NON-ITERATIVE DETERMINATION OF LATITUDE AND LONGITUDE FROM EITHER TIME OR PHASE DIFFERENCES

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Abstract

A method for the determination of latitude and longitude from hyperbolic coordinates is presented in this article. The hyperbolas may be expressed in either time differences or phase differences. The solution may be for two systems of curves having a common master station, or it may be for two independent systems each having its own master station. The new solution is compared to the iterative solution and, like this solution, an initial approximation for position is also required. Unlike the iterative solution, the initial approximation has little effect on the results in the non-iterative method, providing that the approximation is within three miles. More accurate transformations are obtained with less computer time.

Introduction

The current pratice in long range electronic positioning systems is to transform the time or phase difference values into latitude and longitude. The present conversion technique uses a computational procedure of successive approximation which is known as iteration. The iterative method has several shortcomings which the non-iterative solution will overcome. The principal disadvantage of the iterative solution is the computational time of the electronic computer. The time of the solution is contingent upon the desired accuracy. The desired accuracy is stated in terms of a difference limit such that when the difference between the last iterative solution and the observed value is within this limit then computation ceases. The time and phase difference curves represent variable distances in the hyperbolic network, and as a result of this, a stated time or phase limit for computational cutoff results in variable accuracy. The final answer in the iterative solution is further dependent upon the initial approximation of position; change the intial value and the final result will also change. The initial approximation in the new method should be within three miles for maximum accuracy; however, good results can be obtained even when the approximation is five miles in error. The solution presented in this article is both more accurate and more economical than the presently used iterative solution. The iterative solution could be made to achieve the same accuracy as the non-iterative method, but only at a great expense of computer time.

In the event of ambiguity of position for given hyperbolic coordinates, the operator in either method must resolve the uncertainty. Ambiguities occur in the area of baselines and baseline extensions, and rarely do they cause any inconvenience, since they are seldom in an area of interest.

Hyperbolic System with Common Master Station

Figure 1 is the general figure for both time and phase difference illustration where the following definitions are used :

M is the master or center station in the hyperbolic network :

X and Y are the slave or end stations;

H_x and H_y are the X and Y hyperbolas passing through point P;

P is an arbitrary point for which time of phase difference values have been observed;

P′ is an approximate position within several miles of P;

B_x and B_y are the respective geodetic baseline values, expressed in time or distance;

a, b and c are the geodetic distances from P to each station as shown;

a', b' and c' are the geodetic distances from P′ to each station as shown;

a_x and b_x are the values for a and b when P lies on B_x;

b_y and c_y are the values for b and c when P lies on B_y.

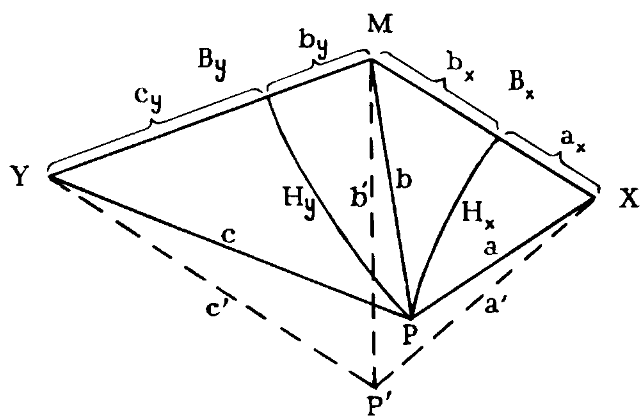

FIG. 1

Observation Equations

The technique by which a ship positions itself at sea by means of measurements with respect to shore based radio transmitting stations is analogous to a trilateration operation whereby several distance measurements are made simultaneously. In the trilateration operation the distances are measured directly, while in the time and phase difference operation the distances are measured indirectly. The observed hyperbolic coordinates correspond to the fixed though unknown distances from the point to each of the transmitting stations.

The general form of an observation equation to be used is :

$$x \sin \alpha_{P'\text{-}I} + y \cos \alpha_{P'\text{-}I} + I - S = 0 \tag{1}$$

where :

x and y are the differential corrections to be applied to P′ so that the position approaches P;

α is the geodetic azimuth from the south from P′ to either M, X or Y;

I is the distance a', b' or c' of figure 1;

S is the distance a, b or c of figure 1.

In the observation equation, I — S normally constitutes the residual. Now since S is the distance from P to each of the three stations, this distance is never known. As a result of the unknown value S, a unique observation equation was devised whereby part of the residual is treated as an unknown. Before the observation equations can be set up, the time or phase difference parameters are transformed into units of distance. The transformed values are then constants, for a given solution.

