangulation chain by running a traverse through it, most of the lines are likely to have large clearances in mid-ray, and be unsuitable for precise tellurometer measurement. The best route up the chain must therefore be selected with care, preferably by flying over and looking along the actual lines. If ground swings prove excessive, they can often be reduced to within acceptable limits by lowering or satelliting

(*) The first part of the article is reproduced; it is devoted to distance measuring. The other part, on azimuths and altitudes, was published in the *Empire Survey Review*, No. 118, Vol. XV, October 1960.

the instruments, as described by KELSEY (*); but the tellurometer is unlikely to develop its maxmum accuracy along lines which were originally selected for triangulation.

Lines should of course be measured twice. If the second measurement is made with the master instrument at the opposite end, the pattern of the ground swings is surprisingly different, and there is less likely to be a significant error in the mean of the selected transit times.

Selecting the correct transit time from the ground swing graph

If there is much ground swing, the correct transit time will always be in doubt, so it is better to keep the ground swing small by measuring suitable lines than to rely on rules for selecting the best value. But rules are necessary, and the results of an experiment carried out by the Ordnance Survey (**), in which all the lines of the Caithness base extension were measured with a tellurometer, suggest that the following may be adopted with some confidence :

a) Whenever possible, take thirty fine readings, evenly up or down the scale, say from 3 to 17 ½, by halves.

b) If the scatter of the readings appears random and no cyclic structure is apparent on the graph, accept the mean of all the readings.

c) If there is some cyclic structure, however irregular, and at least half a cycle seems to have been developed (that is to say, there is at least one clear maximum and one clear minimum), then discard readings from the ends of the graph until an exact number of half cycles remain, and accept the average of the rest. For the field check it is sufficient, and indeed better, to select the axis of the graph by eye. In the office, the average should be worked out arithmetically. The latter value should generally be slightly the better, but if the two values differ significantly, the graph should be inspected to see why. Because of meteorological uncertainties, it is seldom worth quibbling over the adopted value to less than 0.1 mμs, about half an inch.

d) If it appears that less than half a cycle has been developed, then it is impossible to select a precise transit time with any confidence. A sloping graph may be caused by a change in the meteorological conditions, but this will be obvious from the observations. The only remedy is to try raising or satelliting the instruments until at least half a cycle is developed, failing which the traverse must be realigned. On primary traverses, such lines seem fortunately to be rare, but it is advisable to make the forward tellurometer measurement as soon as one arrives at a station, before much other work has been done.

(*) *A method for reducing ground swing on tellurometer measurements*, by Major J. KELSEY, R. E., *Empire Survey Review* No. 112, vol. XV, April 1959.
This article is reproduced in the present Supplement to the *International Hydrographic Review*.

(**) *The use of the tellurometer by the Ordnance Survey in 1957 and 1958*, by Major J. KELSEY, R. E.

Meteorological measurements

The tellurometer does not measure distances; it measures transit times, which have to be converted into distance with the help of measurements made with thermometers and barometers. These measurements can only be made at the ends of each line, yet they are assumed to be representative of the whole ray. On precise work, lines on which such an assumption is almost certainly false should be very carefully avoided. On a coastal traverse, for example, lines should preferably run from hill top to hill top a few miles inland, not from hill top to sea shore alternately, and certainly not, on a calm day, from headland to headland over the sea.

The most critical measurement is the depression of the wet bulb thermometer, and it is essential to take out dew points at once, before continuing with fine readings. In countries like Australia, it is hard to believe that large masses of water vapour arrive or depart unheralded, and in practice, the indicated dew point usually remains constant within 2°F during the measurement of a line. If a range of 2° is exceeded, rereading the thermometers will usually show there has been a bad measurement. Again, if two terminals are at similar sites, their dew points should be similar, and if they are not, it is as well to check the thermometer readings. Sometimes the vapour pressure does change, as when a sea breeze springs up, or dew evaporates and dissipates into the upper air; but if the indicated dew points at a station range erratically over more than 2°F, the observer should regard it as a warning: either the thermometers are not being read properly, or the meteorological conditions are not ideal.

