THE MEASUREMENT OF DISTANCES OVER WATER

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There is a fairly widespread belief that the tellurometer cannot, of at least should not, be used to measure distances over water. This belief may stem from the fact that a water surface may act as a reflector. A strong reflected ray will create a " swing " (*), and hence errors in measurement. Perhaps one should say "water swing". However, the more familiar term "ground swing" may be used no matter what the actual surface material. A water surface is not necessarily a good reflector and, therefore, not necessarily a source of error. This fact is attested to by engineers who have measured lines over water from the Bahamas to the Baltic and from Central Africa to Sarawak.

The reliability of measurements made across water becomes questionable when one or both of the terminals are located at a considerable elevation above the water surface. In figure 1-A the situation shown is where both terminals are at a considerable elevation above sea level. The angle of incidence (grazing angle) is large. The indirect (reflected) ray path is appreciably longer than the direct ray path. The indirect ray, however, may have about the same signal strength as the direct ray. This combination leads to excessive ground swing and unavoidable error in the distance measurement.

In figure 1-B the terminals are relatively close to sea level. The angle of incidence is small, and there is little difference between the lengths of the indirect and direct ray paths. In addition, much of the indirect ray signal strength may be dissipated in random reflections. This combination leads to little or no ground swing, and it does provide good measurement accuracy. The expression "considerable elevation" cannot be defined precisely, because the elevation must be considered in connection with the length of the line being measured.

(*) Ground Swing is the variation in transit time during a measurement caused by stray reflections from objects in the microwave beam.

Even with the two terminals relatively close to sea level, the water surface conditions have an effect on the quality of measurement. Flat-calm sea conditions may produce anything from a rather weak, but readable, signal to a completely unreadable display. If a light breeze picks up, it ruffles the water, breaks up reflections, and the display improves. If the sea builds up to a condition of choppy waves two or three feet high, conditions again become bad. The display is fuzzy, it varies in size, and it has an ill-defined break which is virtually unreadable. These are more or less general statements. In any given situation, the only real test is to try to measure the line. Conditions which are impossible at one time may prove feasible at another.

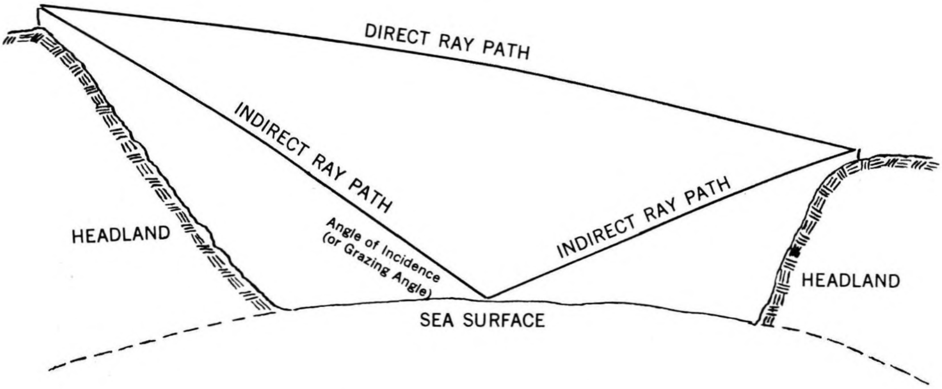

FIG. 1-A. — Ray paths with elevated terminals.

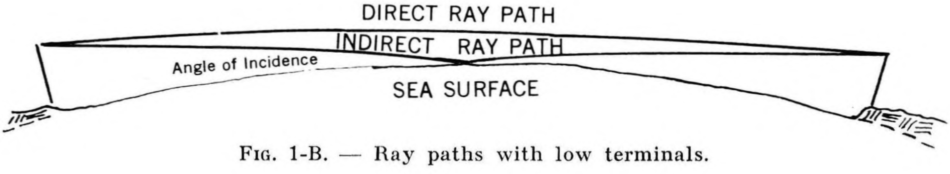

FIG. 1-B. — Ray paths with low terminals.

In 1961, geodetic survey parties of the U. S. Naval Oceanographic Office ran some 500 miles of tellurometer traverse through the central Bahama Islands. These traverse lines were almost exclusively over water. Many stations were sited on off-lying rocks, or on small cays. The greatest elevation did not exceed 130 feet. Closures on these lines were highly satisfactory, and no serious trouble was encountered in the measurements.