Phase-Difference Constants

The determination of constants to be used in the residuals for a phase-difference system follows :

$$a_x - b_x = \mathrm{XLW}\,(\mathrm{MH}_x - \mathrm{H}_x) - (\mathrm{B}_x - a_x) = k_x \tag{2a}$$

$$a - b = k_x$$

$$c_y - b_y = \mathrm{YLW}\,(\mathrm{MH}_y - \mathrm{H}_y) - (\mathrm{B}_y - c_y) = k_y \tag{3a}$$

$$c - b = k_y$$

where :

XLW and YLW are lane width values in metres along the baseline;

MH_x and MH_y are maximum phase difference values for X and Y along the baseline;

B_x and B_y are baseline lengths;

k_x and k_y are values in metres.

Time-Difference Constants

The determination of constants to be used in the residuals for a time-difference system is a follows :

$$a - b = \text{TDX}_o - \text{B}_x - \psi_x - \psi_a + \psi_b - \delta_x = k_x\,\mu\text{s} \qquad (2b)$$
$$a - b = (299.6929\ \text{m}/\mu\text{s})\,(k_x\,\mu\text{s})$$
$$a - b = k_x$$
$$c - b = \text{TDY}_o - \text{B}_y - \psi_y - \psi_c + \psi_b - \delta_y = k_y\,\mu\text{s} \qquad (3b)$$
$$c - b = (299.6929\ \text{m}/\mu\text{s})\,(k_x\,\mu\text{s})$$
$$c - b = k_y$$

where :

B_x and B_y are baseline lengths expressed in time;

ψ is the total phase distortion for composite path;

δ is the coding delay;

TDX_o and TDY_o are observed time-difference values;

$k_x\mu\text{s}$ and $k_y\mu\text{s}$ are values in microseconds;

k_x and k_y are values in metres.

Determination of Residuals

The following solution is the same for both systems.

The primed values a', b' and c' are all geodetic distances whose values are known, while the unprimed values are unknown. The three unknown values are listed as a single unknown b.

$$\text{I} - \text{S} = \qquad (4)$$
$$a' - a = a' - (b + k_x) = (a' - k_x) - b \qquad (4a)$$
$$b' - b = b' - b = b' - b \qquad (4b)$$
$$c' - c = c' - (b + k_y) = (c' - k_y) - b \qquad (4c)$$

Three observation equations are developed as follows :

$$x \sin \alpha_{\text{P}'-\text{X}} + y \cos \alpha_{\text{P}'-\text{X}} - b + (a' - k_x) = 0 \qquad (1a)$$
$$x \sin \alpha_{\text{P}'-\text{M}} + y \cos \alpha_{\text{P}'-\text{M}} - b + b' = 0 \qquad (1b)$$
$$x \sin \alpha_{\text{P}'-\text{Y}} + y \cos \alpha_{\text{P}'-\text{Y}} - b + (c' - k_y) = 0 \qquad (1c)$$

The three equations in simplified notation follow :

$$\text{A}x + \text{B}y + b + \text{C} = 0 \qquad (1\text{a})$$
$$\text{D}x + \text{E}y + b + \text{F} = 0 \qquad (1b)$$
$$\text{G}x + \text{H}y + b + \text{J} = 0 \qquad (1c)$$

Therefore :

$$y = \frac{\text{A}\,(\text{J} - \text{F}) + \text{C}\,(\text{D} - \text{G}) + \text{GF} - \text{DJ}}{\text{A}\,(\text{E} - \text{H}) + \text{B}\,(\text{G} - \text{D}) + \text{DH} - \text{GE}} \qquad (5)$$

and

$$x = \frac{C - F + y\,(B - E)}{D - A} \tag{6}$$

West longitude and north latitude are considered positive and east longitude and south latitude are considered negative. If it is desired to consider east longitude positive then the minus sign of x in (8) becomes positive.

The values x and y are converted to latitude and longitude in the following manner :

$$d\varphi'' = \frac{y}{M} \sin 1'' \tag{7}$$

$$d\lambda'' = -\frac{x}{N} \cos \varphi \sin 1'' \tag{8}$$

where :

$d\varphi''$ and $d\lambda''$ are the differential corrections, in seconds, to be added algebraically to P′ to obtain the geographic position of P;

M is the radius of curvature in meridian for P′ and is obtained by the equation :

$$M = \frac{a\,(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{3/2}} \tag{9}$$

where a is the semi-major axis of the earth and e is eccentricity;

N is the radius of curvature in the prime vertical for P′ and is obtained by the equation :

$$N = \frac{a}{(1 - e^2 \sin^2 \varphi)^{1/2}} \tag{10}$$

φ is the latitude for point P′.

