USE OF LORAC IN GEODETIC SURVEYING

by B. W. Koeppel
Seiscor Manufacturing Company, Tulsa, Oklahoma

Report presented at the Symposium on Distance-Measuring Equipment,
Washington, May 1959

Purpose of project

The experimental results reported in this paper were obtained between February and May 1956 within the framework of a project drawn up by the U.S. Army Engineer Research and Development Laboratories.

The objective of the project was to investigate the possible application of a phase comparison radio system to the accurate measurement of distance. A preliminary investigation, which indicated relatively good prospects of success in an actual field test, was completed in December 1955. The project field test was, as a result, scheduled for early 1956. The Louisiana Gulf Coast was selected as a test area.

The project plan of tests specified the types of lines to be measured as follows : three lines were to extend entirely over water between existing offshore oil well platforms. One of the offshore lines was to be not less than eight miles from land at any point and at least one hundred statute miles in length. Three lines were to be entirely over land of various terrain characteristics to determine these effects on the accuracy of measurement. A variety of lines was to be measured over various combinations of land and water to provide information relative to these effects. A number of lines of varying length was to be measured extending to such distances that the practical range could be determined relative to the specified measuring radio frequency.

The programme of lines thus proposed by the plan of tests is shown in figure 1.

Fundamental method of distance measurement

In a phase comparison position determining system the geographic positions of the system transmitters are determined from existing precise survey information to a high degree of accuracy. From this information the earth surface baseline *orientation* and *length* between any pair of transmitters in the positioning system may be computed. The baseline

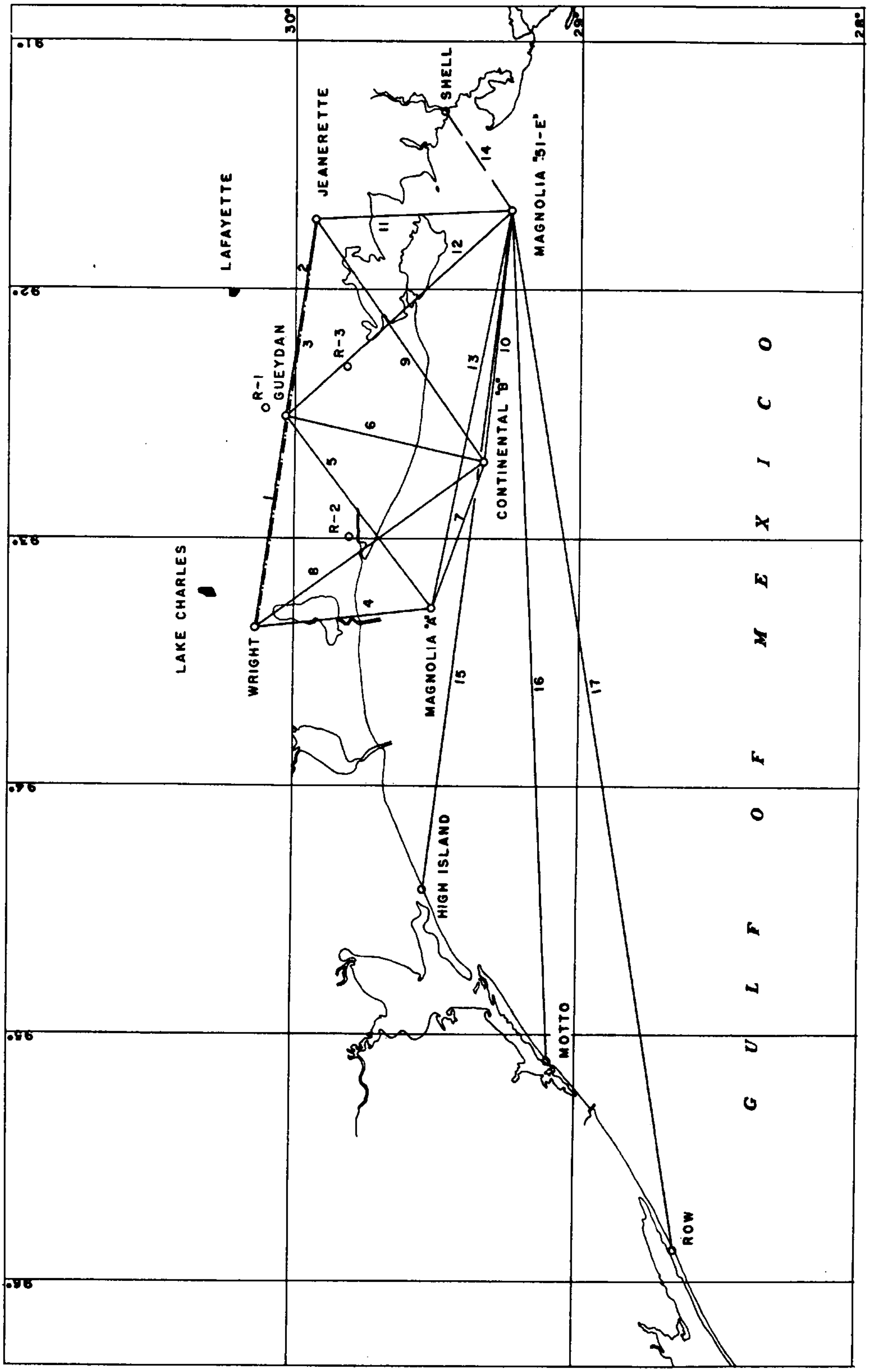
30°
29°
28°
91°
92°
93°
94°
95°
96°
LAFAYETTE
JEANERETTE
SHELL
MAGNOLIA "51-E"
GUEYDAN
R-1
R-2
R-3
LAKE CHARLES
WRIGHT
MAGNOLIA "A"
CONTINENTAL "B"
HIGH ISLAND
MOTTO
ROW
GULF OF MEXICO
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

FIG. 1

length is then divided into N intervals each equal to one half wavelength of the positioning system's basic radio frequency. The half-wavelength unit is called a *lane* and is one unit or revolution of the phase comparison measuring instrument. In this way the *number of lanes* N in a baseline becomes known.

In the distance measuring problem, the geographical positions of the transmitters on the ends of the baseline are known only approximately from astronomic observations for position or from the scale of existing maps. The width of a *lane* can be known to an accuracy which is dependent upon the accuracy to which the basic radio frequency of the baseline transmitters and the velocity of propagation of electromagnetic radiation in the earth's atmosphere are known. The number of lanes in the unknown baseline is determined by the ratio of the true (but unknown) baseline length to the known lane width. The baseline length may then be determined if the number of lanes in the baseline can be counted.