In September 1961, the Royal Danish Geodetic Institute made an interesting application of the tellurometer; they measured lines over water about 70 kilometres long. This was done by the line-crossing method. Two sets of tellurometers were used; the masters were installed in a vessel, and the remotes were located at the shore stations. Distances from the vessel to each shore station were observed simultaneously, while the vessel approached and crossed the line between the stations. The minimum distance, properly corrected, was compared with direct-distance tellurometer measurements from shore station to shore station. The mean difference between direct and line-crossing measurements was 18 centimetres.

Similar tests, using a Hydrodist installation, were made about the same time by the Swedish Hydrographic Department. In these tests the position of an off-lying island was determined by trilateration from three stations ashore. The island to island distances, varying from 42 to 57 kilometres, were measured by the line-crossing technique.

A fourth line about 90 kilometres long was attempted, but the line-crossing trial was not successful. Even though both terminals of the line were less than 45 metres above sea level, and thus well below the radio horizon, VHF radio telephone communication was established between them. A master unit was rushed to one end of the line, and tellurometer distances were measured along the line. The probable explanation of this unusual measurement was the existence of a low-lying bank of thick fog just above the surface of the water and extending to the approximate elevations of the terminal stations. This condition must have created a tropospheric duct between the two terminals. The duct might also account for the lack of success in the line-crossing attempts. Efforts to repeat this measurement on succeeding days were not successful.

In this test, the three lines measured by line-crossing and the one line measured directly were included with the triangulation previously done in a rigorous adjustment. The position of the station on the off-lying island was determined with a radial standard deviation of ± 1.0 metre.

In May 1961 the Hydrographic Establishment of the British Admiralty made some tellurometer distance measurement tests from a fixed point on shore to a survey launch. These were quite successful with the launch either tied up to a buoy or underway at speeds less than four knots. Errors on the order of one to two feet were reported.

For several years, the U. S. Naval Oceanographic Office has used Two-Range Decca and Decca/Lambda electronic positioning systems to control hydrographic surveys. Before operations commence these systems must be calibrated in order to remove fixed errors inherent in each pattern. To eliminate these errors, a distance must be measured from the electrical centre of the shore station to the electrical centre aboard ship. Until 1960, this measurement was done optically. Three theodolites were set up over known points on shore and a series of angles observed to position the ship by triangulation. About 12 hours of daylight were required to observe the angles necessary to fix the position of the ship at several points in the vicinity of one station. The computations occupied another 48 hours. To calibrate one ship at both shore stations often required four to six days.

To reduce this non-productive time, a new method of calibration was developed. In 1959 and 1960 experiments were made to check the feasibility of calibration by tellurometer. These experiments were successful and the new method was used in the calibration in the summer of 1960. Since then, this method has been used more than 20 times, each time with satisfactory results.

In addition to saving time, this method also saved manpower. It was possible to reduce the calibration party from four men to two. Time was not only saved in observing, but the computation time was drastically reduced. Because the tellurometer measured the desired distance directly, the only computation required was to translate the tellurometer travel time into the "lane count" of the phase meters for direct comparison. There were further savings because the tellurometer could carry on calibration during periods of fog or haze that precluded visual work. The system can be calibrated with the vessel further out to sea. This advantage reduces inaccuracies from the effect of the land/water boundary on the low frequency signals.

The trials made in April 1960 were from a barge at anchor. The barge was positioned by sextant over a period of several hours. The distance from the barge to a station on shore was measured by tellurometer. The distance determined from the mean of the sextant was 1 232.8 metres ± 12.2 metres. As determined from a mean of the tellurometer measurement it was 1 236.1 metres ± 6.7 metres. This agreement was considered adequate. The standard deviation of 6.7 metres is equivalent to 0.016 Decca red lanes. This is about the same as the standard deviation of a Decca fix. Consequently these trials indicated the feasibility of Decca/Lambda calibration by this method. However, there is no indication of the real accuracy of the tellurometer measurements over water to a moving vessel.

To find out what kind of accuracy could be obtained between shore and a moving vessel, tests were made in the Patuxent River, Maryland. The observations were made on the 20 and 22 of June, 1962 by a party from the U. S. Naval Oceanographic Office. The results were analyzed by the writer.

Field observations were made with three azimuth instruments set up at convenient points on the shore. They were used to position a sounding launch by triangulation. The tellurometer master unit was in the launch. The remote unit was set up at 3.00 metres eccentric from the centre azimuth station. Figure 2 shows the layout of these test measurements.

At each fix, the angles were read to the sounding launch from the three stations. Simultaneously, the tellurometer readings were made in the same manner as had been used in the ship/shore calibration. The sequence of readings was A, D, C, B, A, and the difference between the first and last A readings was pro-rated for use with B, C and D.

