THE USE AND EVALUATION OF HI-FIX IN A CANADIAN HYDROGRAPHIC COASTAL SURVEY(*)

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Introduction

During the past decade the outstanding success of the Decca 6f system for positioning offshore hydrographic surveys has emphasized the need for an even more accurate system which could be used inshore with smaller vessels. The ideal equipment would be capable of providing positions accurate to about \pm 6.5 feet (\pm 2 metres), and greater accuracy than this would hardly be needed at the charting scales used. The desirable range for Canadian waters is about 25 nautical miles (46 kilometres), although greater ranges would of course be acceptable. Lighter weight equipment for portability is also an objective.

To meet this need the Decca Company has produced the Hi-Fix system, and two sets of this equipment were purchased in 1961 by the Canadian Government to assist our hydrographic survey operations. One was used successfully in the Canadian Arctic by the Polar Continental Shelf Project; the other was sent in 1962 to Bedford Institute of Oceanography for evaluation and subsequent use on standard hydrographic surveys along the Nova Scotia coast or elsewhere in the region.

This is a report of the trials made with the latter set in 1962 by the Canadian Survey Launch Party Anderson in charge of Mr. L. P. MURDOCK, Hydrographer. It includes only brief descriptions of the system and of the technical problems concerning operation and maintenance. For these details, reference is made to a report (890-1052/TR, July 1962) published by Computing Devices of Canada Limited, P. O. Box 508, Ottawa, Canada.

The system used was a Hi-Fix Duplex, Lane Identification Chain.

^(*) This report was issued by the Bedford Institute of Oceanography, Dartmouth, Nova Scotia, Canada, and has received only limited circulation.

"Duplex " in the name refers to the duplication of equipment in order that lane identification may be achieved, as will appear later.

The Hi-Fix frequencies are in the two megacycle range, in our case specifically 1 900 kc/s and 1 710 kc/s, and it is noted that these frequencies are considerably higher than the 70 to 130 kc/s range which is used for the Decca 6f and Lambda 12f systems.

The equipment is well designed and constructed, and the entire assembly for one slave station can be carried in a small truck or van.

Measuring by Hi-Fix

It is well known that radio waves in the lower frequencies tend to follow the earth's surface, and the range at which they may be used is not limited to the intervisible distance between stations. At the other end of the spectrum, very high frequency waves travel in nearly straight lines and their effective range is line-of-sight. Waves of intermediate frequency have in this regard intermediate characteristics. This explains why the effective range of Hi-Fix is shorter than that of the Decca 6f or 12f systems.

Hi-Fix is a high precision radio location system using the phase comparison of master and slave signals to measure the distance between them. Thus, concentric circles about the slave represent constant phase differences between the two signals. Unlike similar radio location systems, Hi-Fix basically requires only one operating frequency since successive slaves are interrogated on a time-sharing basis. The second frequency in the Duplex system is used for lane identification as described later, but it can also be used if necessary for measuring.

Another innovation is the use of an automatic phase comparison process for the two transmissions through the action of low-inertia servo-goniometers capable of generating sufficient torque, and of measuring without error of linearity a phase difference of about one degree. The readout, or measure of phase difference, appears on a digital counter where unity is equivalent to half a wavelength or, as it is called, a lane. The smallest division of the counter (0.01 lanes) is equivalent to about 2.5 feet (0.76 metres).

If maximum accuracy is to be secured from the system, corrections have to be applied for the non-uniform speed of propagation of radio waves in the groundwave mode, and for fixed phase shifts. The full expression for measurement of the distance between the ship and a slave station therefore becomes:

$$d = \left(\lambda c \frac{f}{2}\right) (\varphi - \alpha - \psi)$$

where d is the distance from the electrical centre of the ship to the midpoint between the receiving and transmitting aerials at the slave station, $\lambda cf/2$ is the lanewidth in metres for the appropriate pattern, assuming free space velocity, φ is the observed Hi-Fix reading (whole lane number plus fraction), α is the locking constant and ψ is a correction to the free-space

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value of the speed of propagation depending upon the phase lag with distance, which results from absorption of energy by an imperfectly conducting earth.

The locking constant is the name given to the overall phase shift due to the close proximity of the receiver to the master transmitter (placing the former in the induction field) and, at the slave station, a possible fixed displacement from the nominal zero phase-difference condition that is assumed to exist between the received master signal and the outgoing slave transmission. The value of the locking constant for each pattern is found at the start of a survey by observations at known distances from the slave, and is thereafter subtracted from all observed Hi-Fix readings.