The lane counting process may be accomplished by circulating a phase comparison lane counting receiver from one *baseline extension* to the opposite baseline extension. The actual path traced by the receiver in passing from one baseline extension to the opposite extension is entirely arbitrary as long as the receiver stays in range of the baseline transmitters.

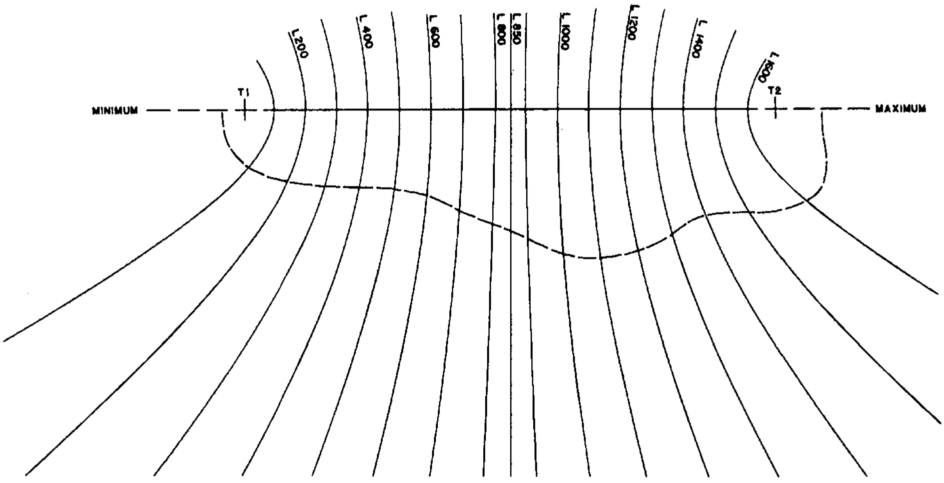

Fig. 2

The word picture drawn above is shown graphically in figure 2. In this figure the baseline transmitters are labeled T_1 and T_2. The line L between these transmitters is the baseline of unknown length. The curved lines which converge to cross the baseline are *hyperbolic lanes* of constant phase value or lane count. The dotted line from the baseline extension extending to the left of station T_1 to the baseline extension extending to the right of station T_2 is seen to cross each lane regardless of the route selected to pass from the one point to the other. A lane counting receiver following the path shown would provide an integrated lane count plus a fraction for the baseline length L.

For a particular selection of radio frequencies, the phase comparison receiver will indicate a minimum reading on the baseline extension to left of T_1 and a maximum reading on the extension to the right of T_2. The difference in lanes between the minimum and maximum phase comparison readings is just equal to the number of lanes in the unknown baseline. The fact that phase comparison observations are actually made at only two points in the system has led to the development of methods of line measuring where it is not necessary actually to trace out the long path from one extension to the other. This method employs multiple measuring frequencies and will be described in greater detail later.

The unknown line length may be expressed in terms of the variables upon which it depends in simplified form as follows :

$$L = \frac{2\,\mu_a\,f\,(\psi_{max.} - \psi_{min.})}{c} \qquad (1)$$

In this formula $\psi_{max.}$ and $\psi_{min.}$ are the phase comparison receiver readings on the respective baseline extensions. The refractive index of the earth's atmosphere at the surface of the earth is denoted by μ_a. The measuring radio frequency is f and the velocity of propagation of electromagnetic radiation in free space is represented by the factor c.

As one might suspect, a number of natural factors is actually operating at all times which tends to disturb the simplicity of the above equation by the introduction of more complex variables. This is particularly true when precise results are expected for L. The factors operating to distort the results of eq. 1 will be divided into three groups as follows :

1) Instrumental
2) Propagational
3) Geometrical

These groups of factors are the subject of the next section.

Accuracy of phase comparison measurements

In considering the various groups of factors which affect the accuracy of a line length measurement, it will be necessary to return from time to time to the basic operation of a phase comparison system itself. To facilitate these investigations a block diagram of a phase comparison line measuring system is shown in figure 3.

The operation of the phase comparison system in figure 3 may be briefly described as follows : Radio-frequency carriers f_1 and f_2, which are different by only a very small frequency, are radiated from transmitters T_1 and T_2. Since these radiations are nearly equal in frequency they are readily received by receiver R_1 in the phase comparison measuring unit. As a result of a process known as heterodyning or beating the radio-frequency carriers within the receiver, the signal which emerges from R_1 is actually the difference frequency $(f_1\text{-}f_2)$. This signal is usually more conveniently represented by the letter n_p.

The frequency n_p is arbitrarily chosen, usually in the range from 100 to 600 cycles. The phase of the difference frequency signal n_p is dependent directly upon the difference in phases of the two radiated carriers f_1 and

f_2. The phases of these two carriers are, to a high order of approximation, linearly related to the transmission distances r_1 and r_2 in figure 3. In this way it can be seen that the phase of the signal n_p is directly proportional to the difference in distance (r_2-r_1). Before the phase of the difference signal n_p can be measured, it is necessary to provide a *reference signal* of the same frequency as n_p and constant in phase. With the reference signal at hand it is possible to *compare* the position sensitive signal n_p with the reference signal and display the result of this comparison.

