COMPUTATION OF HYPERBOLIC LATTICE FOR NAVIGATION CHART

Japanese Hydrographic Office

1. — Introduction

In 1959, the Japanese Hydrographic Office published tables and charts of Loran position lines (2 S 1 and 2 S 2) in the East of Japan, prepared from data computed with an IBM 704 electronic computer. Recently, the Office has completed the computation for a hyperbolic chart for a Hi-Fix survey system Type A in Tokyo Kaiwan, with a HIPAC 103 electronic computer. The Transverse Mercator projection being employed in this computation, the solution of hyperbolae was obtained on the grid of the projection and the longitude λ and latitude φ were transformed from x and y on the grid. Though x and y should be directly plotted on a UTM grid chart when such a chart is available, in this case they were plotted on one of the charts published by the Office in ordinary Mercator projection.

2. — Grid coordinates of the Master and two Slave stations

If the meridian through the master station is taken as the central meridian, then the Transverse Mercator coordinates, x and y, the scale k and the convergence γ, are given by the following equations of THOMAS (1952) (*) :

$$x = N \left[\Delta \lambda \cos \varphi + \frac{1}{6} \Delta \lambda^3 \cos^3 \varphi (1 - \tan^2 \varphi + \delta \cos^2 \varphi) + \right.$$
$$\left. + \frac{1}{120} \Delta \lambda^5 \cos^5 \varphi (5 - 18 \tan^2 \varphi + \tan^4 \varphi) \right] \quad (1)$$

$$y = S_\varphi + N \left[\frac{1}{2} \Delta \lambda^2 \sin \varphi \cos \varphi + \frac{1}{24} \Delta \lambda^4 \sin \varphi \cos^3 \varphi (5 - \tan^2 \varphi) \right] \quad (2)$$

$$k = 1 + \frac{1}{2} \Delta \lambda^2 \cos^2 \varphi (1 + \delta \cos^2 \varphi) + \frac{1}{24} \Delta \lambda^4 \cos^4 \varphi (5 - 4 \tan^2 \varphi) \quad (3)$$

(*) THOMAS, Paul D. : Conformal projections in geodesy and cartography. U. S. Coast and Geodetic Survey Special Publication No. 251, Washington, 1952.

$$\gamma = \Delta\lambda \sin\varphi \left[1 + \frac{1}{3}\Delta\lambda^2 \cos^2\varphi\,(1 + 3\,\delta\cos^2\varphi) + \frac{1}{15}\Delta\lambda^4\cos^4\varphi\,(2 - \tan^2\varphi)\right] \quad (4)$$

$$\delta = \frac{e^2}{1 - e^2} \quad (5)$$

φ = geographical latitude from the equator

$\Delta\lambda = \lambda - \lambda_0$ = the longitudinal difference from the central meridian λ_0 (6)

$N + a\,(1 - e^2\sin^2\varphi)^{-\frac{1}{2}}$ = the radius of curvature normal to the meridian at latitude φ (7)

a = the equatorial radius of the spheroid used

e = the eccentricity of meridian

S_φ = length of the meridian arc from the equator to the latitude φ

$$S_\varphi = \int_0^\varphi a\,(1 - e^2)\,(1 - e^2\sin^2\varphi)^{-\frac{3}{2}}\,d\varphi$$

$$= a\,(1 - e^2)\left[A\,\varphi - \frac{1}{2}B\sin 2\varphi + \frac{1}{4}C\sin 4\varphi - \frac{1}{6}D\sin 6\varphi\right] \quad (8)$$

In the case of BESSEL's spheroid :

$A = 1.0050373060$

$B = 0.0050478492$

$C = 0.0000105638$

$D = 0.0000000206$

The values employed in this computation are shown as follows, all of them being based on the BESSEL spheroid :

	Master (Tsurugi Saki)	Slave (Kannon Saki)	Slave (Okino Shima, Tateyama)
φ	35° 08' 17" . 0 N	35° 15' 00" . 5 N	34° 59° 17" . 0 N
λ	139 40 50 . 0 E	139° 45' 01" . 3 N	139 49 42 . 0 E
x	0 metre	6 352 . 21 m	13 391 . 72 m
y	3 889 552 . 57 m	3 901 958 . 47 m	3 872 893 . 04 m
Distance d between the master and the slave stations		13 964 . 33 m	21 413 . 56 m
Azimuth angle θ*		+ 65° 56' 92" . 0	- 50° 56' 57" . 5

(*) The angle θ is reckoned from the grid axis through the Master station, the angle northward from the grid axis being positive, and southward negative.