The sum of experience so far points to a mean wave velocity close to $299\,650$ km/s over sea water transmission paths, but this may fall as low as $298\,400$ km/s over land of low conductivity. In what follows we have used for distance measurements the expression :

$d = (\lambda E f/2) (\varphi - \alpha)$

where $(\lambda E f/2)$ is a close approximation of the mean wave speed in the area under study and the other symbols are the same as above.

Lane identification

As noted before, two frequencies, 1 900 kc/s and 1 710 kc/s, are used in the Hi-Fix Type B Duplex for obtaining Lane Identification. The 1 710 kc/s frequency is 10 percent less than 1 900 kc/s and this provides the lane identification by way of a computer in the following manner.

Consider a distance of one lane on the higher frequency. This distance equals 0.9 lane on the lower frequency. Similarly, 5 lanes at 1900 kc/s equals 4.5 lanes at 1710 kc/s, and 10 lanes at 1900 kc/s equals 9 lanes at 1710 kc/s, etc. Thus, for any given distance, subtracting the 1710 kc/s readout from the 1900 kc/s readout gives a value of one tenth of the higher frequency reading. In this answer by moving the decimal one place to the right the lane reading of the higher frequency pattern is obtained. This in fact is what the Lane Identification Computer does and the answer appears on the digital readout of the Lane Identification unit.

But the computer is only concerned with lane fractions, as this is the only information recorded automatically by the receiver. Thus the resolution of ambiguity is restricted to 10 lanes (2 587 feet - 0.79 kilometres). In other words, should a failure occur, it is necessary to know the vessel's position within half a mile, or else obtain a fix by visual or other means before the correct number of integral lanes can again be set into the counters.

Area of operations

The place chosen for the work lies on the southwestern part of Nova Scotia between Yarmouth and Cape Sable Island (figure 1). Here rocks

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and shoals abound, dangerous tide rips prevail and, during summer months, fog is the order of the day. A major survey ship could not safely operate inshore from Seal Island but, since it lies about 15 nautical miles (28 kilometres) off the mainland, this island afforded good survey control for the evaluation trials.



FIG. 1. — Area of Operations

Survey vessel

The C. S. L. Anderson, a 45-foot (13.7 metres) launch, was used for the work (figure 2). This shallow draft vessel encountered no difficulties other than bad weather, and could operate safely in these shallow waters. Powered by twin diesel engines she makes about 9 knots in good weather.

Configuration

It was decided to test the Hi-Fix in the two-range mode as this produces somewhat better accuracy than the hyperbolic form. The two-range mode is also somewhat easier for the hydrographer to use as the position lines are concentric circles centred on the slaves and a simple diagram can be used for plotting positions.



FIG. 2

A comparison of the accuracy of repeatability in the two modes, as computed from theoretical considerations, is shown by figures 3 and 4.

However, anticipating the future need for using the system in hyperbolic form in order to operate multiple survey vessels, a third slave site was positioned at Point St. Ann (figure 4) and is available when needed.

Slave stations

A first consideration for any survey using electronic positioning methods is the choice of good slave sites. They should be close to the shore to minimize the wavepath over land and, so far as possible, there should be clear wavepaths over water from the slaves to all parts of the survey area. The sites should have good grounding qualities to avoid fluctuations in signal strength during wet or dry weather, and power lines which may cause electrical interference must be avoided. Easy access to the slaves by road is of course an advantage.

Good sites, which met these requirements as well as possible, were located and occupied at Chebogue Point (Slave 1) and West Head (Slave 2), in grassy fields where the soil would retain some moisture and not dry out too quickly. Slave 1 was about 75 feet (23 metres) and Slave 2 about 25 feet (7.7 metres) above sea level (figure 3).

The slave stations could have been left unattended for several hours

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FIG. 3. — Two Range Operation

FIG. 4. — Hyperbolic Operation

at a time but it was found more practical to have a man at each to charge batteries, switch equipment on and off as required, and report meter readings or failures to the surveyor by radio telephone.

Area of Coverage

The layout of a typical slave station is illustrated in figure 5.

Master station

In the two-range mode the master is placed aboard the survey vessel. On our small wooden vessel this involved a certain amount of experimenting and electronic adjustment before adequate signal strength could be obtained.