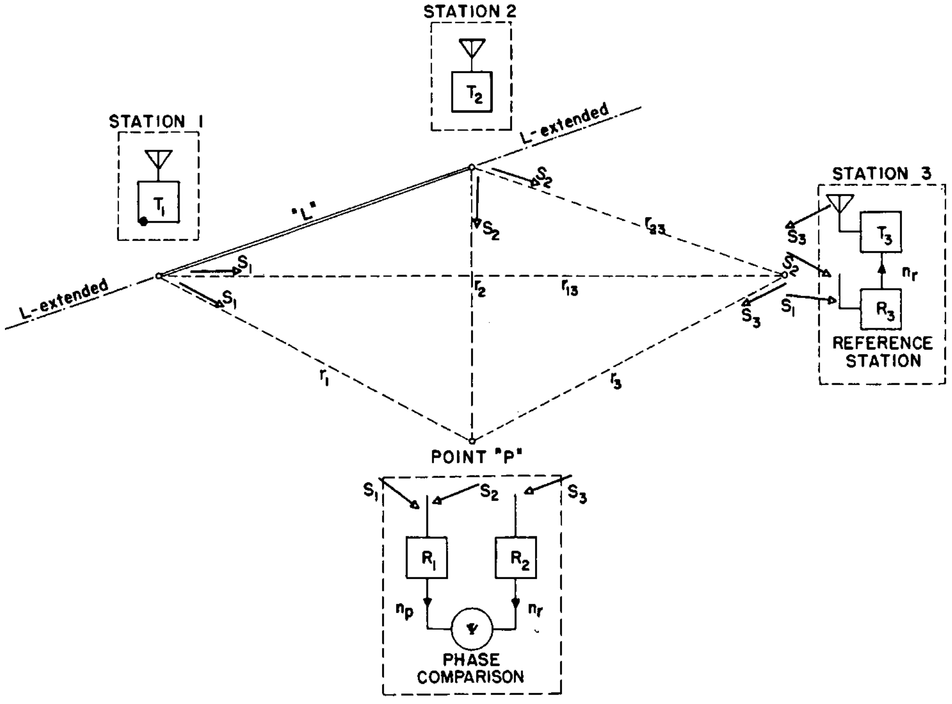

FIG. 3

Referring to figure 3, it can be seen that a difference frequency signal n_r will be developed in the output of receiver R_3, which is identical to receiver R_1, and that the phase of this signal will be constant since the transmission distances to the *reference* station receiver R_3 are held constant. The constant-phase reference signal n_r is impressed on the carrier of the transmitter T_3 and radiated to the receiver R_2 at the phase comparison measuring unit. Although it is not feasible to radiate the signal n_r directly, its conduction to receiver R_2 by means of the carrier f_3 is for practical purposes equivalent to the direct radiation of n_r itself. Now, the frequencies f_1 and f_2 are usually in the order of magnitude of several millions of cycles per second, while n is in the order of hundreds of cycles per second; consequently the difference in wavelength between f_1 and n is very large. The transmission of the signal n_r from reference station 3 thus provides a reference signal at the output of receiver R_2 which is virtually insensitive to variation in the position of the phase comparison measuring unit.

The phase sensitivity of the position signal n_p can be estimated from the following considerations : If the measuring frequency f_1 is 2 megacycles, one wavelength or 360 electrical degrees of phase of this carrier is 492 feet. For $n = 100$ cycles per second one wavelength is 9 835 705 feet. Since changes in position of the phase comparison measuring unit produce approximately equal and opposite phase changes in the two carriers from stations T_1 and T_2, the difference phase change is twice the phase change produced in one carrier. Thus when the position of the measuring unit changes by an amount equal to one-half wavelength of the carrier f_1, the phase of the signal n_p changes one full revolution or 360 electrical degrees. Thus a lane is 246 feet wide on the baseline using a measuring frequency of 2 megacycles. The device for phase measuring is usually graduated into one hundred units so that each unit represents 2.46 feet on the baseline. The special line measuring phase comparison receiver was modified to read to the nearest one-thousandth of a lane or .246 foot. This amounts to about three inches or 250 micro-micro seconds if converted to a direct time measurement.

With the above explanation of the operation of the phase comparison radio system, we proceed with the examination of factors which must be considered in precise line length measurements.

Instrumentation

(a) *Frequency measurement control*

In eq. 1, given previously, the measuring frequency f_1 occurs, and the line-length L depends directly on this factor. During the project tests the frequency f_1 was developed by a quartz crystal oscillator of routine design. Oscillators of this type may be depended upon to hold frequency within plus or minus one cycle per megacycle per degree centigrade of ambient temperature. In order to reduce the measuring frequency variation to a minimum amount, the project crystal oscillators were operated in thermostatically controlled ovens so that temperature was maintained to within plus or minus one degree centigrade. In this case a measuring frequency of 2 megacycles, a two-cycle variation from the absolute frequency, amounts to a plus or minus variation of $.4918 \times 10^{-3}$ foot per lane. In a line length enclosing 2 000 lanes, or about 100 miles long, the variation in measuring frequency would produce a variation of about ± 1 foot in the measured line length. Further improvement in line-length measurement results was achieved by comparing the project measuring frequency continuously against one of the primary standard frequencies emitted by radio station WWV at Beltsville, Maryland. The result of the measurement was the absolute value of the measuring frequency to ± .1 cycle per second. Variations of this order produce variations of $\pm .2459 \times 10^{-4}$ foot per lane or about ± .05 foot in 2 000 lanes. As a result of the control exercised over the measuring frequency, errors due to this source were considered to be reduced to an insignificant value.

(b) *Beat frequency control*

While the *beat frequency n* or (f_1-f_2) does not appear in eq. 1, it does appear in some low ordered terms of the fully expanded form of eq. 1, which was used for actual line-length computations. Furthermore if n is allowed to vary, differential phase variations at the final phase comparison point may appear due to the dissimilar paths followed by signals from the baseline transmitters to the outputs of receivers R_1 and R_2. For these reasons the beat frequency was monitored at station T_2 and compared with a standard signal of frequency n. The standard signal was developed by a vibrating reed capable of maintaining frequency to within one part in 20 000. With the beat frequency controlled to this accuracy, differential phase shifts in the instrumentation become negligible. Since the sensitivity of the line-length measurement to variations in n is extremely small compared to the sensitivity of the line-length result of f_1, errors contributed by the beat frequency n will be insignificant.

(c) *Station T_3 phase control*

The two halves of the receiving system at the phase comparison point are identical and consequently have phase characteristics, relative to most variables, which produce very small differential phase errors. The reference signal n however must be developed in receiver R_3 at station T_3, go through the process of modulation onto the carrier of transmitter T_3 and finally radiate to receiver R_2 at the phase comparison point. Now, while the control of the beat frequency n to within narrow frequency tolerances does much to eliminate phase errors due to variation of the frequency n in the circuitry of station T_2, some phase shifts may occur for other reasons. To reduce these errors to a minimum, a sample of the signal radiated from the transmitting antenna at T_3 was demodulated and phase compared with the signal out of receiver R_3. Any phase variations produced by the circuitry in station T_3 for reasons other than frequency variations were thus detected and eliminated by automatic equipment or by the station operator.

Propagation of electromagnetic energy

The propagation of the signals emitted by transmitters T_1 and T_2 has been assumed to be a linear function of distance in considerations up to this point and in eq. 1. Actually the radio frequency carriers experience effects produced by the earth's atmosphere through which they must pass and the earth's surface materials over which they must travel. Due to the tedious mathematical details which one must consider to appreciate fully the effects produced in a radio carrier being propagated over the earth's surface, the various effects will be mentioned and described only.

(a) *Free space velocity of EM propagation*

The limiting velocity of electromagnetic radiation is its velocity in free space. This velocity has been the subject of a great many investigations in past years and continues to be a subject of concern to the present time. During the past ten years the free space velocity determinations have all been in good agreement and a number of these determinations has been combined with a group of other, measured and inter-related, physical constants in a least square determination of the best values for these constants. The value thus computed for the free space velocity of EM Radiation is as follows :

$$c = 299\,792.9 \pm 1.6 \text{ km/sec.}$$

This value for c is used throughout all computations.