3. — Computation of hyperbolae

Computation becomes rather complicated, since the traverse axis of a hyperbola generally intersects the axis of coordinates at an arbitrary angle θ. Therefore, in order to make the computation feasible a new orthogonal (right hand) coordinate system (ζ, η), having the Master M as origin, is established, the ζ axis being taken through the origin and the Slave S. Then we have only to obtain the (ζ, η) of points on a hyperbola in these new coordinates, and to transform (ζ, η) back to (x, y) merely by rotating the axis through angle θ (Fig. 1).

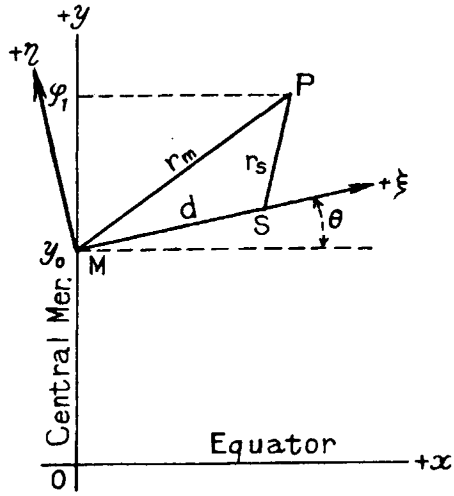

FIG. 1

Putting :

P = an arbitrary point on the hyperbola

$d = MS$

$r_m = MP$

$r_s = SP$

then,

$$r_m - r_s \mp C = \text{Constant} \qquad (9)$$

due to the nature of the hyperbolae, and

$$(\zeta^2 + \eta^2)^{\frac{1}{2}} - \left((\zeta - d)^2 + \eta^2 \right)^{\frac{1}{2}} = C \qquad (10)$$

From (10) we have :

$$\zeta = \frac{d}{2} \pm \frac{C}{2} \left(1 + \frac{4\,\eta^2}{d^2 - C^2} \right)^{\frac{1}{2}} \qquad (11)$$

or :

$$\eta = \pm \left[\frac{d^2 - C^2}{C^2} \left\{ \left(\zeta - \frac{d}{2} \right)^2 - \frac{C^2}{4} \right\} \right]^{\frac{1}{2}} \qquad (12)$$

C in the above equations is connected with the lane number as follows: let the lane number on the base line extension from the Master towards the opposite side of S (i.e. on the line $\zeta < 0$, $\eta = 0$) be 0, and then determine the lane number by the following formula, so that the lane number may increase in value towards the Slave from the Master :

$$d + r_m - r_s = \Lambda L \tag{13}$$

where :

L = lane number, and
Λ = wave length

Then from (9) and (13), we have :

$$C = \Lambda L - d \quad \text{or} \quad L = \frac{C + d}{\Lambda} \tag{14}$$

In the computation, Λ corresponds to 165.128 metres, which is the wave length established at the Hi-Fix frequency of 1 815 kc/s.

x and y are given by the coordinate transformation :

$$\left.\begin{aligned} x &= \zeta \cos \theta - \eta \sin \theta \\ y &= y_0 + \zeta \sin \theta + \eta \cos \theta \end{aligned}\right\} \tag{15}$$

where y_0 means y for the Master station, which is 3 889 522.57 metres in the actual case described in section 2.