In a good breeze on a cloudy day, there is little doubt what temperatures to record. But in the tropics, there is often not a cloud in the sky, and in the absence of wind, the ground at each terminal heats the air, and the indicated temperatures are too high. Luckily, such conditions are unstable: every few minutes a convectional up-draught stirs the air and both wet and dry temperatures fall several degrees, the dry more than the wet, leaving the dew point unchanged. It seems reasonable that the lowest temperatures that can be recorded during a gust of wind are the ones most representative of the whole ray, and it is worth waiting for them and recording them.

Small aneroids are adequate for measuring the pressure, but they are notoriously unreliable, and it is well to record the mean of three, and to calibrate them against a mercury barometer at every opportunity. Because of curvature of the earth, the mean pressure at the terminals is always less than the mean pressure along the line, but the error only exceeds 1 part per million (p.p.m.) in a line longer than 30 miles [*]. The correction is insignificant on a single line, but it is systematic, and might be worth applying on a transcontinental traverse.

(*) If L is the length of a line, R the radius of the earth, and dP the fall in pressure with altitude, the error is :

$$\frac{2}{3} \cdot \left(\frac{L}{2}\right)^2 \frac{1}{2R} \cdot dP.$$

If R is 21×10^6 feet, dP is one inch of mercury per thousand feet, and 0.1 inch of mercury is equivalent to 1 p.p.m. in the refractive index, then the correction reaches 1 p.p.m. when L is 30 miles.

The best check on the meteorological measurements is to make the second measurement under as different conditions as possible. If the second measurement is always made on a different day, so that the vapour pressure has had a chance to change; and at a different time of day, so that temperatures differ by 10° or 20°F; then, if reduced distances consistently agree closely, there is likely to have been little wrong with the observations, or with the reduction formulae (see paragraph below on computations).

The zero error

The zero error of a tellurometer is the distance between the electrical centre of the instrument, from which transit times are measured, and the physical centre, from which the plumb bob hangs. So far as the author is aware, none of the instruments so far manufactured has been proved to have a zero error of significant size. But zero error causes a constant error in every line; there is no guarantee that it is negligibly small on every set; and it is necessary to prove the fact beyond doubt. Its determination is not easy. If the zero error is not to be lost among the meterological errors, it can only be determined by measuring short lines, whose lengths are known accurately from tape measurements. Short lines which are easy to tape are usually highly reflective, and yield most unsatisfactory ground swing graphs, often with less than half a cycle of swing, from which it is impossible to obtain a transit time to the precision required.

This problem has also been solved by the Ordnance Survey. To develop a full cycle of swing on a short line, the reflected path length must be increased relative to the direct path length, by increasing the clearance of the ray. The Ordnance Survey mounted their tellurometers on two 50 foot towers 260 metres apart on a disused railway embankment, which was ideal, except that something more permanent is required. It might be possible to find a natural site which was suitable without towers. The essentials are: that the base must be over 500 feet long and capable of being accurately taped; that the reflected path must be at least one metre, and preferably 2 metres, longer than the direct path; and that the ground in between these paths should be free from irregular reflecting surfaces, not only on line, but also to either side.

If the site is suitable, at least one symmetrical cycle of swing will be produced, with an amplitude of about 3 mμs, from which it is possible to derive a precise transit time, and hence a measure of the zero error. The error must be determined for every combination of master and remote instruments used in the field, and the determinations should be repeated between field seasons, so that their accuracy and permanency may be assessed.