(b) *Refractive index of the earth's atmosphere*

The effective wavelength, or lane width, of the phase comparison radio signals in the earth's atmosphere depends on the free space velocity of propagation of electromagnetic radiation, frequency, and the index of refraction of air. A value for the free space velocity of propagation was considered in the last section. The measurement of the signal frequency was considered in an earlier section [(*a*) Frequency measurement control]. This section deals with the effective refractive index of the earth's atmosphere.

The index of refraction of any material may be defined as the ratio of the velocity of propagation of electromagnetic radiation in free space to the velocity of propagation in the material. The properties of air on which the index of refraction depends are the air pressure, temperature, and water vapor content. The equation relating these quantities and the index of refraction is shown in the following :

$$(\mu_{t,p} - 1)\,10^6 = \frac{77.6}{T}\,p + 4810\,\frac{\omega}{T} \tag{2}$$

where

p = total air pressure in millibars
ω = partial water vapor pressure in millibars
t = temperature in degrees centigrade
T = temperature in degrees absolute
μ = actual numerical value of the index.

The development of this form is given in Research Paper 2385 of the Bureau of Standards.

Air pressure, temperature, and humidity were measured continuously during the project field tests at five locations. Three of these locations were at stations T_1, T_2, and T_3 which are shown in figure 3. The remaining two locations were on the baseline extensions at the points where baseline extension crossings were made by the phase comparison receiver unit. From the measured atmospheric or meteorological data, average values of the index of refraction may be computed for any time along all the pertinent paths in figure 3. The inclusion of the index of refraction in computational forms so far has been only in the linear relationship of a radio carrier phase with distance or the effective wavelength. Instantaneous values of refractive index are used in the determination of effective wavelength as the linear phase angle of a carrier is most sensitive to small changes in the refractive index of the atmosphere. The index of refraction is also required in the equations of *non-linear phase delay* with distance. These equations also take into account the electrical characteristics of the earth's surface.

Since the equations for nonlinear phase delay are relatively insensitive to changes in the index of refraction, a mean index of refraction is used at this point.

Since the air pressure, temperature, and water vapor content all vary with altitude, the index of refraction of the atmosphere is a function of altitude. While the index of refraction profile with altitude varies considerably from hour to hour and day to day, the index of refraction generally decreases with increasing altitude.

The radiation emitted from the transmitting antennae in the phase comparison system may be represented as a series of concentric spherical wave fronts. Lines perpendicular to these wave fronts and passing through the radiation origin are called rays. Since the refractive index of a moist atmosphere decreases with altitude, the upper portions of the wave front surface move with a greater velocity than the nearly horizontal parts. This has the overall effect of causing the rays to be bent downward so that radiation emitted at low angles of elevation eventually returns to the earth's surface at some distance from the transmitter. This curvature of the rays by the atmosphere is called vertical refraction.

In an average atmosphere and in the lowest few kilometres where most short wave propagation takes place, it may be assumed that the decrease of refractive index with height is linear, though the rate of decrease depends on weather conditions. It can be shown by further analysis that a linear lapse rate of refractive index with altitude may be absorbed in calculations by using a modified value for the radius of the earth. By introducing a modified earth radius it is possible to eliminate the refraction effects produced by vertical refraction, and the earth's atmosphere may be treated as if it were homogeneous.

The modified earth radius will be referred to again relative to geometric corrections to line measurement data. The modified earth radius factor also appears in the non-linear phase delay equations but a mean value for this factor is used in these equations.

(c) *Non-linear carrier phase delays*

Due to interaction between electromagnetic radiation in space and conducting boundaries such as the earth's surface, adjustment is constantly taking place in the phase of the radiation as it progresses along the boundary face. As a result, the phase of an individual carrier is not a linear function of distance as was assumed in eq. 1 given previously.

The interaction effects mentioned above are manifest in the carrier by causing it to require more time in transmission between two points than the transit time given by $\frac{r}{v}$. The actual delay in time or equivalent lag in phase angle is a function of :

— the vertical lapse rate of refractive index of the earth's atmosphere,
— the earth's surface value of atmospheric refractive index,
— the electrical conductivity and dielectric constant of the earth's surface,
— the frequency of the radiated energy,
— the distance over which radiation takes place.

Since the development of applicable mathematical forms is well covered in the literature (*), such information will not be repeated here. Generally the non-linear function of phase-lag versus distance is exponential in nature and becomes linear with distance for large distances.

A problem which arises in the application of phase comparison radio systems to distance measurement is that the conductivity and dielectric constant of the earth's surface vary, radically at times, from one region to another. Several solutions to this problem have been given (**). Where a multiplicity of different regions occurs in the signal paths, all with different or alternating electrical constants, reduced accuracy can be expected in the non-linear phase lag corrections computed for such paths. The accuracy of the correction remains good however for most mixed path conditions.

Geometrical corrections

Due to the fact that all transmitting and receiving antennae in the phase comparison radio system are elevated above the surface of the earth, the measured line must be transferred from the phase centres of these antennae to the earth's surface. Since corrections of this type depend only on the geometric shape of the earth and the antenna system, they are considered separately as geometric corrections.

(a) *Altitude correction*

No mention has been made so far of the means used to transport the phase·comparison receiver from one baseline extension to the other. In six separate surveys made to determine various baseline lengths, three have been conducted using surface ships and three have been made with aircraft. In passing across the baseline extension it is usually the case that the phase comparison receiver antenna does not exactly intersect the baseline extension but passes near the extension, either slightly above it, or possibly below in some cases. The non-intersection of the baseline extension is equivalent to not counting all the lanes in the baseline or neglecting small fractions of the lane on each end of a baseline. The correction which arises from this source is very small and is insensitive to the factors involved in its computation. Since the correction arises from the altitude of the receiving antenna above the earth's surface it is called the *altitude correction*. The correction becomes sensitive with increasing altitude and short distances from the extension crossing point to the nearest phase comparison transmitter. For these reasons, extension crossings were made approximately ten miles out from the nearest transmitting station and at an altitude of 500 feet.

(*) *Terrestrial Radio Waves*. H. Bremmer. Elsecier Press. 1949.
The Calculation of Ground-Wave Field Intensity Over A Finitely Conducting Spherical Earth. K. A. Norton. Proc. of IRE, vol. 29, no. 12, December 1941.
Phase of the Low Radio Frequency Ground Wave. J. R. Johler, W. J. Kellar, L. C. Walters. National Bureau of Standards Circular 573.

