TELLUROMETER CYCLIC ZERO ERROR(*)

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Introduction

Cyclic variation of the tellurometer zero error was suggested in 1960 by Poder, Lehn and Bedsted [1]. The cyclic portion of the error was assumed to be a sine curve, with a period of 50 units of fine reading or 50 millimicroseconds. Thus a fine reading of 100 represents 720°, and the error may be expressed as :

$$E = b_1 + b_2 \sin (7.2A)^\circ$$

The first check indicated values of +3 cm and +8 cm for b_1 and b_2 respectively. A somewhat more thorough test followed, resulting in the values + 1.8 and +4.8. Further work, as reported by Bedsted in 1962 [2], confirmed the existence of the cyclic error, and gave results differing only slightly from the previous values.

Besides the Danish work, investigation of the matter has been carried out by the Geographical Survey of Norway, the Swedish Geographical Survey, the Ordnance Survey of Great Britain, and the Geodetic Survey of Canada, but very little appears to have been published. A preliminary report of Canadian work was included in a paper by C. D. McLellan [3], and a report of the British work was presented to the Commonwealth Survey Officers' Conference, 1963 [4]. The purpose of the present paper is to give a more complete account of the Canadian work, including some tests carried out since the publication of McLellan's paper.

Instruments and Bases

The Canadian investigation has been carried out with three sets of instruments and on three base lines. The instruments may be designated I, II, III, as follows ;

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I : instruments MA 117 and RA 153, tellurometers Model 1;
II : instruments MA 283 and RA 397, tellurometers Model 1;
III : instruments MRA 573 and 708, tellurometers Model 2.

All were equipped with ovens for control of crystal temperature. In general the two instruments of set III were used alternately as master and remote; there was no indication that the result obtained for the length of line depended on which particular instrument was used as master.

The three base lines may be designated A, B, C, as follows :

A — lengths 394 to 408 metres in steps of 2 metres;
B — lengths 192 to 208 metres in steps of 2 metres;
C — lengths 240 to 247 metres in steps of 1 metre.

Bases A and B were laid out on fairly level grass land, and in measuring these lines by tellurometer the instruments were about 5 feet above ground level. Base C consisted of the roofs of two buildings approximately 50 feet above ground level, and perpendicular to each other. Any possible reflections of rays from the intervening ground surface were blocked by parapets on the two roofs, and there were no vertical surfaces in the vicinity such as walls of buildings which could be expected to reflect the tellurometer rays. To ensure that the parapets would block the possible reflection of rays from the ground surface, the instruments were set low — 49 inches and 28 inches above the two roofs. The different lengths were obtained by moving both instruments along the roofs parallel to the parapets so that all measured lines were parallel.

Control Measurements

The control measurements of Bases A and B were made by invar tape. Actually these two bases, of lengths 400 and 200 metres respectively, had been laid out previously for a different purpose and additional markers were placed to indicate additional lengths required for this investigation. Three of the lines of Base C were measured using two Model 4 geodimeters, with good agreement between the results obtained with the two instruments. Further checks were provided by angular measurements and by tape measurement of the short distances along the two roofs.

Tellurometer Measurements

Tellurometer measurements were made by our standard first-order procedure, involving the measurement of each line at 37 cavity settings spread all across the range of carrier frequencies. The mean of the 37 readings was accepted as one measurement of the line. The adopted tellurometer length of the line, used to study the cyclic zero error, was the mean of several such measurements, taken on different days. Meteorological readings were taken at the master station only, which was quite justifiable in view of the shortness of the line.

For any one line measured with any one set of instruments, individual measures which differed from the mean by more than 10 centimetres were rejected. Following rejections, the simple mean of accepted values was taken as the adopted length for Bases A and C. For Base B a system of weighting was used, values close to the mean being given greater weight than values differing from the mean by several centimetres. As a result of these procedures we have one adopted tellurometer length corresponding to each set of instruments on each line. Thus we have three sets of adopted tellurometer lengths for the lines of Base A, since the lines of this base were measured with all three sets of instruments.

On Base A with instrument set I, 7 measures were made of each of 8 lines in the autumn of 1960. In the autumn of 1962 one additional set of measurements was made of the 8 lines, and one or two extra measurements were made on some of the lines. Applying the standard of rejection mentioned above, it was found that several rejections would be necessary for measurements taken on two specific days (October 5 and 24, 1960). Practically all the measures made on these two days, even if within the rejection limits, were appreciably shorter than other measures of the same lines. Consequently all the work done on those two days (one measurement of each of the 8 lines on each day) was discarded. No other discards were necessary and the adopted tellurometer lengths were the means of from 6 to 8 measurements of each line.