Hyperbolic System with Separate Master Stations

When the master station is not common to both systems of hyperbolas, we have the result shown in figure 2. This presents no difficulty. An extra observation equation is developed and the following solution results :

$$x \sin \alpha_{P'-X} + y \cos \alpha_{P'-X} - b + (a' - k_x) = 0 \tag{1a}$$

$$x \sin \alpha_{P'-M} + y \cos \alpha_{P'-M} - b + b' = 0 \tag{1b}$$

$$x \sin \alpha_{P'-N} + y \cos \alpha_{P'-N} - d + (c' - k_y) = 0 \tag{1c}$$

$$x \sin \alpha_{P'-Y} + y \cos \alpha_{P'-Y} - d + d' = 0 \tag{1d}$$

In simplified notation :

$$Ax + By + b + C = 0 \tag{1a}$$

$$Dx + Ey + b + F = 0 \tag{1b}$$

$$Gx + Hy + d + J = 0 \tag{1c}$$

$$Ux + Vy + d + W = 0 \tag{1d}$$

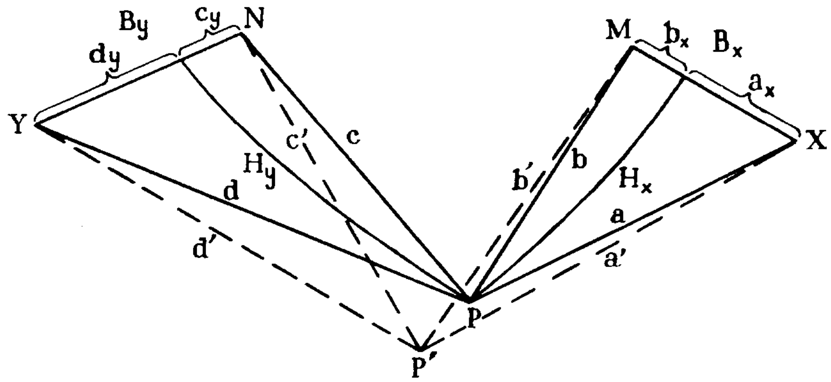

FIG. 2

Therefore :

$$y = \frac{(A - D)(J - W) + (C - F)(U - G)}{(A - D)(V - H) + (B - E)(G - U)} \tag{11}$$

and

$$x = \frac{C - F + y(B - E)}{D - A} \tag{6}$$

The remaining solution is then by (7) and (8), as before.

Comparison of the Non-Iterative Method with the Iterative

A comparison of the iterative process and the suggested solution was made on 109 navigational fixes taken from an actual ship's track during survey operations. The data consisted of fixes obtained by a time-difference electronic system at two-minute intervals for a period of several hours. An initial approximate position was given for the first position in the ship's track, and each successive required approximation was that of the final position of the previous fix.

The data from the ship's track were chronologically reversed and recomputed by both methods. This procedure maximizes the difference for the initial approximation of position for each computation.

A computational time comparison between the two methods is contingent upon the stated accuracy for computational cutoff in the iterative solution. The iterative accuracy, in the comparison, was 0.1 microsecond; the new solution resulted in accuracy of 0.003 microsecond. The non-iterative solution required 25 percent less computer time in this comparison.

The average differences of position obtained by comparing the positions computed with the same time-difference values between the ship's track and the ship's track reversed were as given in the tabulation, in units of nautical miles.

Differences for iterative method :

$$M \Delta \varphi = 0.076$$

$$M \Delta \lambda = 0.099$$

Differences for non-iterative method :

$$M \Delta \varphi = 0.002$$

$$M \Delta \lambda = 0.002$$

When the initial approximation is five miles in error, the accuracy of the new method is 0.05 of a nautical mile. When the approximation error is between five and fifty miles, then the new solution should be iterated one time. The tabular values listed for the 109 comparisons contained approximation errors of less than three miles.

Conclusion

The non-iterative method offers significant advantages in both economy and accuracy. The problem of obtaining variable accuracy, contingent upon location within the network, is eliminated. The initial approximation for position is not nearly as critical as it is in the iterative process. The method has no restrictions for operational range, other than those mentioned in the introduction.