GEODETIC POSITIONING OF DISTANT NON-INTERVISIBLE ISLANDS

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INTRODUCTION

The determination of the position of islands has always been of concern in the development of hydrographic charts. Islands have in the past been positioned astronomically when the distances involved have been such as to prohibit the use of conventional triangulation technique. The position of islands so determined would perhaps be correct to within one-half mile; island position appearing on nautical charts served the mariner quite well. There was no need for greater accuracy.

Since the end of World War II to the present, increasingly more use has been made of electronic navigational and surveying systems. In order to obtain optimum benefit from the electronic systems, the transmitting stations must be located so as to produce geometrically favorable lines of intersection for position determination. Frequently an island occupies an ideal, though not accurately determined, location for the establishment of an electronic station — first for geodetic and hydrographic surveying and later for electronic navigation. Before an electronic positioning network, which includes such an island as location for one of its transmitters, can be of practical use, the position of the island must be accurately determined and related to a common geodetic datum.

DEVELOPMENT OF TECHNIQUE FOR ISLAND POSITIONING

(See figure 1)

Definition of a hyperbola

A hyperbola is the locus of a point which moves so that the difference of its distances from two fixed points (foci) is a constant.

In a hyperbolic system the transmitting stations M (Master) and S (Slave) are foci of a family of hyperbolae, where eccentricity ranges from one, at the baseline extension, to infinity, at the midpoint of a baseline $\overline{MS}$.

$$1 \leqq e \leqq \infty$$

The range of eccentricity seems to violate the requirements for hyperbolae, whereby the eccentricity shall be greater than unity. Actually the requirements remain inviolate, since at each extreme the hyperbolae degenerate into straight lines. The degenerate hyperbolae in hyperbolic networks are of great interest, as will be seen later.

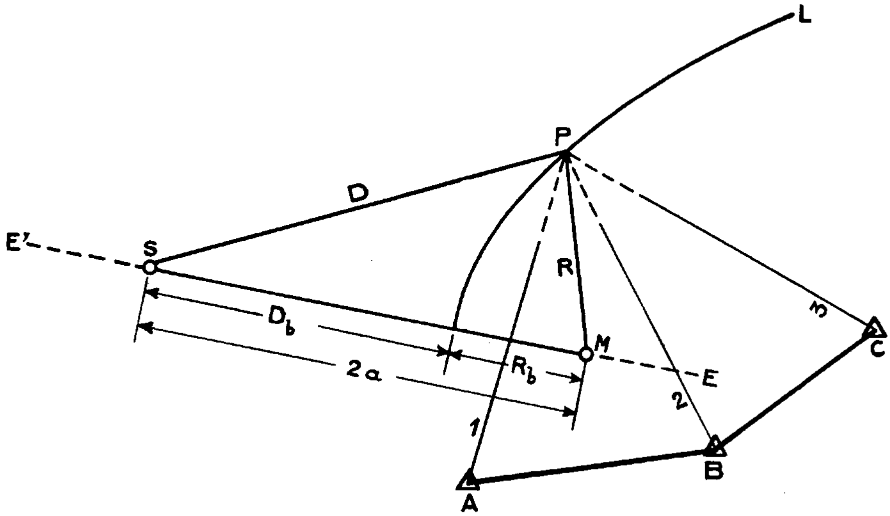

FIGURE 1

Baseline length (2a) and lane width (LW) determination

The baseline is the distance between foci, the foci being the location of the transmitting stations in a hyperbolic network. Lane width is a unit of distance along baseline from the intersection of one integral hyperbola to the next. The length of a baseline is, of course, a constant in any given system. Lane width varies as a function of both speed of propagation and transmitting frequency. Transmitting frequency can be very well controlled within certain allowable limits; for practical purposes lane width will be considered to vary as a function of the speed of propagation.

a) *Time difference system*

In a time difference system an arbitrary unit of time, such as a microsecond, is used to divide the baseline into hyperbolic intersections. The unit of time taken as a distance is then equal to the lane width (LW). The baseline distance ($2a$) can be measured by occupying a position E on the baseline extension. A signal is transmitted simultaneously from S and M with the resulting measurement :

$$\overline{SE} - \overline{ME} = \overline{SM} = 2a \ (*)$$

(*) In actual measurement of baselines and lane values allowance is made for a correction resulting from the nature of the terrain for the individual transmission paths. (See Bibliography 1 and 2).