(**) *Mixed Path Ground Wave Propagation*. J. R. Wait. Journal of Research of the National Bureau of Standards. Vol. 57, no. 1, July 1956, and vol. 59, no. 1, July 1957.
Ground Wave Propagation Over an Inhomogeneous Smooth Earth — Part 2. G. Millington and G. A. Isked. Proc. Inst. Elec. Eng., vol. 97, Part III, July 1950.

The geometry involved in the computation of this correction is shown in figure 4. Straight line ray paths between the transmitting and receiving antennae are based on the modified earth radius factor *k*, which appears in figure 4 multiplied by the earth's radius. This factor was discussed previously.

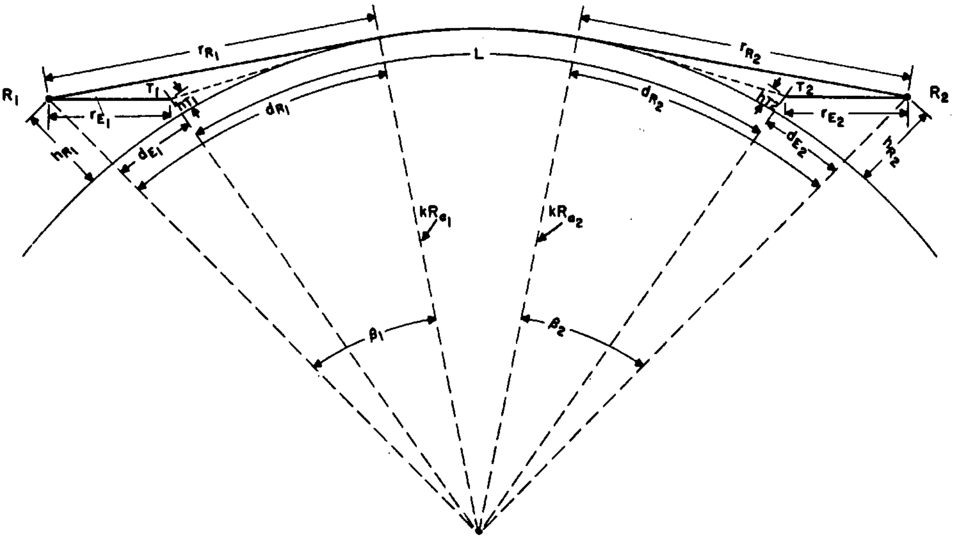

Fig. 4. -- Altitude correction.

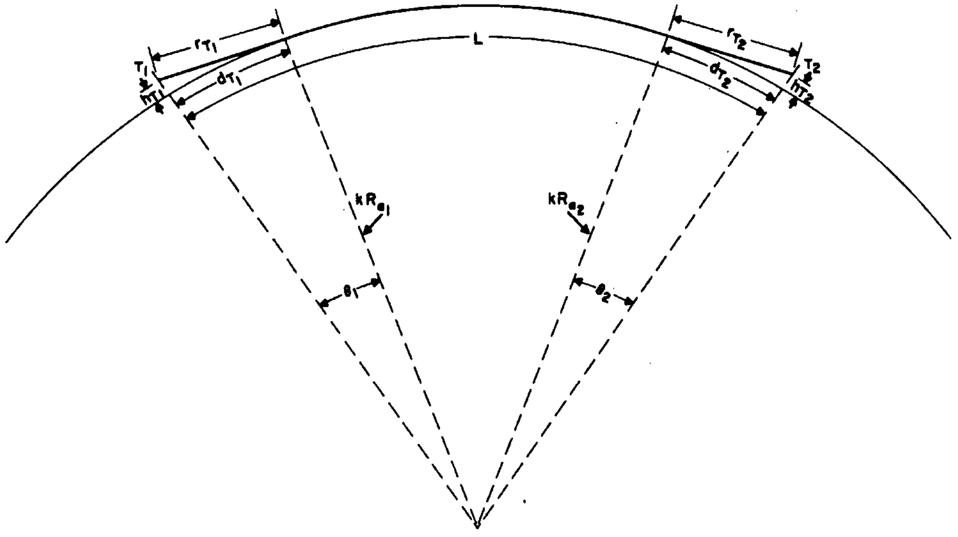

Fig. 5. — Sea level correction.

(b) *Sea level correction*

When all corrections described so far are applied to the line measurement data the line length turns out to be the distance from the phase centre of one transmitting antenna to the phase centre of the opposite transmitting antenna. To reduce the measured line to sea level a correction is determined which is the difference between the transmitting antenna phase centre to horizon distance and the distance along the surface of the earth to the horizon. The geometry involved in this computation is shown in figure 5.

DESCRIPTION OF FIELD METHODS AND RESULTS

Equipment

The project test line programme is shown in figure 1.

The phase comparison receiving system used to make precise measurements of the number of lanes in a given baseline has been air transported in all cases to date. Several types of aircraft have been used in this service. Those in the project field tests conducted in Louisiana were Canadian made De Havilland Otters and were assigned to the project by the U. S. Army. A twin engined Beech Bonanza and a single engined Beech Bonanza were used in a later USA-ERDL line measuring project conducted in Arizona.

The phase comparison receiver was a standard Lorac positioning receiver converted to line measuring service. One Lorac indicator indicated lanes and fractions down to one one-hundredth of a lane. The other indicator was modified to repeat the hundreths of a lane indication and gave in addition fractions of lane down to one one-thousandth of a lane.

Operations carried out

The number of lanes contained in a line between the phase centres of the transmitting antennae may be found by dividing the line length by a lane width. Conversely, if a line is determined by two transmitting antennae, the length of the line is determined if the number of lanes contained in the line can be found. Line measuring by phase comparison consists of measuring the number of lanes contained in the unknown line.

If the baseline determined by two Lorac transmitting stations is extended beyond the transmitting station on each end of the baseline, the extensions so generated are referred to simply as the baseline extensions.

Space surfaces which have a constant phase comparison indicator reading are, to a high order of approximation, hyperbolae of revolution. The special hyperbolic sheet which passes through the mid-point of the baseline is a plane perpendicular to the baseline.

If a Lorac receiver starts from a point on a baseline extension and moves along any arbitrary path to a point on the opposite baseline extension, it must traverse every lane in the Lorac system. It is only necessary for the receiver to produce a permanent record or count of the individual lanes traversed to know the total number of lanes in the baseline.