On the same base, with instrument set II, 4 measurements were made of each of the 8 lines in the autumn of 1960, and 4 more measurements of each line, with two or three extra measurements of 2 lines, were made in the autumn of 1962. One rejection was necessary, and the adopted tellurometer lengths were based on 8 to 10 measurements of each of the 8 lines.

Using instrument set III on the same base, 6 measures of each of the 8 lines were made in the autumn of 1962. No rejections were necessary.

On Base B, 4 measures of each line were made with instrument set I in the autumn of 1960, 4 measures with intrument set II in the spring of 1962, and 2 measures with instrument set III in the autumn of 1962. No rejections were necessary on any of these measurements.

Base C was measured with instrument set III only, in the spring of 1963. From 6 to 8 measurements were made of each of the 8 lines. Only one rejection was necessary.

Analysis

The data gathered from Base A were analysed rather thoroughly, the measurements with the three sets of instruments being treated separately. The analysis was based on the assumption that the curve of error versus

A-reading was a sine curve with a period of 50 millimicroseconds. Whereas PODER, LEHN and BEDSTED tacitly assumed that the mean value of the error curve corresponded to an A-reading of zero, we did not include this assumption, but expressed the error as :

$$E = b_1 + b_2 \sin [7.2 (A + b_3)]$$

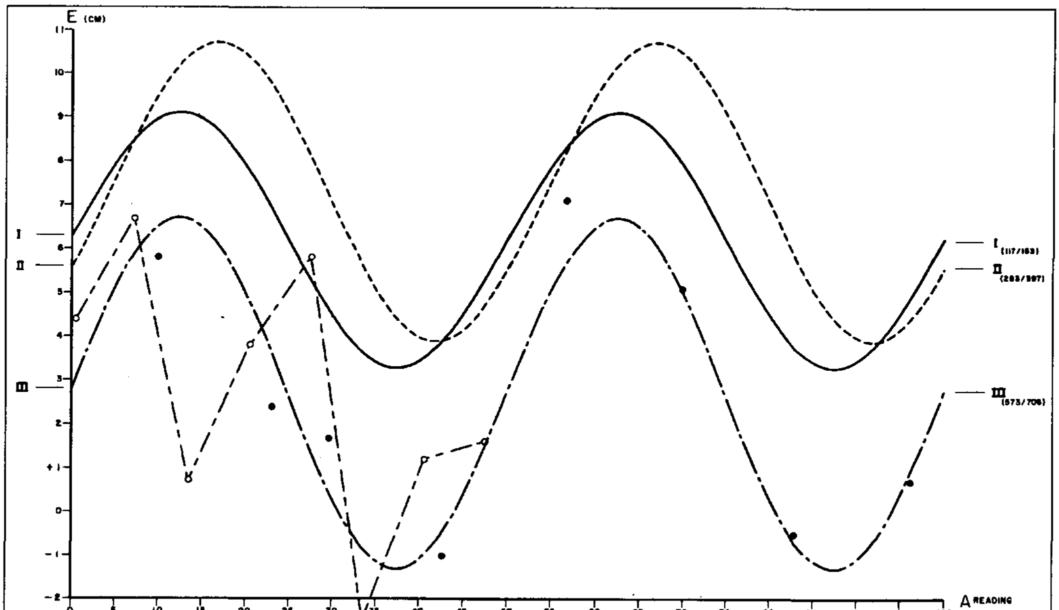
Values of b_1, b_2 and b_3 were obtained by the usual least squares treatment, after transformation of the observation equation as follows :

$$E = b_1 + b_2 \cos 7.2b_3^\circ \sin 7.2A^\circ + b_2 \sin 7.2b_3^\circ \cos 7.2A^\circ$$
$$= x + y \sin 7.2A^\circ + z \cos 7.2A^\circ.$$

Normal equations were set up and solved for x, y and z, after which b_1, b_2 and b_3 were determined as follows :

$$b_1 = x$$
$$b_3 = [\tan^{-1} (z/y)]^\circ/7.2$$
$$b_2 = y \sec 7.2b_3^\circ = z \operatorname{cosec} 7.2b_3^\circ$$

The solution was carried through for the three sets of instruments separately, with the results shown in Table I. These results are also shown graphically in figure 1. Individual errors from which the curve for instrument set III was derived are shown as solid circles.