A high transmitter aerial was required and this 32-foot (9.8 metres) mast had to be stiffened by using an outboard spreader. The matter of properly grounding the wooden hull was eventually overcome by fixing a copper plate, 4 feet by 8 feet (1.2 metres \times 2.4 metres) in size, to the hull directly under the transmitter aerial.

These problems were considered normal, however, as it was the first time the Hi-Fix Duplex system had ever been used in two-range form and the set was made operational without much delay.

The placement of equipment aboard the vessel is shown in figure 6 and it is noted that the electrical centre was assumed to be a point slightly forward of the mid-point between receiving and transmitting aerials. This coincided with the position of the vessel's sounding tanks. The choice of



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this point is arbitrary as its true location is difficult to determine, and an allowance of ± 1 foot (± 0.3 metres) has been made in the results for possible inaccuracy.

Survey control

The electrical centre at slave stations is taken to be the mid-point between transmitting and receiving aerials, and upon the positions of these points with relation to the other survey control in the area would depend the value of all our determinations.

The triangulation net here is based on a coastal tellurometer traverse which started and ended at stations previously established by the Canadian Geodetic Service. This traverse was subsequently balanced, closure was good and it proved to be of Second Order accuracy (errors not exceeding 1 part in 20000). The supplementary triangulation interconnecting the



FIG. 6

slave stations and the survey stations on Seal Island satisfies Third Order requirements (errors not exceeding 1 part in 10 000).

As a further safeguard, all the survey data were submitted for assessment to the Nautical Geodesy Section of Canadian Hydrographic Service. From this review it was concluded that the probable error in the distances as surveyed between the boat, when off Seal Island, and the slave stations is about ± 3 feet (± 0.91 metres), but it was noted that the maximum error might be three times this amount. In cases where the boat was relatively close to the slaves, the probable error in the surveyed distances was estimated to be in the order of ± 2 feet (± 0.61 metres), with possible maximum errors again three times this amount.

Procedure

First the survey control for slave sites was completed. Then, while the slaves were being erected, and electronic equipment installed, the field

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sheet was plotted and the position circles drawn in preparation for the work which was to follow evaluation trials.

The installation, adjustment, improvement and final testing of the Hi-Fix required about two weeks. When it was finally ready and declared operational by the technicians, it performed extremely well and was almost entirely trouble-free until the conclusion of the work.

Range of Hi-Fix

In August, 1963, a distance run to test the equipment was finally managed. During a period of good weather Mr. T. B. SMITH, Hydrographer, proceeded offshore until the launch was 65 nautical miles (120 kilometres) equidistant from the slaves. At this distance the signal strength was still fair and no loss of lock was experienced.

Plotting

For a half wave or lane we may write:

Lane width = $\frac{\text{Wavespeed}}{2 \times \text{frequency}}$.

For plotting purposes the use of wavespeed 299 650 km/s is satisfactory and this, together with frequency 1 900 kc/s, gives a lane width of 258.711 feet (78.855 metres).

The plan was plotted at natural scale $1/36\ 000$ and circles in multiples of 30 lanes radii were scribed around the slave stations. For plotting intermediate positions a small astrafoil diagram was used. On this was scribed a circle equal to 30 lanes and this in turn was subdivided into single lanes. Thus a plotting accuracy of about 1/20 of a lane was achieved, which was considered adequate for the work.

A study of the results which follow shows clearly that no plotting errors of significance at this scale were introduced by using these values, and no changes or corrections to the plotting sheet were necessary.

Only two survey officers are required to conduct sounding operations, one to plot and the other to read the counters and record notes. A seaman operates the sounding machine.

Distance measurements and computations

During May and June as weather allowed, nine observation series were completed (figure 7). With the vessel at anchor, the distance from its electrical centre to the mid-point between slave aerials was measured by tellurometer. These measurements were checked by theodolite angles from a shore base and, simultaneously, the Hi-Fix readings were recorded for comparison. Each series consisted of ten trials which were meaned, and the results are shown in Table 1. INTERNATIONAL HYDROGRAPHIC REVIEW SUPPLEMENT



FIG. 7. — Ship Station Locations

Hi-Fix distances with respect to each slave depend on two values, the lane width and the locking constant. The latter is different for each slave, whereas the former may be assumed to be the same for both slaves and looked upon as constant when no landpath is involved. If the distances being measured are small, a slight error in the assumed wavespeed will not be significant. This is not the case, however, for very long distances. For our purposes a wavespeed of 299 650 km/s, which is known to be a fair approximation, was assumed for determination of the locking constants, and the wavespeed was computed from available observations. Then, using the new value of wavespeed, the lane width was re-computed and this more exact value applied to the original Hi-Fix measurements. The Hi-Fix and survey measurements were then compared.