TABLE I

Computation of Line Length

Line No 7	Date 3-31-56		Date 3-31-56		Date 3-31-56		Date 3-31-56	
Measurement No	①		②		③		④	
Maximum Phase Meter Reading	7134	3889	1300	3801	7134	4048	1300	4151
Minimum Phase Meter Reading	6215	7344	381	7282	6215	7344	381	7282
Measured Lanes	918	6545	918	6519	918	6704	918	6869
Propagation of Phase Correction in Lanes	-	1104	-	1104	-	1104	-	1104
Third Frequency & Reference Phase Correction in Lanes	+	0872	+	0872	+	0861	+	0862
Total Corrected Lanes	918	6313	918	6287	918	6461	918	6627
Lane Width in Metres	66.3618903		66.3616253		66.3616328		66.3613594	
Semi-Corrected Line Length in Metres	60,962	110	60,961	694	60,962	855	60,963	706
Altitude Correction in Metres	-	049	-	049	-	049	-	049
Line Reduction to Sea Level in Metres	-	082	-	082	-	082	-	082
Total Corrected Distance	60,961	979	60,961	563	60,962	724	60,963	575

TABLE II

LINE	MEASURED LANES	REF. PHASE CORR.	PHASE CORR.	CORR LANES	LANE WIDTH	SEMI-CORR. LENGTH	ALT. CORR.	SEA-LEVEL CORR.	LINE LENGTH
1	1255.3939	-.0010	-.5984	1254.7945	66.3644156	83,273.704	+.043	-.049	83,273.698
2	1164.1293	+.0057	-.4639	1163.6711	66.3634127	77,225.185	+.052	-.048	77,225.189
3	2418.7879	+.1789	-.6133	2418.3535	66.3642696	160,492.264	-.231	-.048	160,491.985
4									
5	1419.2242	+.1152	-.3749	1418.9645	66.3634374	94,167.362	-.025	-.066	94,167.271
6	1188.5330	+.0845	-.4186	1188.1989	66.3630301	78,852.479	+.005	-.067	78,852.417
7	918.6545	+.0872	-.1104	918.6313	66.3618903	60,962.110	-.049	-.082	60,961.979
8	1650.0871	+.1173	-.3943	1649.8101	66.3615922	109,484.025	-.015	-.067	109,483.943
9									
10	1483.8769	+.1314	-.1658	1483.8425	66.3622262	98,471.092	-.046	-.081	98,470.965
11	1145.0349	+.1183	-.2641	1144.8891	66.3633125	75.978.633	+.019	-.066	75,977.586
12	1782.9793	+.1844	-.3958	1782.7679	66.3637712	118,311.201	-.095	-.065	118,311.041
13	2386.8318	+.2814	-.2579	2386.8553	66.3617902	158,395.991	-.061	-.082	158,395.848
14									
15	4031.5015	+.2832	-.3836	4031.4011	66.3604570	267,525.619	-.139	-.059	267,525.421
16	5032.5633	+.2704	-.5506	5032.2831	66.3594632	333,939.605	-.135	-.069	333,939.401
17	6233.4072	+.2683	-.6116	6233.0639	66.3603837	413,628.512	-.163	-.078	413,628.271

TABLE III

LINE NO.	NUMBER OF MEASUREMENTS	GEODETIC LENGTH	ELECTRONIC LENGTH	DIFFERENCE	RELATIVE ACCURACY
1	3	83,272.604	82273.520	+ .916	1: 90,900
2	5	77,226.033	77225.481	- .552	1: 139,900
3	4	160,498.074	160493.469	-4.605	1: 34,850
4		(68,453.275)		No Measurement	
5	4	94,165.810	94166.894	+1.084	1: 87,790
6	4	78,852.410	78852.760	+ .350	1: 225,290
7	4	60,960.493	60962.460	+1.967	1: 30,990
8	4	109,484.039	109484.305	+ .266	1: 411,590
9		(114,849.546)		No Measurement	
10	4	98,472.960	98470.812	-2.148	1: 45,840
11	4	75,975.358	75977.378	+2.020	1: 37,610
12	4	118,307.045	118309.749	+2.704	1: 47,750
13	4	158,395.545	158394.626	- .919	1: 172,360
14	Deleted From Project				
15	3	267,525.000	267520.914	-4.086	1: 65,470
16	1	333,947.945	333939.401	-8.544	1: 39,090
17	2	413,629.482	413629.784	+ .302	1:1,369,630
		2,130,712.798	2,130,701.553		

All Lengths In Meters

In the field applications of the line measuring technique described above, the phase comparison receiver is transported by means of an aircraft in order to complete one traversal of the Lorac system of hyperbolic lanes in the shortest possible time. At least five complete trips are usually run in order to ensure proper functioning of all equipment and to obtain a series of independent measurements. Since the aircraft cannot remain stationary on a baseline extension while a reading is being observed, the observed reading is obtained during the time the aircraft transits the baseline extension. A general description of the baseline crossing technique follows :

As the Lorac receiver leaves one baseline extension, the position indicator turns positively, or clockwise, and begins adding lanes; so this baseline extension is called the *minimum baseline extension.* On leaving the opposite baseline extension, the position indicator will have reached a maximum count and will turn negatively or subtract lanes. This extension is called the *maximum baseline extension.* As either baseline extension is approached, the position indicator approaches an extreme reading and reverses its direction of rotation as the extension is crossed. This reversal in the direction of rotation of the position indicator is easily readable by an observer and the extreme meter reading is recorded. This operation is not difficult since near the baseline extensions the lane expansion factor approaches infinity and the smallest graduations on the Lorac position indicator (one hundredth of a lane for example) may occupy at least one quarter of a mile on each side of the baseline extension. Thus, for the last one hundredth of a lane to the extreme meter reading and back to the same meter reading will require one half mile on the ground, which at a ground speed of 120 miles per hour gives 15 seconds or one quarter of a minute for the observer to note the extreme indication.

Repeated crossing of a baseline extension provides a series of extreme position indicator readings. The group of observations so obtained is screened by a statistical method to eliminate errors and to give an average value calculated for the group.

The average minimum extension readings are subtracted from the average maximum extension readings and the result of this subtraction is the uncorrected measured number of lanes in the baseline. The corrections discussed previously are applied to the measured lane count and the corrected measured lane count is then multiplied by the effective lane width to obtain line length. Examples of the results obtained during the Louisiana line measuring project are given below.

Line numbers are identified in figure 1 given earlier. In table I all computed corrections are given for all complete line measurements made on the sample line. This example shows the variation of the corrections throughout the period of time required to make a number of measurements.

In table II the first measurement on all lines measured is given with the exception of lines on which a measurement was not completed. This example shows the variation of correction magnitude from one line to another. The effect of various types of terrain on correction magnitude can be seen in this table. Finally in table III the results of all line measurements are compared with the USC&GS length as standard. This table shows that a high degree of accuracy was achieved in all cases.