Crystal calibration

This is generally a much smaller source of error, of the order of 1 or 2 p.p.m., but large errors have sometimes been caused by instabilities, not in the crystal, but in the associated circuitry. Fig. 1 illustrates two examples. On MA 154, the graph has moved sideways about 11°C, or 20°F, because

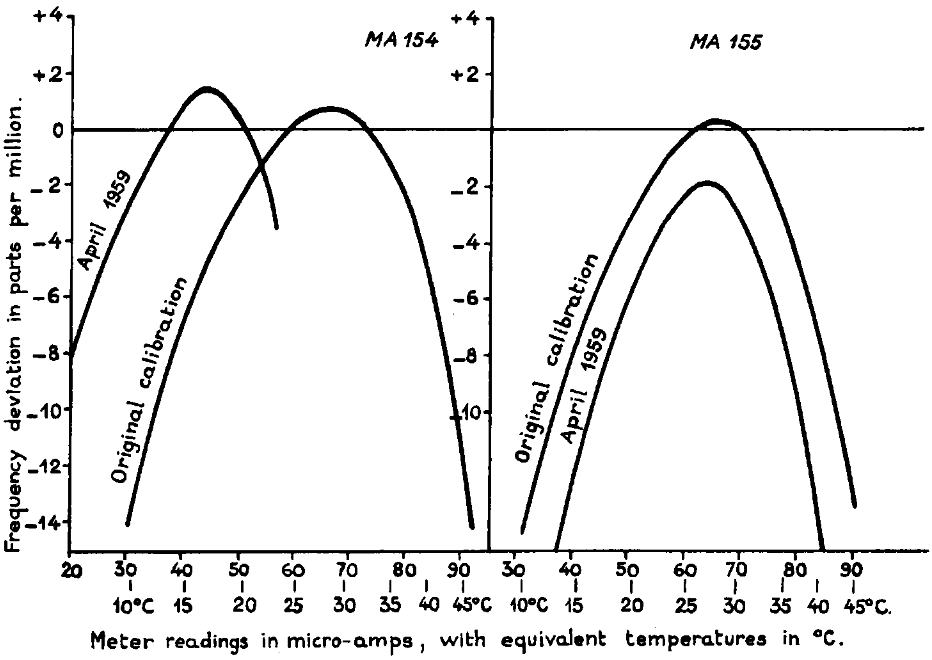

FIG. 1. — Changes in crystal calibration on two master tellurometers after a year's hard work.
(The changes are exceptionally large, and illustrate particular faults).

one of the resistances in the thermistor bridge was unstable, resulting in erroneous meter readings: a line measured on a hot day with this set might have been 50 p.p.m. in error. By contrast, on MA 155 the graph has moved downwards about 2 p.p.m., possibly because the trimmers had gradually drifted out of adjustment. A third set, MA 124, had calibration changes that could have accounted for errors of 10 p.p.m. under normal operating conditions, and although the changes on another four sets were trivial, it is clear that precautions must be taken.

The second measurement of each line should as a general rule be taken with a different master; on sets where crystal is not continuously heated, measurements should only be made when the crystal temperature is near the peak of the graph, so that the indicated correction is less than 2 p.p.m.; and crystals should be recalibrated immediately after every field season. If there has been a significant calibration change, a comparison of the measures made by the two masters may enable one to detect when it took place.

The frequency of the B, C and D crystals at normal operating temperatures should also be checked between field seasons, and if necessary adjusted, to ensure that there need be no difficulty in breaking out (*) the coarse readings during transit.

(*) The conversion of the coarse readings into the approximate transit time is so simple that it can be done at a glance when understood; but it is a novel arithmetical process, for which the term *breaking out* is now in general use.

Coarse readings

It is hard to believe that errors can be made in the coarse readings, but on at least four occasions, incorrect distances have been adopted from coarse readings which broke out ambiguously. Coarse readings must be broken out at once, before proceeding with fine readings. If they break out badly, the drill is:

a) Resynchronize the crystals. If the crystals cannot be properly synchronized, it is no use proceeding further with the measurement, and one set must be cooled (or heated) until proper synchronization is possible.

b) Recentre the trace. If one particular crystal breaks out badly, recentre the trace when switched to that crystal.

c) Ensure that the correct trace is being read, not the circular side band with the weak and deformed pulse.