Details of these computations appear in Tables 2, 3 and 4. It is noteworthy that the locking constants, derived from ship stations close to the slaves, were slightly greater than the values obtained from more distant points, and the former were adopted. Also, the wavespeed, computed from four short lines and nine fairly long lines where no landpath was

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involved, was remarkably consistent and differed only slightly from the value originally assumed.

The data are, admittedly, not extensive enough and, since the probable measurement errors from survey and from Hi-Fix are of the same order, comparison is difficult. However, the various possible errors are listed below and they are all charged to the Hi-Fix system. By this device it is believed that an estimate of the greatest possible error of the electronic system has been stated which makes due allowance for all contingencies.

Probable error	Maximum error		
\pm 3 feet	\pm 9 feet		
\pm 2.2 feet	± 6.6 feet		
\pm 1.0 feet	\pm 2.0 feet		
\pm 3.0 feet	\pm 9.0 feet		
\pm 4.9 feet (1.5 metres)	± 14.5 feet (4.4 metres)		
	Probable error \pm 3 feet \pm 2.2 feet \pm 1.0 feet \pm 3.0 feet \pm 4.9 feet(1.5 metres)		

Conclusions

In two-range mode, under carefully controlled conditions where the wavepaths are over sea water and free from land, it appears that the Hi-Fix will deliver the suggested accuracy of \pm 6.5 feet (\pm 2 metres) over distances up to 15 nautical miles (28 kilometres). Under less well-controlled conditions, as when sounding, it is estimated that errors should seldom exceed 20 feet (6.1 metres).

The range of this equipment is certainly adequate for inshore work. As shown by the test run, it is effective up to at least 65 nautical miles (120 kilometres), and the indications are that it may be even better than this.

For relatively short distances, over sea water paths at Hi-Fix frequencies, the wavespeed is very close to 299 650 km/s and, it is noted that this is the value which has been used throughout our Canadian work. More measurements of the wavespeed over long sea water paths are needed so that a better assessment of phase lag with distance can be made and a suitable wavespeed value determined for such conditions.

More study could well be given to the effect on measurements of various percentages of land in the wavepaths. The position of such land, relative to master and slave, may also be significant.

The Hi-Fix equipment is rugged, well designed and reasonably portable. Electrical storms affected operations, but rain and salt spray in moderate amounts had no appreciable effect. The results were considered even more accurate than those which are obtainable with the Decca 6f and 12f systems although the maximum range is less. In general, Hi-Fix is considered to be an excellent aid for hydrographic surveys, particularly in areas where the highest possible accuracy is required.

TABLE 1

The measurements

Ship station	Surveyed (Fe	Distances eet)	Hi-Fix Readings (Lanes)		
	Slave 1	Slave 2	Slave 1	Slave 2	
Chebogue Pt. (June 23)	16 221.00	167 775.46	62.917	648.754 (*)	
Chebogue Pt. (June 23)	11 191.09	150 687.82	43.443	582.791 (*)	
West Head (June 21)	155 288.11	7 458.46	600.509 (*)	28.998	
West Head (June 27)	156 414.32	8 738.59	604.823 (*)	33.958	
Seal I. (West) (May 28)	122 499.30	104 189.61	473.670	402.935 (*)	
Seal I. (East) (May 28)	122 867.54	92 272.57	475.098	356.817	
Seal I. (East) (June 19)	124 352.46	92 195.52	480.846	356.532	
Seal I. (East) (June 28)	123 000.25	92 079.68	475.628	356.093	
Seal I. (East) (June 28)	123 373.51	92 621.76	477.044	358.184	

Note 1 :

(*) Lines with some landpath involved.

Note 2 :

Hi-Fix was referenced to zero. No locking constants were applied. Note 3:

Each value here is the mean of a ship station series of ten trials. Note 4 :

Hi-Fix frequency, 1 900 kc/s.