As the intersection of the baseline extension is approached the difference of distance from S to the point minus the distance from M to the point approaches the maximum $\overline{SM}$. This is the case of the degenerate hyperbola, where eccentricity equals one.

b) *Phase comparison system*

In a phase comparison system one-half the wave length of the mean transmitting frequency determines the lane width, and it may be computed by :

$$LW = \frac{\text{Speed of propagation}}{2 \times \text{mean frequency}}$$

To determine the baseline length a mobile receiver crosses the baseline extension at E where a minimum value of lanes is recorded. The receiver is then transported to E′ where a maximum value is recorded. The baseline distance is determined by :

$$2a = (LW)\ (\text{max. lane value} - \text{min. lane value})\ (*).$$

With the mobile receiver acting as a lane counter, a phase comparison system must be in continuous operation in order to measure the number of lanes in the baseline.

Coordinates of P and distance R

Theodolite stations A, B, C and the master transmitting station M are triangulation stations on a common geodetic datum. The three theodolite observers at stations A, B and C make direction pointings 1, 2 and 3 upon the receiving antenna of the survey ship at point P. By the process of adjustment for an intersected station, the most probable coordinates for P are computed.

The residual for the condition equation is obtained from a length equation comparing the two fixed lengths through the directions 1, 2 and 3. The condition equation resulting is :

$$0 = \text{residual} + av_1 + bv_2 + cv_3$$

where the residual is in units of six places of logarithms. a, b and c are the values for the difference between the log sine of the angle for the direction angles 1, 2 and 3 and the log sine of one second of angle.

The function u to be rendered a minimum is :

$$u = v_1^2 + v_2^2 + v_3^2 - 2C\ (\text{residual} + av_1 + bv_2 + cv_3)$$

differentiating with respect to the v's in succession and equating to zero, there results :

$$v_1 = aC$$
$$v_2 = bC$$
$$v_3 = cC$$

Substituting into the condition equation results in the determination

(*) Same as preceding footnote.

of C, which then makes it possible to compute v_1, v_2, v_3 which are the corrections to the observed angles.

The two triangles are solved and the position of P is computed. The distance and azimuth of R are then computed by inverse, where $\overline{PM} = R$.

Determination of D

For any point P on a given hyperbola L

$$D - R = k \tag{1}$$

Any value L intersects the baseline, where

$$R_b = L(LW) \tag{2}$$

Also

$$D_b = 2a - R_b \tag{3}$$

Substituting

$$D_b - R_b = k = 2a - 2L(LW) = D - R$$

Solving for D

$$D = 2a + R - 2L(LW) \tag{4}$$

which is the parametric equation for the determination of electronic distance measurement D.

Determination of S

Given three sides of a triangle $2a$, D, and R, the three angles are computed. The side R is fixed in azimuth and the geodetic position of S is computed.

APPLICATION OF TECHNIQUE IN MIDWAY ISLANDS AREA

In the Spring of 1959 the USNHO conducted a geodetic and hydrographic project in the vicinity of the Midway Islands that was probably the first of its kind. The project evolved as a result of need for a hydrographic survey in the area. The geographic location of islands in the area is such that a hyperbolic system involving three stations, which would create two families of intersecting hyperbolae, could be used ideally. The problem was that the positions of the three islands were unknown with respect to each other. The islands involved are at the western end of the Hawaiian chain, and they are approximately in a straight line. The island farthest west is Kure, about 55 miles from Midway; Pearl and Hermes Reef at the other end is about 90 miles from Midway. The locations of the islands make it obvious that neither triangulation nor trilateration could be used to determine the positions of the three islands with respect to a common geodetic datum. The solution involved simultaneous measurements for trilateration and triangulation to accomplish what neither electronic distance measurements nor optical instruments could perform alone.

The solution (see figures 2 and 3)

A phase comparison system of Lorac manufacture was used, and the hyperbolic network was put into operation. The green slave station was erected on Kure Island, the centre or master on Midway, and the red slave