On some lines, coarse readings do break out badly at first but they can always be made to break out perfectly (unless the calibration of one or more of the crystals has drifted badly out of adjustment, in which case the instrument is unserviceable until readjusted in a laboratory), and there need never be any doubt about the correct reading. Nevertheless, it is reassuring to have the second measurement made by a different observer with a different instrument, quite independently. It is also advisable to take one or two sets of coarse readings with the *Reverse* switch on the remote instrument depressed throughout. The figures are entirely different but they break out in the same way.

Routine at the master station

Thirty fine readings are taken, in three sets of ten, with wet and dry temperatures, pressure, crystal temperature and coarse readings before and after each set, four measures in all. Fine readings are best made at equal intervals straight up or down the scale. It might seem better to try to establish the top and bottom of a swing more accurately by taking a large number of fine readings close together at the peaks, with fewer in between; but in practice, when studying the graphs of a large number of lines, it is well to have them all over the same range, at the same interval, and at the same scale. Peculiarities then leap to the eye, and it is easier to be consistent in the selection of the final transit time.

Fine readings are recorded to 0.2 mμs. If the trace is bad, it may not be possible to read to this accuracy, but it often is, and it makes for easier reductions than reading quarters and halves.

On a long or difficult line, it is sometimes impossible to read the trace throughout the frequency range, in which case one of the sets of ten may be repeated once or twice over the same frequencies. But it is desirable to read over as great a range of frequency as possible, and always to take at least four sets of meteorological readings. On some sets it is not possible to peak the crystal current at the ends of the frequency range. On a single reading the resulting error may not be apparent, but the study of a large number of graphs suggests that when the crystal current cannot be peaked, fine readings are best not taken, or discarded.

A complete measurement on these lines can be made comfortably within an hour.

Organization in the field

On a traverse, the various conditions suggested above can be satisfield, without wasting time, by giving the angle party a remote instrument, and both the forward and rear light parties a master. If the parties are moving forward by helicopter every morning, the rear measurement should be made in the cool period before departure, and the forward measurement in the heat of the afternoon just after the vertical angles. When the traverse is progressing smoothly at the rate of a station a day, forward and back measurements will be separated in time by over forty hours, and the refractive index may differ by 20 p.p.m. or so. The measurements are automatically made in opposite directions, by different observers with different masters, and are about as independent as it is possible to make them.

Field checks

The second measurement should be checked against the first, at once, on the hill, before the instruments are taken down. A slide rule is recommended: it cannot convert millimicroseconds precisely into feet, but it can reduce transit times by the meteorological and crystal factors well within 1 p.p.m., and it can convert short satellite distances into millimicroseconds when necessary. If the booker has reduced the fine readings during the measurement, the rest of the check can be done in twenty minutes.

If the two reduced transit times fail to agree within 10 p.p.m. and no mistake can be found in the working of the checks, the line must be remeasured. Whenever possible, there should be a delay, to allow the meteorological conditions to change yet again. But it may not be considered worth holding up the traverse for a complete day, and the third measurement may have to follow within an hour of the second. Then conditions being so similar, second and third measurements will usually agree to 2 or 3 p.p.m., and it will be necessary to examine the graphs and meteorological readings of all three measurements before deciding what figure to adopt.

Results

Fig. 2 shows the lengths of thirty consecutive lines measured to these specifications in 1959 in NW Australia. Fig. 3 shows the misclosures between first and second measures in feet and in parts per million. Third measurements are not shown. Only the two lines which misclosed by 0.58 and 0.67 feet in 6.3 and 12.3 miles respectively are unacceptably bad. Both these lines ran from the same station, which was on the end of a peninsula and only 28 feet above sea level. Both rays crossed the coastline at least four times; the ground was hot, the sea was cool, there was little wind, and the errors were obviously due to unrepresentative meteorological readings. Such lines must be avoided.