LINE MEASURING USING MULTIPLE FREQUENCIES

Explanation of method

This second method of line measuring was partially tested during a second line measuring project in Arizona. For the purposes of describing this method, suppose the following :

(*a*) A given line is to be measured whose length is known approximately. The accuracy to which the length is known need not be any better than that obtained by inverse computation from astronomic positions at the ends of the line.

(*b*) A multiplicity of measuring frequencies is available where the multiplicity is not less than two and preferably three or more. If the measuring procedure described in the previous paragraphs is used, the number (whole number and decimal fraction) of lanes in the line to be measured can be obtained successively for each of these frequencies.

Consider a nomogram constructed for a given line measurement as follows : Starting from the left-hand end of a straight line as the origin, lay out a distance to scale which represents the fraction of a lane obtained in the line measurement. If the fraction of a lane is multiplied by a lane width, then the scale on the line will be directly in feet or metres. From the terminal point of the scaled length representing the measured fraction of a lane, lay out a series of distances to scale, each equal to an integral lane width. Continue this series of line segments until a number is obtained equal to the measured number of integral lanes in the line.

The scaled fraction plus the measured integral lanes is now a graphical representation of the line-length measurement. To improve the appearance of the graphic plot, short vertical risers of uniform length are constructed at the origin and terminus of each line segment on the plot. The base of each vertical riser is connected by a straight line to the top of the next riser on its right. This construction creates a saw-toothed plot where each saw tooth represents one Lorac lane. By constructing similar plots for the remaining independent line measurements on line bases which are parallel to one another and with vertical alignment of the origins, a comparison may be made between the line measurements. In this respect, since the same line is being measured each time, the vertical risers for all plots will be in vertical alignment at the right-hand end of the line plots.

Since each line measurement is made on a different frequency, individual saw teeth in each measurement plot will have a different width. If the vertical alignment of the risers is compared from the end of the line plot toward the origin, a progressive misalignment will be noted. After a certain distance along the graphical plot from the end of the line towards the origin, a second vertical alignment may be noted.

It can be seen at this point that all that is actually necessary to plot the nomograms described above is (1) the fraction of a lane in a line measurement, and (2) the frequency used during the measurement. Out of these two requirements, only (1) must be measured in the field. The frequency used to perform the measurement is an instrumentation factor.

The integral lane count may be determined by the vertical alignment characteristic of the graphical plots. The correct vertical alignment may be selected out of all such alignments that occur by the known approximate length of the line. This information suggests the following modification in the line measuring technique : Instead of one Lorac receiver performing the line measurement, two would be used. These receivers would be checked against one another at a common point so that it is known that they indicate identically when identical points are occupied. A line measurement would begin by having the two receivers on station near the baseline crossing point and in communication with each other. At a pre-set time, both receivers would begin a series of baseline extension crossings. After a prescribed number of crossings had been made, the data so collected would be considered as one line measurement. The process would be repeated several times to obtain the best possible measurement. After a sufficient number of line measurements had been collected, the baseline transmitting stations would change to a new measuring frequency and the line measuring procedure would be repeated. The line measurement would be complete when all allotted measuring frequencies had been used. The basic line measurement data would be processed as before, and a chart constructed to permit the selection of the proper lane coincidences.

Numerical example

The observed field data are corrected for this example in exactly the same way as described previously. The three measuring frequencies used were frequencies assigned to the project without any previous planning relative to the multiple frequency line measuring project and consequently do not represent the best selection of frequencies for this service.

The integral lane counts obtained during the line measurement have been discarded for this example and only the fractional lane count retained. These data appear as follows :

Frequency	Lane Fraction
1 798 315 cps	.3948
2 042 315 cps	.6834
2 082 315 cps	.7294

Using these data, a graphical plot was constructed and is shown in figure 6. The plot relative to each frequency is shown to the right of the frequency listed along the left-hand edge of the construction. Neglecting the numbers in parentheses and the plots opposite these numbers, the above data are plotted in the bottom three plots. The entire left-hand part of the construction has been deleted in figure 6 as it is not actually necessary, and only that part of the construction carried out which falls near the expected length of the line. The approximate known length of the line was determined from astronomic observations for position of the two baseline stations. The computed astronomic length is 112 547.59 metres. In constructing the plot shown in figure 6, it was assumed that the astronomic length might possibly be in error by as much as ± 1 000 metres. Consequently, the constructed plot covers this band on each side of the *arrow* which indicates the computed astronomic length.

Some further information is available from the line measurements

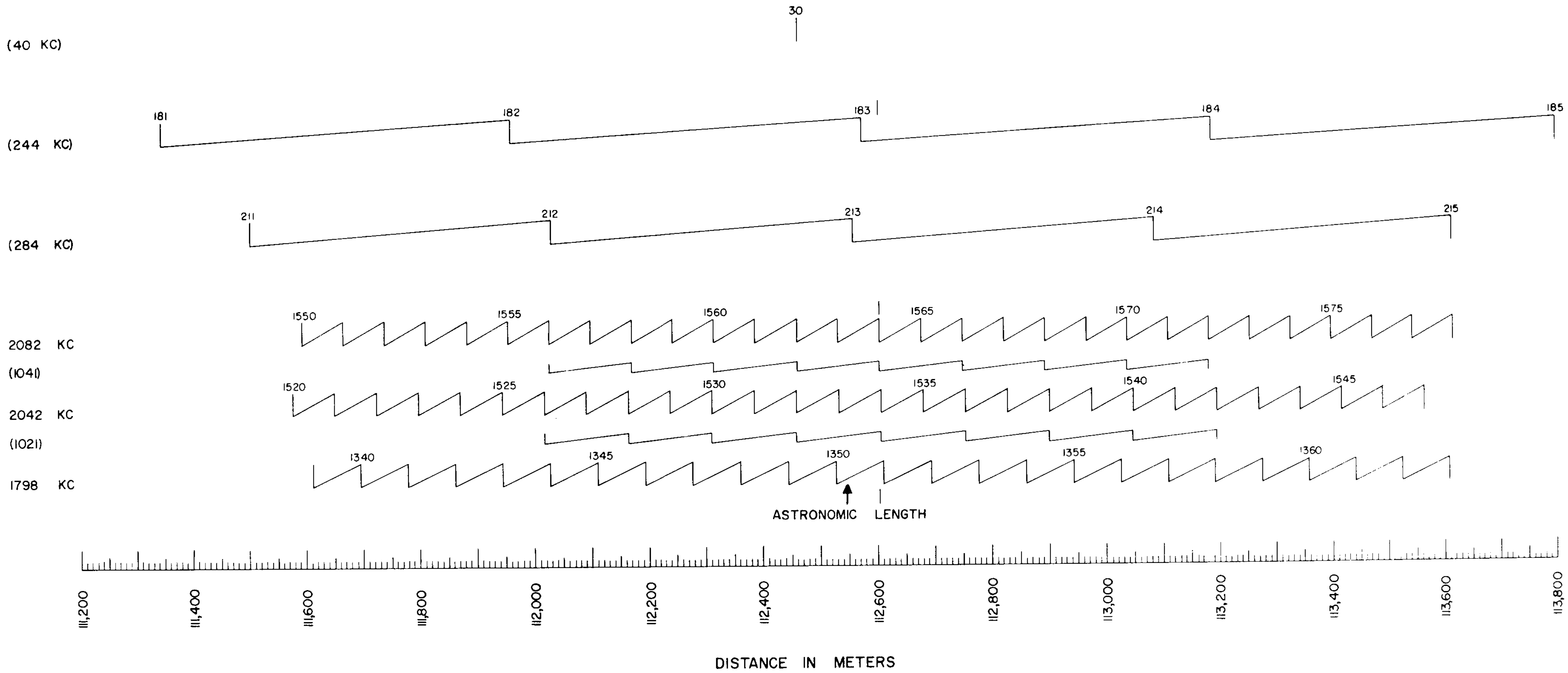
30
(40 KC)
181
182
183
184
185
(244 KC)
211
212
213
214
215
(284 KC)
1550
1555
1560
1565
1570
1575
2082 KC
(1041)
1520
1525
1530
1535
1540
1545
2042 KC
(1021)
1340
1345
1350
1355
1360
1798 KC
ASTRONOMIC LENGTH
111,200
111,400
111,600
111,800
112,000
112,200
112,400
112,600
112,800
113,000
113,200
113,400
113,600
113,800
DISTANCE IN METERS