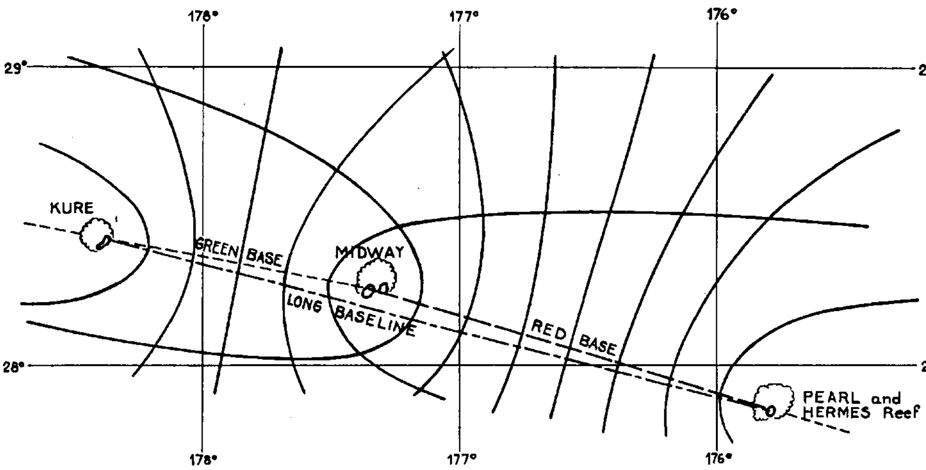

FIGURE 2

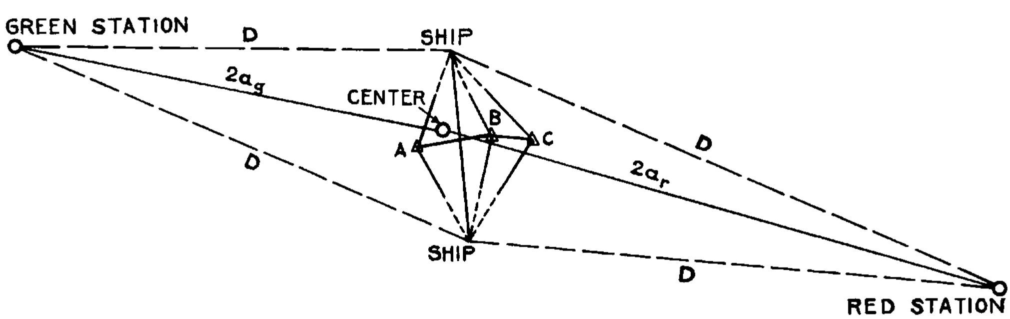

FIGURE 3

station on Pearl and Hermes. The position of the master station was referred to an astronomically determined position which established the Midway datum, and it was the only position of the three which was known.

Measurement of baselines

Two survey ships, the USS *Rehoboth* and the C&GS Ship *Pioneer,* served as mobile receivers to measure the minimum and maximum lane values of both the red and green systems.

$$2a = (\text{LW})\ (\text{max.} - \text{min. lane value})\ ^{(*)}$$

Since any change in the speed of propagation or in the mean frequency affects the LW, these properties required consideration. The speed of

(*) Same as preceding footnote.

propagation is dependent upon weather conditions, and may be computed from the following formula :

$$\mu = \frac{77.6}{T}\left[p + 4\,810\ \left(\frac{\omega}{T}\right)\right]$$

where μ is the index of refraction, T is temperature in degrees absolute, p is total air pressure in millibars, ω is partial water vapor pressure in millibars.

The speed of propagation is determined by :

$$\text{Speed of propagation} = \frac{299\,793\ \text{km/sec}}{1 + \text{N}\ 10^{-6}}.$$

Both of the survey ships and the weather station at Midway acted in obtaining the necessary weather information, so that the speed of propagation could be computed for the individual lane measurements. The speed of propagation for the given data varied from 299 681 to 299 693 km per second.

The crystals of each station control the frequencies and prevent them from varying beyond ± (.002 Kc). For greater accuracy each station frequency should be monitored in order to determine the exact frequency at the time of observation; however, the frequencies were not monitored. The maximum error that would result from this source would be ± 0.2 metres.

The independent determination of the lengths of the red and green baselines by the two ships compared favorably. The differences were of the order of less than one metre, and henceforth the baselines were considered fixed.

Coordinates of P and distance R

Three theodolite triangulation stations A, B and C were set up on the Midway Islands, these on Midway datum. These stations were chosen to obtain the maximum possible distance apart on the two small islands that make up the Midway group. On two successive days the USS *Rehoboth* occupied several positions to the north of Midway and several positions to the south. While the *Rehoboth* occupied these positions, the hyperbolic net was in operation and phasemeter readings were observed aboard ship at the times of simultaneous theodolite pointings on the ship's mast, coordinating observations by radio. The ship's mast was selected as a target for the theodolites rather than the Lorac antenna which was not easily visible. The antenna was treated as an eccentric target of one metre distance from the mast, and the true bearing of the ship for each observation was used to accomplish the reduction.