4. — Geographic coordinates of the points on a hyperbola from grid coordinates

The following inverse formulae (THOMAS, 1952) were applied :

$$\varphi = \varphi_1 + \tan \varphi_1 \left[-\frac{1}{2} \frac{x^2}{R_1 N_1} + \frac{1}{24} \frac{x^2}{R_1 N_1^3} (5 + 3 \tan^2 \varphi_1) \right] \tag{16}$$

$$\lambda = \lambda_0 + \sec \varphi_1 \left[\frac{x}{N_1} - \frac{1}{6} \left(\frac{x}{N_1} \right)^3 (1 + 2 \tan^2 \varphi_1 + \delta \cos^2 \varphi_1) + \right.$$
$$\left. + \frac{1}{120} \left(\frac{x}{N_1} \right)^5 (5 + 28 \tan^2 \varphi_1 + 24 \tan^4 \varphi_1) \right] \tag{17}$$

$$k = 1 + \frac{1}{2} \left(\frac{x}{N_1} \right)^2 (1 + \delta \cos^2 \varphi_1) + \frac{1}{24} \left(\frac{x}{N_1} \right)^4 (1 + 6 \delta \cos^2 \varphi_1) \tag{18}$$

$$\gamma = \tan \varphi_1 \left[\frac{x}{N_1} - \frac{1}{3} \left(\frac{x}{N_1} \right)^3 (1 + \tan^2 \varphi_1 - \delta \cos^2 \varphi_1) + \right.$$
$$\left. + \frac{1}{15} \left(\frac{x}{N_1} \right)^5 (2 + 5 \tan^2 \varphi_1 + 3 \tan^4 \varphi_1) \right] \tag{19}$$

where :

$$N_1 = a (1 - e^2 \sin \varphi_1)^{-\frac{1}{2}} \tag{20}$$

$$R_1 = a (1 - e^2) (1 - e^2 \sin \varphi_1)^{-\frac{3}{2}} = N_1^3 \frac{1 - e^2}{a^2} \tag{21}$$

λ_0 = longitude of the Master station

φ_1 = foot point latitude, which is obtained by the following equation (*)

$$\varphi_1 = Ay + B \sin 2\,Ay + C \sin 4\,Ay + D \sin 6\,Ay + E \sin 8\,Ay \qquad (22)$$

where :

$$A = \frac{1}{a}\left(1 + \frac{1}{4}e^2 + \frac{7}{64}e^4 + \frac{15}{256}e^6 + \frac{579}{16384}e^8\right) \qquad (23)$$

In the case of the BESSEL spheroid, we then have :

$A = 1.5706619151 \times 10^{-7}$ rad.

$B = 0.00251127324$ rad.

$C = 0.00000367879$ rad.

$D = 0.00000000738$ rad.

$E = 0.00000000002$ rad.

5. — Computation and chart construction procedure

The entire computation for a pair of Master and one of the Slave stations, was carried out in the following steps :

a) Determine the grid coordinates x and y for each of the Master and Slave stations through equations (1) - (2) and compute the mutual distance d and the inclination θ of the ζ-axis to the grid axis through the Master station. Thus we have the basic values for the computation.

b) Next compute the coordinates of the points on a hyperbola in the (ζ, η) system through equation (9). Here the numerical computation was carried out for values of C corresponding to every five lanes and for every 500 metres for either ζ or η. For the parts of the hyperbolae farthest from their foci, computation was made for every two lanes.

c) Transform the coordinates from (ζ, η) to (x, y) through the equation (15); thus we have the coordinates (x, y) of each point on a hyperbola.

d) From x, y and φ_1, the foot point latitude given by equation (22), we finally obtain the geographic longitude λ and latitude φ of the points on the hyperbola following equations (16) and (17). We can also obtain the scale k and convergence γ from equations (18) and (19) respectively.

All these computations have been carried out with an electronic automatic computer for the final results λ and φ, with the final decimal reduction to 0′.001, and the errors contained in the final results are expected to be about ± 0′.001.

Using λ and φ of each lane obtained as mentioned above, hyperbolae were drawn and printed on the chart with a scale of 1/52 000, each range of hyperbolae for a respective pair of Master and Slave stations being distinguished from the other pair by different colours.

(*) The equation is due to KUROIWA (1952), Asia Koku-Sokuryo Co. (Private communication).

6. — Remarks

This computation and chart construction is only experimental, and it has been found that our present method is rather inconvenient for mapping, since the final results λ and φ are given at unequal intervals and this makes the drawing of hyperbolae quite complicated. This point is to be studied in the future.

In future, a chart with a scale of 1/10 000 giving one lane interval is to be prepared for special survey projects.