Tellurometer Cyclic Zero Error Curves
Individual errors shown for Instruments 573/708

FIG. 1

The data gathered from Base B with instrument sets I and II were treated similarly, and the results are shown with the Base A results in Table I. Agreement between the results from the two bases and with the three sets of instruments is satisfactory. Measurements on Base B with instrument set III were insufficient to warrant careful study, but the same type of curve was indicated with a mean error of +5.6 centimetres.

Data gathered on Base C were not analysed by least squares, but the results are shown in Table II. The mean error, corresponding closely to b_1, is —8.9 centimetres, compared with +2.7 centimetres as determined for the same instruments on Base A. All the results from Bases A and B agree together much more closely than this. The cyclic portion of the error as determined from Base C is shown by open circles in figure 1. To obtain the amended errors so plotted, all errors as tabulated were increased by 11.6 centimetres to give a mean of +2.7 centimetres, agreeing with the plotted sine curve. These errors as plotted do not agree with the curve, but are slightly suggestive of a sine curve with a period of 25 millimicroseconds. Thus results obtained from Base C do not agree with work on Bases A and B.

TABLE I

Deduced Tellurometer Errors (cm)
Bases A and B

Base A

Inst.	I :	$E = +6.2 + 2.9 \sin [7.2 (A + 0.1)]^\circ$
"	II :	$E = +7.3 + 3.4 \sin [7.2 (A - 4.3)]^\circ$
"	III :	$E = +2.7 + 4.0 \sin [7.2 (A + 0.2)]^\circ$

Base B

Inst.	I :	$E = +2.2 + 3.6 \sin [7.2 (A + 0.0)]^\circ$
"	II :	$E = +3.5 + 3.7 \sin [7.2 (A + 0.1)]^\circ$

TABLE II

Tellurometer Errors
Base C, Inst. III

A-reading	*E* (cm)
0.5	— 7.2
7.3	— 4.9
13.6	— 10.9
20.6	— 7.8
27.5	— 5.8
33.6	— 14.1
40.6	— 10.4
47.4	— 10.0

Tests of Results

A table of tellurometer zero corrections was prepared, based on the results of analysis of data from Base A, and corrections were applied to results of measurements on other lines. Twenty-five lines had been measured with instrument set I, and also by tape or geodimeter. Nineteen of these lines were under 500 metres in length and 6 lines were longer than this (the longest 21.1 kilometres). On the whole, application of the cyclic zero correction increased the tellurometer errors. There was no sharp distinction between long and short lines. Seven lines had been measured with instrument set II and also by tape or geodimeter. Of these 7 lines, 4 were under 400 metres in length and 3 lay between 13 and 21 kilometres. Application of the cyclic correction resulted in an improvement in the zero errors. These tests are shown in tabular form in Table III.

TABLE III

Tests of cyclic zero corrections

	E_1	E_2
Instrument Set I, 25 lines :		
Maximum error	+ 11.7 cm	— 13.9 cm
Mean error, considering sign	+ 2.0	— 3.7
Mean error, without regard to sign	5.0	5.2
Instrument Set II, 7 lines :		
Maximum error	+ 18.8 cm	— 9.5 cm
Mean error, considering sign	+ 5.2	— 2.6
Mean error, without regard to sign	6.7	5.5

E_1 : error without application of cyclic correction.
E_2 : error after application of cyclic correction.

Ground Swing

The data gathered with instrument set III on Base A was studied further from the point of view of ground swing. The term " ground swing " is here used to indicate the variation of A-reading with change of carrier frequency. Theoretically, for a short line close to the ground the amplitude of swing should be zero. The mean ground swing for the 6 measures on each line was computed and the swing graphs are shown in figure 2. No systematic shift or change in swing curve from line to line which might explain the cyclic error can be detected. There is, however, a noticeable difference between the ground swing for the two halves of the carrier frequency range (cavity settings 1 to 5 $\frac{1}{2}$ and 5 $\frac{3}{4}$ to 10). Consequently the errors for the individual lines were recomputed on the basis of the two halves separately, and the results were plotted. This graph is not reproduced here. The indications are that b_1 would become + 2.2 and + 3.2 cm for the lower and upper halves respectively as compared with + 2.7 for the whole cavity range, and that b_2 and b_3 would not be greatly changed. Agreement between results from the lower and upper halves of the cavity range is much better than reported by BAKKELID [5], but his report referred to tellurometers Model 1, possibly without thermostats.