On these tests regular meteorological observations were not made, because of meteorological equipment troubles. Corrections for index of refraction were taken from bihourly data logged at the control tower of the Naval Air Station adjacent to the operation. In the usual ship/shore calibration meteorological data are taken on board ship only at the beginning and end of a series of observations. In observations made ashore it is customary to take the mean of observations made at both ends of the line. In the case of ship/shore measurements, the data at the ship end is considered to be more nearly representative of conditions existing over the entire water path.

In figure 2, the distance from S to T was determined in two ways : first, by direct tellurometer measurement, (D_T) and second, by computation, (D_C). The common side, Z, of the two triangles was computed from each triangle, and a mean value derived for each measurement. D_C was computed from the expression :

$$D_C = \sqrt{Z^2 + 9 - 6Z\cos(180 - B)}$$

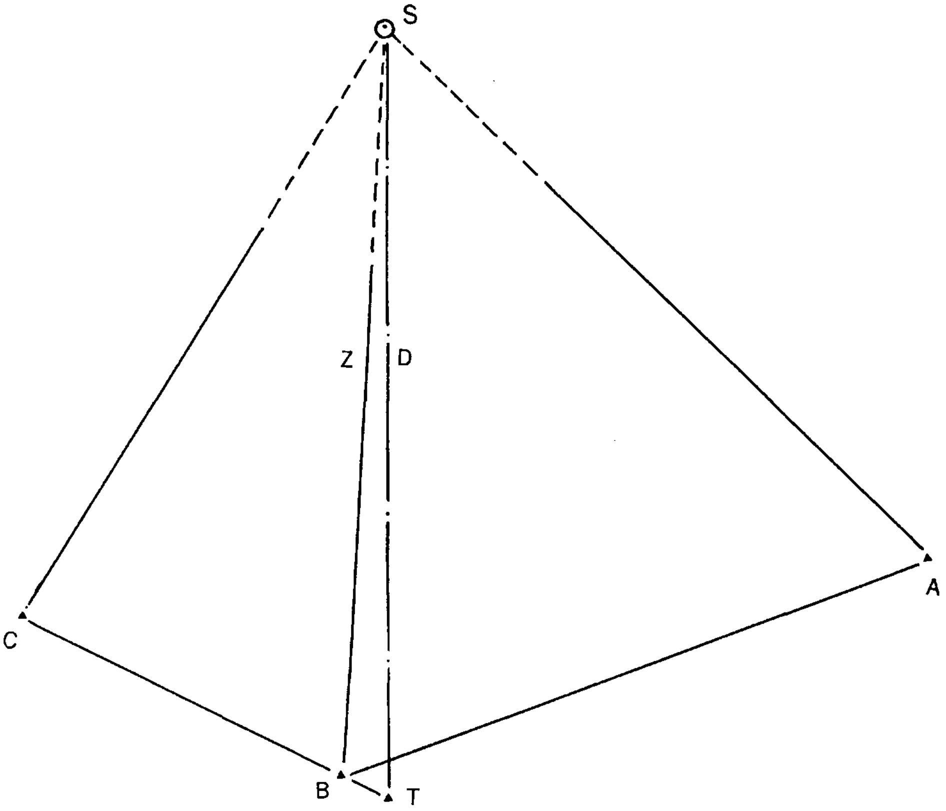

FIG. 2. — Layout of test measurements.

The angles A, B and C are observed; the distance ST = D_T is measured by Tellurometer; the distance SB = Z is computed from each triangle and a mean value of Z is used to compute D_C from the expression :

$$D_C = \sqrt{Z^2 + 9 - 6Z\cos(180 - B)}$$

The Tellurometer setup 3.00 metres from B on the extension of line CB. Line CB = 1 348.308 metres; line AB = 2 391.487 metres.

For each observation a D_C and a D_T are obtained. The most significant function of these two measurements is their difference. Therefore, the difference D_C minus D_T was derived from each observation.

A preliminary analysis was made of the data without correction for index of refraction. This analysis showed that some of the readings contained ambiguities. Some of these ambiguities could be resolved by application of 100, 1 000 or 10 000 millimicroseconds. Ten observations were rejected. Two of these ten were rejected because an azimuth angle was missed; the other eight were rejected because of unsolved ambiguities. Ambiguities were resolved for nine observations. The mean difference, without correction for index of refraction, was found to be about 0.9 metre.