The determination of a single position for P was the result of 3 direct and 3 reverse angles at one minute intervals. One minute was considered ample time to plunge and reverse the theodolites between the direct and reverse readings, and the one minute interval between observations made it convenient to coordinate the readings at the four stations. Each position determined was the result of an adjustable quadrilateral; if the adjustment corrections were excessive the position was rejected. Of thirty-four such positions eighteen remained after rejections. Following is a list of the v corrections in seconds of arc for the 18 acceptable positions :

Positions N of Midway			*Positions S of Midway*		
v_1	v_2	v_3	v_1	v_2	v_3
0.1	—0.4	0.3	—0.8	1.5	—0.7
—0.4	1.1	—0.8	1.0	—1.9	0.9
1.6	—4.7	3.8	0.9	—1.7	0.7
1.5	—4.4	3.5	—0.1	0.2	—0.1
1.7	—4.1	2.9	1.0	—2.0	0.9
—0.6	1.5	—1.1	—2.4	4.9	—2.3
—1.0	2.4	—1.7	—1.8	3.7	—1.7
0.5	—1.3	0.9	—0.3	0.7	—0.3
			2.3	—4.8	2.2
			—0.3	0.6	—0.3

Determination of D

Employing equation (4)

$$D = 2a + R - 2L(LW)$$

At the instant of the theodolite observations the red and green phasemeter readings were recorded. The six readings for both red and green, which comprise a set, were merely meaned. A determination for D was then computed for both the red and green systems which would correspond to the positions determined by theodolite.

Each value of L was corrected prior to being used in (4) (*).

Determination of geodetic positions for end stations

In order to obtain stronger triangles for the determination of position for the end stations, it was decided not to use the sides R as the side for which the azimuth had been fixed. Instead inverses were computed from the 8 stations of the north to the 10 stations of the south. It was a technique similar to expanding a baseline; it was for this reason that positions both to the north and south were obtained. Eighty unique baselines resulted from the procedure, which produced 80 triangle computations for both the red and green position determinations.

Each value of D was considered to be potentially biased, and the criterion for determining bias was to use each value of D to the north with every combination of D to the south in the determination of position for each end station. If bias existed in a certain value of D then every time this value was used to determine a position, the resulting position would be shifted in a characteristic manner from the main group. In examining values of D for bias in the determination of the green station, three values

(*) Same as preceding footnote.

of D to north were rejected as was one value of D to the south. This left a combination of forty-five acceptable triangles for the determination of the green position. In the red system, two values of D to the north were rejected and five values of D to the south, which left thirty acceptable triangles for the determination of the red position.

The probable error of the green station was ± 1.7 metres.

The probable error of the red station was ± 4.7 metres.

The computed position of the green station was shifted 0.6 metres in both northing and easting to agree with the previously fixed green baseline. The shift to northing and easting of the red station was 3.5 metres. This shift is probably justified since the baseline can be measured with greater care than any random lane observation. In examining figure 3, it becomes obvious why the probable error of the green station is lower than that of the red station. The strength of figure of the red triangles is considerably weaker than that for the green.

LONG BASELINE MEASUREMENT

In the measurement of the long baseline, the red station was used temporarily as a master station and the green station as its slave. The *Pioneer* measured the distance electronically, in terms of hyperbolic lanes. The measured value of this long baseline was 252 194.5 metres. An inverse between the newly computed and adjusted red and green stations produced a computed distance of 252 193.5 metres. The value of the long baseline was not used in the computations determining the positions, but was reserved as a check. The difference of only one metre between the two values was highly encouraging, but should be considered as a check for consistency rather than an evaluation of accuracy.

CONCLUSION

The positions of the three islands were related to a common geodetic datum, which enabled the construction of hyperbolic lattice charts which then permitted the hydrographic operation to get under way. From the Hydrographic Office viewpoint, the island positioning technique was economical. The requirements in excess of a normal hydrographic operation were slight. Three theodolite observers, in radio communication with each other and with the ship, provided the necessary observations from the shore. One desk calculator and four days were required to compute the positions of the end stations and to construct the hyperbolic lattice charts. The technique presented here is reasonably straightforward, and anyone acquainted with electronic surveying equipment and the problems of hydrographic control should find this a practical solution to the problem of island positioning.

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