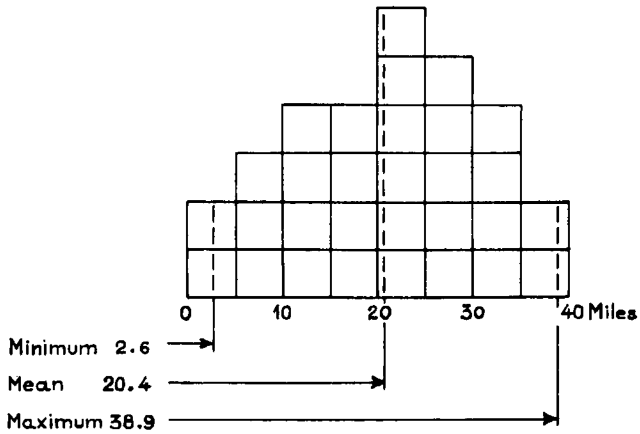

FIG. 2. — Lengths in miles of 30 consecutive tellurometer traverse lines measured in NW Australia in 1959.

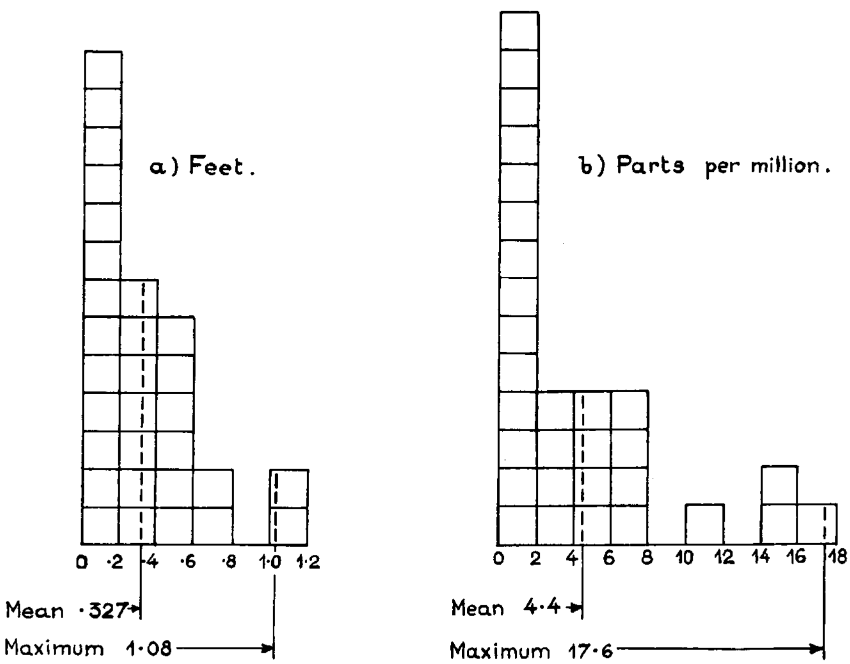

FIG. 3. — Misclosures between first and second measurements of the 30 lines shown in fig. 2.
(Third measurements are not shown).

Details of the worst misclosures are :

a)

Feet	Miles	ppm
1.08	26.4	7.7
1.00	38.9	4.9

b)

ppm	Miles	Feet
17.6	6.3	0.58
14.8	4.4	0.35
14.1	2.6	0.20
10.3	12.3	0.67

Lines varied from 2.6 to 39 miles in length. Twenty miles is usually considered ideal, but in the stable conditions common in Australia, longer lines over 30 miles are usually satisfactory. Short lines waste time, and each contributes its basic 2 inches of error, but there is no objection to an occasional short line which provides the simplest route through difficult country. Short lines are traditionally unacceptable to the angle party, but this difficulty can be overcome by observing stellar azimuths, as their errors are not cumulative.