Fig. 6

other than what is plotted directly in the three bottom plots in figure 6. By subtracting the measuring frequency of 1 798 315 cycles per second from the frequency of 2 082 315 cycles per second, an apparent measuring frequency of 284 000 cycles per second is obtained. The fraction of a lane which would be measured if this frequency was actually used to measure the example line would be the difference between the first and last lane fraction shown in the table above or .3346 lanes. Similarly, for the difference between frequencies of 1 798 315 cycles per second and 2 042 315 cycles per second, an apparent measuring frequency of 244 000 cycles per second is available. Subtracting the two highest frequencies gives a still lower apparent measuring frequency. The fractions of a lane obtained by subtracting corresponding fractions in the above table give a second table as follows :

Apparent Frequency	Apparent Lane Fraction
284 000 cps	.3346
244 000 cps	.2885
40 000 cps	.0460

Using the fractions of a lane obtained through the subtraction of lane fractions of the original data and lane widths corresponding to the apparent frequencies given above, three new plots were constructed and are shown at the top of figure 6. The lane width of the 40 000-cycle per second apparent frequency is so wide that only one vertical riser appears in figure 6. The next risers on either side of lane No. 30 fall off the plot altogether. The length of the line being measured is then indicated to be in the vicinity of lane No. 30 of the 40 000-cycle per second plot. If the plots relative to 284 000 cycles per second and 244 000 cycles per second are considered, it can be seen that lanes Nos. 213 and 183 align vertically fairly well and that progressive misalignment in opposite directions is indicated between the plots to the right and left of the vertical risers Nos. 213 and 183. This vertical alignment is further evidence of an agreement relative to the length of the measured line. The variation in the alignment of the vertical risers for the apparent frequency group is due to errors of measurement in the basic data from which the apparent frequencies were obtained. Since the various measurements on a line cannot be expected to agree perfectly, variations in the apparent frequency results are quite normal.

Now between lanes Nos. 30, 183, and 213 and the indicated astronomic length of the line we would expect to find a vertical alignment on the higher frequency plots in the general vicinity of these points. Noting lane No. 1565 (2 082 kilocycles), lane No. 1535 (2 042 kilocycles), and lane No. 1352 (1 798 kilocycles), a gentle misalignment to the right may be seen which increases progressively to the right. Considering lane No. 1562, lane No. 1532, and lane No. 1349 taken in the same order as before, a gradual misalignment to the left is noted which again becomes more severe progressively to the left. The line length then apears to be indicated by lanes Nos. 1563, 1533, and 1350 or lanes Nos. 1564, 1534, and 1351. At this point it is not a simple matter to make a choice between the two groups of lane numbers selected above. This condition arises because the *range in frequencies* used to measure the sample line was not originally selected for best resolution of the vertical alignment problem.

In order to show the effect of at least a wider range in measuring frequencies, two fictitious frequencies are supposed which are shown in parentheses immediately beneath the 2 082 315 - cycle per second and 2 042 315 - cycle per second plots. The phantom frequencies selected are one half 2 082 315, or approximately 1 041 kilocycles, and one half 2 042 315, or approximately 1 021 kilocycles. From the plots of these frequencies, it can be seen that it would be difficult even in the presence of relatively large measurement errors to obtain a vertical alignment such as lanes Nos. 1563, 1533, and 1350, since the plot for frequencies 1 041 and 1 021 kilocycles has no vertical risers in this group. The lanes Nos. 1564, 1534, and 1351 would be selected and this is indeed the correct group. The true length of the line is indicated by short vertical marks. One of these marks is just above the L in the word Length in figure 6. A second mark is just to the left of lane No. 1565, and the third is to the right of lane No. 183.

Collecting the line lengths which correspond to lanes Nos. 1564, 1534, and 1351 together in a table results in the following :

Lane No.	Line Length
1351	112 612.0639
1534	112 607.5946
1564	112 604.9187

The above measurements combined as a weighted mean become 112 607.4303. Applying a sea-level correction which was previously neglected gives a corrected line length for the example line of 112 603.8123 metres. This differs from the true length 112 614.326 by -.513 metres.

The numerical example given above shows that line measuring by means of multiple frequencies is possible and needs only refinement of techniques and equipment to become a matter of practice.

The modified line measuring method is most directly applicable to all sea-water lines where the baseline extensions are also over sea water. Lorac ground stations would be ship transported in this case and all measurements would be made by the same ship. The need for aircraft would be obviated and the operation thus simplified to a large extent. Since sea-water paths and lines usually yield much more accurate results than land paths and lines, the range in frequencies would not need to be so great as in the case of the example line given above, and quite possibly the frequency range available to measure the sample line would be sufficient for a sea-water line.

CONCLUSIONS

The experimental results obtained during the two line measuring projects show that good accuracy in line length results can be obtained by phase comparison measurements. The prediction of phase delays along all types of lines was accomplished with better than expected accuracy and homogeneous transmission path lines can be predicted with excellent results. Further line measuring projects would make further gains in accuracy through better computational techniques and field practice.