TABLE I

Analysis of Differences in Distance
Computed Minus Tellurometer
(Corrected for Index of Refraction)

	First Day Observations	Second Day Observations	All Observations
Number of Observations	20	42	62
Observations Rejected	3	7	10
Observations Used	17	35	52
Maximum Positive Difference	1.68	2.33	2.33
Maximum Negative Difference	0.65	1.54	1.54
Maximum Spread of Difference	2.33	3.87	3.87
Arithmetic Mean of Differences	0.691	0.369	0.474
Standard Deviation of a Single Observation	0.630	0.907	0.723

A second analysis was made, after corrections for index of refraction had been included. Table I gives the results of this final analysis. It will be noted that the two days have been handled separately. This is because different operating procedures for the survey launch were used on the two days. On the first day, 20 June, the vessel maintained a more or less constant course and speed during observations. Speed was held to between two and four knots. The second day, 22 June, the launch was operated so that a series of observations were made with the launch laying to, after several observations the launch was moved to a new position. No observations were made during the runs between each series.

A plot of the recorded differences, distributed by the length of the line, is shown in figure 3. The observed differences were sorted by the computed distance into 100 metre groups, 500 to 599, etc. Again a distinction has been made between the first and second day's work. The dashed line is drawn as a rough mean between the plotted points. Distribution of the points is believed to be random in nature.

Although the mean value of the second day's observations is lower, the spread of observations is much greater, and, consequently, so is the standard deviation of a single observation. From these data, it would appear that more accurate measurements are made when the vessel maintains a more or less constant course and speed during observations. Because it is impossible to stop a vessel, a predictable, consistent motion gives better results than a smaller, but purely random, motion.

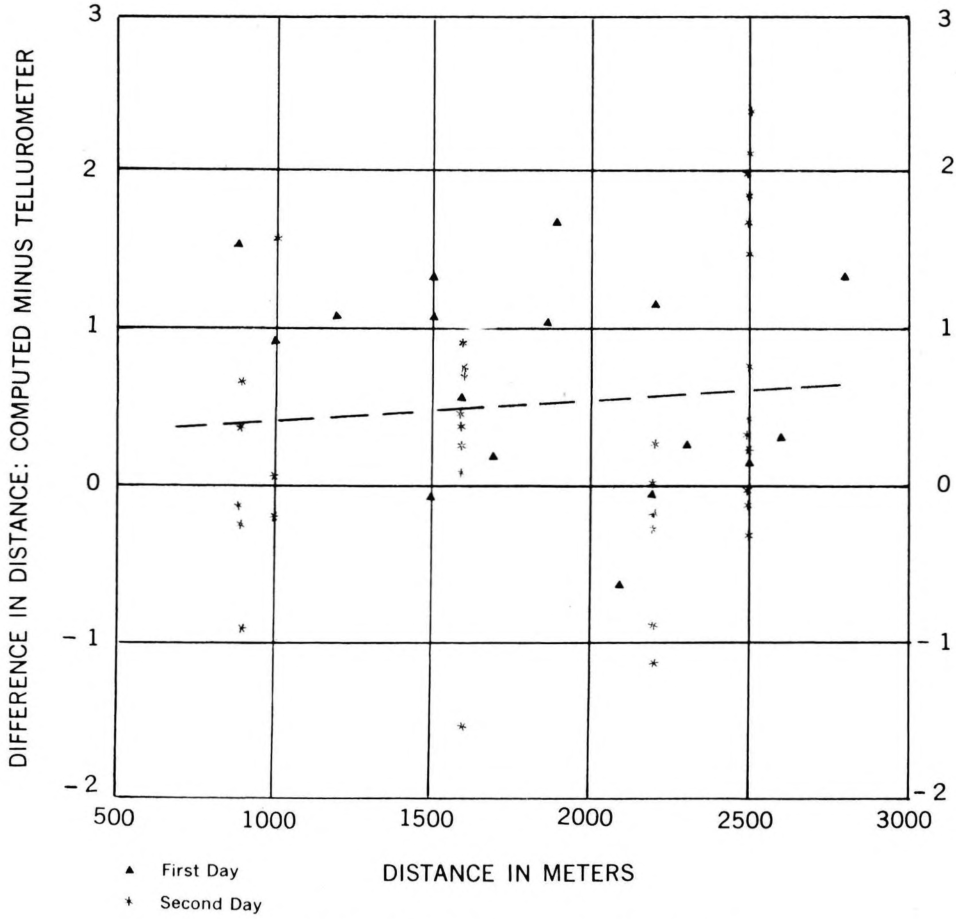

FIG. 3. — Distribution of observations by distance.

In conclusion, two items should be stressed :

First : Accurate tellurometer measurements can be made over water, if (a) care is taken to see that the terminal stations are not too high above the water surface, and if (b) the water surface itself is such as to minimize strong reflections.

Second : That measurements can be made from a fixed station to a moving vessel with a circular error of position of about 0.75 metre. If the usual meteorological corrections are not applied, the error will be higher than this value.

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