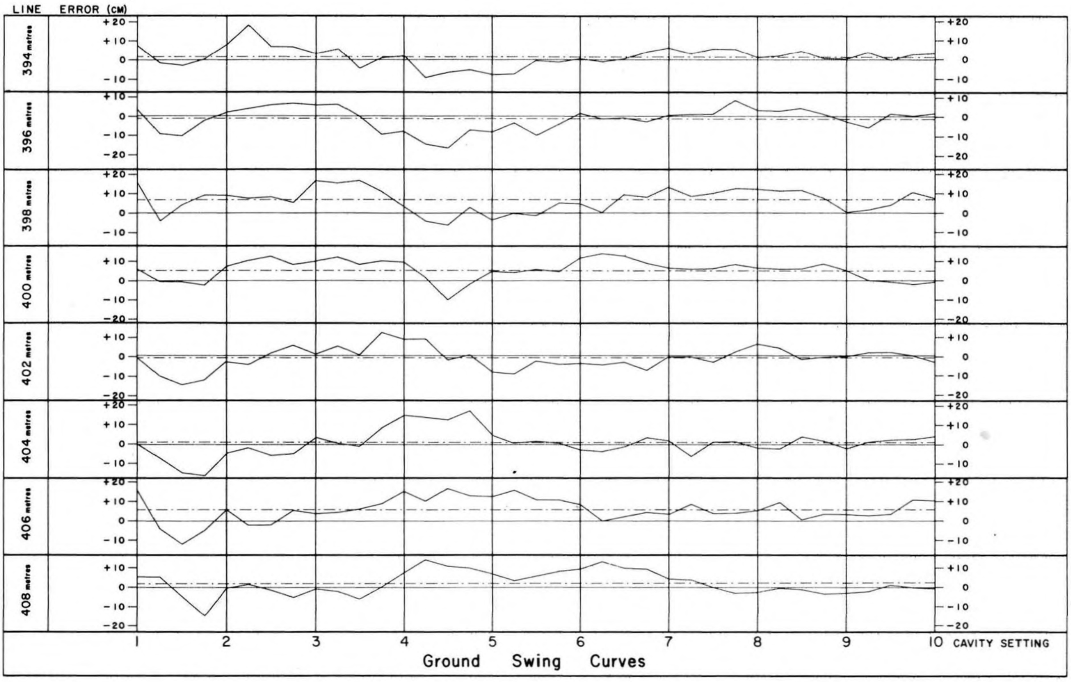


FIG. 2

Tellurometers Model 3

In the spring of 1963 the Army Survey Establishment acquired a pair of tellurometers Model 3. Several lines of known length, including 3 lines of Base C, have been measured with these instruments, and permission has been obtained to include in this paper a brief summary of the results obtained.

Twenty measurements were made on lines whose lengths had been determined with tape or geodimeter. Eighteen of these measures were on lines between 91 and 390 metres in length, and two were on a line about 4 700 metres long. Each measurement consisted of readings at 18 cavity settings spread all across the range of carrier frequency. For 13 measurements Instrument No. 142 was used as master, and for 7 measurements Instrument No. 147 was master. The indicated zero errors were — 5.1 and — 2.1 cm respectively. The greatest error indicated by a single measurement was 9 cm. The measurements taken on the 3 lines of Base C were too few in number to give reliable results, but they indicated that the cyclic error was much smaller than in the case of Model 2 instruments, if it was present at all.

Discussion of Results

This investigation has not produced a formula which may be applied to tellurometer measurements of short lines to give precise lengths. It confirms previous reports that the zero error of the tellurometer is not constant, even for a single set of instruments, but serious doubt is thrown on the assumption that the zero error is truly a periodic function of the A-reading. It appears rather that the zero error may vary unpredictably from one line to another. There is no suggestion that errors are likely to be serious, even by geodetic standards, on lines more than 10 kilometres in length, provided proper measuring procedures are followed, and it appears that errors are not likely to exceed one part in 5 000 on lines longer than 500 metres. The small amount of work done with tellurometers Model 3 suggests that this model has a zero error smaller and more nearly constant than that of Models 1 and 2.

References

[1] Poder, K., Lehn, C.J., and Bedsted, O. : Some Preliminary Results and Considerations of Tellurometer Investigations. IUGG General Assembly, Helsinki, 1960.

[2] Bedsted, O. : A note on the Cyclic Error. Tellurometer Symposium, London, 1962.

[3] McLellan, C.D.: The Tellurometer in Geodetic Surveying. *Canadian Surveyor*, Vol. XV, No. 7, March 1961.

[4] Gardiner-Hill, R.C. : Refinements of Precise Distance Measurement by Tellurometer. Commonwealth Survey Officers' Conference, Cambridge, 1963.

[5] Bakkelid, S. : Preliminary Results of Studies of Some Tellurometer Problems. Geographical Survey of Norway, *Geodetic Publication* No. 13, 1962.