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TABLE	

Determination of locking constants

Lanes	.250 (*)	.336 (*)	.169	.181	.210 (*)	.155	.168	.176	.172
Feet	64.63 (*)	86.89 (*)	43.66	46.73	54.29 (*)	40.08	43.39	45.66	44.54
By Hi-Fix	167 840.09 (*)	150 774.71 (*)	7 502.12	8 785.32	104 243.90 (*)	92 312.65	92 238.91	$92\ 125.34$	92 666.30
By Survey	167 775.46	150 687.82	7 458.46	8 738.59	104 189.61	92 272.57	$92 \ 195.52$	92 079.68	92 621.76
Lanes	.218	.186	.272 (*)	.233 (*)	.172	.177	.185	.194	.167
Feet	56.35	48.11	70.45 (*)	60.32 (*)	44.56	45.76	47.91	50.16	43.24
By Hi-Fix	16 277.35	11 239.20	155 358.56 (*)	156 474.64 (*)	$122\ 543.86$	122 913.30	124 400.37	123 050.41	$123\ 416.75$
By Survey	16 221.00	11 191.09	$155\ 288.11$	156 414.32	122 499.30	122 867.54	$124\ 352.46$	$123\ 000.25$	123 373.51
	By Survey By Hi-Fix Feet Lanes By Survey By Hi-Fix Feet Lanes	By Survey By Hi-Fix Feet Lanes By Survey By Hi-Fix Feet Lanes 16 221.00 16 277.35 56.35 .218 167 775.46 167 840.09 (*) 64.63 (*) .250 (*)	By Survey By Hi-Fix Feet Lanes By Survey By Hi-Fix Feet Lanes 16 221.00 16 277.35 56.35 .218 167 775.46 167 840.09 (*) 64.63 (*) .250 (*) 11 191.09 11 239.20 48.11 .186 150 687.82 150 774.71 (*) 86.89 (*) .336 (*)	By Survey By Hi-Fix Feet Lanes By Survey By Hi-Fix Feet Lanes 16 221.00 16 277.35 56.35 .218 167 775.46 167 840.09 (*) 64.63 (*) .250 (*) 11 191.09 11 239.20 48.11 .186 150 687.82 150 774.71 (*) 86.89 (*) .336 (*) 155 288.11 155 288.11 155 358.56 (*) 70.45 (*) .272 (*) 7 458.46 7 502.12 43.66 .169	By Survey By Hi-Fix Feet Lanes By Survey By Hi-Fix Feet Lanes 16 221.00 16 277.35 56.35 .218 167 775.46 167 840.09 (*) 64.63 (*) .250 (*) 11 191.09 11 239.20 48.11 .186 150 687.82 150 774.71 (*) 86.89 (*) .336 (*) 155 288.11 155 358.56 (*) 70.45 (*) .272 (*) 7 458.46 7 502.12 43.66 .169 156 414.32 156 474.64 (*) 60.32 (*) .233 (*) 8 738.59 8 785.32 46.73 .181	By SurveyBy Hi-FixFeetLanesBy SurveyBy Hi-FixFeetLanes16 221.00 16 277.35 56.35 $.218$ 167775.46 $167840.09(*)$ $64.63(*)$ $.250(*)$ 11 191.09 11 239.20 48.11 $.186$ 150687.82 $15074.71(*)$ $86.89(*)$ $.336(*)$ 155 288.11 $15538.56(*)$ $70.45(*)$ $.272(*)$ 7458.46 7502.12 43.66 $.169$ 156 $474.64(*)$ $60.32(*)$ $.233(*)$ 8738.59 8738.59 46.73 $.181$ 122 249.30 122543.86 44.56 $.172$ $104.189.61$ $104.243.90(*)$ $54.29(*)$ $.210(*)$	By SurveyBy Hi-FixFeetLanesBy SurveyBy Hi-FixFeetLanes16 221.0016 277.3556.3556.35.218 167775.46 $167840.09(*)$ $64.63(*)$.250(*)11 191.0911 239.2048.11.186 1506775.46 $167840.09(*)$ $64.63(*)$.250(*)155 358.11155 358.56(*)70.45(*).272(*) 7458.46 7502.12 43.66 .169156 474.64(*) $60.32(*)$.233(*) 8738.59 8785.32 46.73 .181122 499.30122 543.86 44.56 .172 $92.372.57$ $92.312.65$ 40.08 .155	By SurveyBy Hi-FixFeetLanesBy SurveyBy Hi-FixFeetLanes16 221.0016 277.3556.35.218 167775.46 $167840.09(*)$ $64.63(*)$.250(*)11 191.0911 239.20 48.11 .186 1506775.46 $167840.09(*)$ $64.63(*)$.250(*)155 288.11155 358.56(*) $70.45(*)$.272(*) 7458.46 7502.12 43.66 .169156 414.32156 474.64(*) $60.32(*)$.233(*) 8738.59 8785.32 46.73 .181122 499.30122 543.86 44.56 .172 $104 189.61$ $104 243.90(*)$ $54.29(*)$.210(*)122 867.54122 913.30 45.76 .177 $92 272.57$ $92 2312.65$ 40.08 .155124 352.46124 400.37 47.91 .185 $92 195.52$ $92 238.91$ 43.39 .168	By SurveyBy Hi-FixFeetLanesBy SurveyBy Hi-FixFeetLanes16 221.0016 277.3556.35.218 $167 775.46$ $167 840.09 (*)$ $64.63 (*)$.250 (*)11 191.0911 239.20 48.11 .186 $150 687.82$ $150 774.71 (*)$ $86.89 (*)$.336 (*)155 288.11155 358.56 (*) $70.45 (*)$.272 (*) $7 458.46$ 7502.12 43.66 .169126 414.32156 474.64 (*) $60.32 (*)$.233 (*) $8 738.59$ $8 785.32$ 46.73 .181122 499.30122 543.86 44.56 .172 $92 312.65$ 40.08 .155122 499.30122 543.86 $1774 70 (*)$ $92 272.57$ $92 232.26$ 40.08 .155122 867.54122 913.30 45.76 .177 $92 272.57$ $92 232.65$ 40.08 .155124 352.46124 400.37 47.91 .185 $92 195.52$ $92 238.91$ 45.66 .168123 000.25123 050.4150.16.194 $92 079.68$ $92 125.34$ 45.66 .176