Computations

The refractive index has been calculated by the usual formulae due to ESSEN and FROME given in the Tellurometer Hand-book. Although lines were measured under the most varied conditions obtainable, half of them closed within 2 p.p.m., so there seems to be little wrong with the formulae. For the field check, readings at both ends were all meaned together, but for the office computation it is more enlightening to compute the refractive index separately at each end. Close agreement, or otherwise, proves nothing, as conditions at the terminals may be identical, yet quite unrepresentative of the ray, or vice versa. But the surveyor gains valuable insight into the change of refractive index from place to place, a great asset on future reconnaissance.

Slant distances were reduced to the spheroid by the three well-known approximations:

Slope correction: $\dfrac{d\,H^2}{2\,L} - \dfrac{d\,H^4}{24\,L^3}$

Sea level correction : $\dfrac{L.H_m}{R + H_m}$

Chord to arc: $\dfrac{L^3}{24\,R^2}$

where dH is the difference in height, Hm the mean height of the two stations, L the length of the line, and R the radius of the earth. On a 20 mile line, the second term of the slope correction amounts to 0.1 foot for every 4° of slope; and omitting Hm from the denominator of the sea level correction amounts to 0.1 foot when Hm is 20 000 feet. Although the error is systematic, both terms can usually be omitted. The value of R in the sea level correction is more critical. The rigorous formula is:

$$\frac{1}{R} = \frac{\sin^2 A}{\nu} + \frac{\cos^2 A}{\rho}$$

but accurate values of R have long been required for trig heights and numerous tables of R against latitude and azimuth exist. For the office computation it is advisable to use them. Since R varies almost as much with azimuth as with latitude, using tables of $R = \sqrt{\rho\nu}$ is not advised. For the field computation, taking R equal to 20.9×10^6 feet cannot give an error on a 20 mile line greater than 0.03 foot for every 1 000 feet above sea level. It is also convenient to have the chord to arc correction tabulated, critical tables being convenient when L is less than 1.5×10^5 feet. These formulae are less elegant than those given by BRAZIER (*Empire Survey Review*, No. 109, Vol. XIV, July 1958, page 295), but they have two advantages : they can be computed at once by slide rule in the field if required; and it is easier to see the effect of a given error in height.

Reconnaissance

With the routine described above, it unlikely that a bad measurement will go undetected, but no routine can make a precise measurement of an unsuitable line. The art of precise distance measurement with the tellurometer lies largely in the reconnaissance, in the selection of suitable lines.

Reconnaissance for extensive first order surveys is always hard and sometimes worrying work. In remote and uninhabited areas, much the greater part of the surveyor's time and energy is expended merely in travelling through the country. If reconnaissance is done from the air, there is too little time to record, to memorize, reflect and think. The tellurometer will *measure* almost any line, and in these circumstances, it is tempting to accept the first reasonable route that presents itself. But for precise work, the surveyor must ask himself on every line:

a) Is there unlikely to be much ground swing ?

b) Are meteorological measurements made at the terminals likely to be reasonably representative of the whole ray ?

Near the sea, it may be impossible to give affirmative answers in every case, but the questions must still be asked, and unavoidable errors accepted deliberately, not blindly.

Reconnaissance for a tellurometer traverse is quite different from reconnaissance for a triangulation chain. In order to see 20 or 30 miles, and to enable future work to be joined on easily, stations still have to be on hill tops, but they need no longer be on the highest and least accessible hill for many miles around. Suitable lines can often be found running between low hills close to lines of communication, and stations can often be sited close to where future surveyors are going to need them. A tellurometer traverse can usually be run at reasonable cost through country where triangulation would be physically or economically impossible. Towers are of course necessary on a flat plain, but if azimuths are provided by astronomical observations, control of first-class accuracy can be carried without towers into any country where it is possible to find a single series of lines of sight, averaging ten miles or more in length.