Note 1 :

299 650 km/s (assumed)	1 900 kc/s	258.7115 ft (78.8553 metres).	:	
:		ein		
:	:	her		
:	:	used		
speed	ency	width		
Wave	Freque	Lane 1	e 2 :	

Note

(*) Lines involving some landpath. These values were excluded.

Note 3 : Locking constants adopted are : Slave 1, 0.202 (mean of first two trials which were close to slave 1); Slave 2, 0.175 (mean of third and fourth trials which were close to slave 2).

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TABLE 3

Wavespeed computations

Wavespeed (km/s) = $\frac{2 \times \text{Frequency (kc/s)} \times \text{distance (metres)}}{2 \times \text{Frequency (kc/s)} \times \text{distance (metres)}}$

Lanes

Wavespeed					
To Slave 1 (km/s)	To Slave 2 (km/s)	Remarks			
299 574.5		Locking Constants used were :			
299 761.1		Slave 1, 0.202 Slave 2, 0.175			
	299 715.1				
	299 600.0	Frequency 1900 kc/s			
299 668.8					
299 665.9	299 666.8	Wavepaths obstructed by land were not used.			
299 660.5	299 656.1	-			
299 655.1	299 648.7				
299 671.9	299 652.3				

Note 1 :

Mean of 13 values by least square method is : 299 661 km/s.

Note 2 :

Lane width at this wavespeed is : 258.7209 feet (78.8582 metres).

TABLE 4

Distance comparison

Locking constants	Slave 1, 0.202;	Slave 2, 0.175
Wavespeed	299 661 km/s	
Frequency	1 900 kc/s	
Lane width	258.7209 feet (78.	8582 metres).

Ship to Chebogue Poir (Slave 1)

Ship to West Head (Slave 2)

Distance by Survey (feet)	Distance by Hi-Fix (feet)	Diff.	Distance by Survey (feet)	Distance by Hi-Fix (feet)	Diff.
16 221.00 11 191.09	16 225.68 11 187.35	+ 4.68 3.74			
	-		7 458.46	7 457.11	-1.35 $+ 1.78$
122 499.30 122 867.54 124 352.46 123 000 25	122 496.05 122 865.51 124 352.64 123 002 63	$\begin{array}{c c} - & 3.25 \\ - & 2.03 \\ + & 0.18 \\ + & 2.38 \end{array}$	92 272.57 92 195.52 92 079 68	92 270.73 92 196.99 92 083 42	
123 000.25	123 368.98	+ 2.38 - 4.53	92 679.08 92 621.76	92 624.40	+ 3.74 + 2.64

Note 1:

Lines involving landpath were not used.

Note 2:

Mean square error $\sqrt{\frac{108.286}{13-3}} = \pm 3.3$ feet (1.0 metres); Probable error .67 × MSE = ± 2.2 feet (0.7 metres).