USE OF A RANA H CHAIN IN THE WESTERN MEDITERRANEAN

by Captain Christian PERRIEN
Master Mariner
Monaco Oceanographic Museum
November 1964

Introduction

In 1961 the *Délégation générale à la recherche scientifique et technique* (General Delegation for Scientific and Technical Research) on the recommendation of the *Comité " Exploitation des Océans "* (Committee for Ocean Exploitation) decided while assembling national oceanographic equipment to acquire a radio positioning system capable of determining, with an accuracy far greater than that of current navigational means, the continuous and unambiguous fixes necessary for work at sea. By continuous and simultaneous recording of data (depths, temperature, etc.) and positions, such a system would specifically allow us to determine variations in the factors studied along desired courses or to follow their evolution in time at a given point to which periodically we could return with great accuracy.

The final choice was a hyperbolic Rana chain, type H, with two patterns and three sensitivities for eliminating lane ambiguity, manufactured by the *Compagnie des Compteurs.*

The Oceanographic Institute (Monaco Oceanographic Museum) was given the job of installing and operating this equipment for the benefit of scientific laboratories or organizations asking for it.

As the first step in its program the *Comité d'exploitation des océans* planned the coverage of a zone extending from the Riviera to Corsica and the concentrating of oceanographic and hydrographic research equipment in that region. For intensive operation of the chain we had on hand three receivers, all recorders.

The chain was set up at the beginning of the summer of 1962. Since then it has not been moved because it covers a large zone where its operation continues to be extremely useful. Thus there has been a long period during which the chain has been in almost constant use.

It is not necessary to go back over the characteristics of the Rana equipment or the general operational procedure for all maritime hyperbolic radio positioning systems which readers of this review know well. We give here only a brief account of the chain's history. We will try to bring out its working features and then to draw conclusions from our more than two years of experimentation.

Equipment

This equipment was described in Special Publication No. 39 of the IHB (Supplementary Paper 3). Our transmitting stations are set up on vehicles. Each slave transmitter is installed in a four-wheel trailer (figure 1) pulled by an all-terrain truck on the platform of which a shelter can easily be placed by means of a gantry crane. The shelter contains the free transmitter.

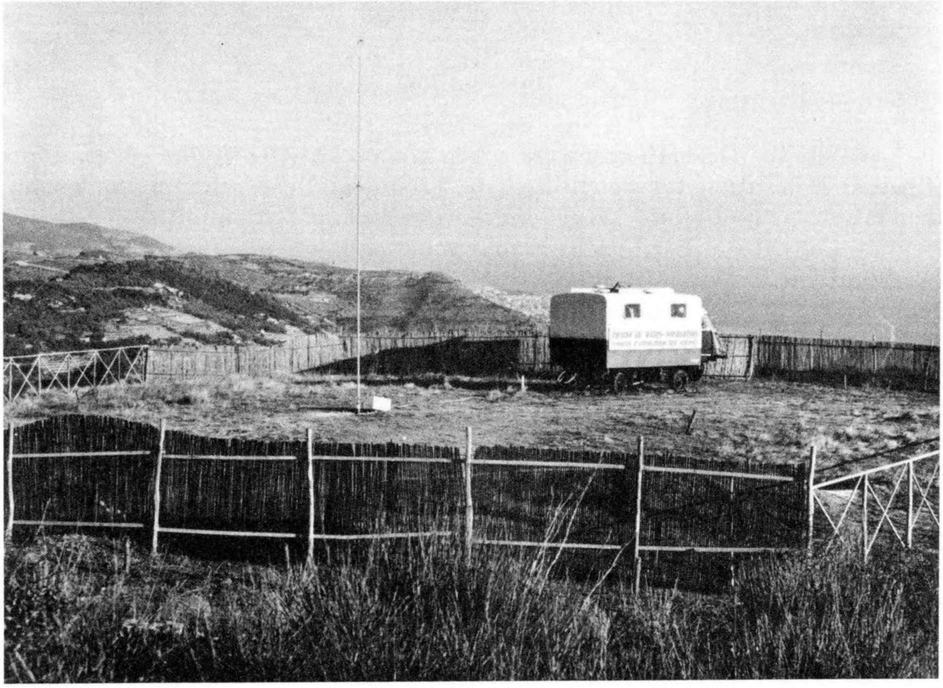

FIG. 1. — View of a slave transmitter.

This solution has advantages, providing of course that it is possible to find sites that can be reached by passable roads and terrain. On the other hand, these all-terrain trucks, whose purchase and maintenance are costly, cannot be used for transport of goods or personnel after their setting up at the sites.

In summer on the Riviera, efficient mechanical ventilation of the trailers must necessarily be added, even though they come equipped with numerous vents.

Staff

One electronic technician takes care of the supervision and another of the maintenance of each pattern. In addition, a technician is responsible for the upkeep and repair of the receivers and sometimes operates them on board. This staff has acquired an excellent knowledge of the Rana equipment, but has proved too few in number for an almost permanent use of the chain.

Choice of sites for the patterns

The very general role assigned to the chain has led to the search for station sites which would allow the greatest coverage possible within the expected radioelectric range and under the best operational conditions for the Rana process. In particular we wanted the " high accuracy zone of intersection fixes " to reach the northern coast of Corsica. In order to avoid the irregularities and anomalies of overland propagation of Hertzian waves, it was decided to choose the sites only along the shore. This has made it impossible to use the Rana chain for positioning near the coast between the two patterns, but that is a region where accurate fixes can easily be determined by other methods.

To get the most from the system, rigorous technical considerations must be taken into account when choosing sites for the transmitting antennae. The rugged relief of the Riviera, the exceptional density of population along the coast and the presence of many causes of radioelectric interference have greatly complicated this choice, in spite of a certain flexibility due to the absolute independence of one pattern from the other.

A systematic and detailed exploration of this whole coast was necessary.

Finally the western pattern (Pattern I) was installed at the mouths of Golfe de Saint Tropez and Golfe de Fréjus, with a baseline of 22 728 metres, oriented to 38° true. The slave transmitter is on Pointe de Capon and the free transmitter on the bluff of Cap Drammont. These sites are particularly favorable since the enslavement path (synchronization path) actually lies entirely over sea water. The great baseline length dictated the placing of the locking receiver as far away as possible to have a sufficient amount of modulation without changing the power of the transmitters. To achieve this we had to use a 2 000-m screened coaxial cable which was difficult to lay and a burden to maintain. But the amount of modulation of this HF-head has always been better than 6 %.

The large distance between the two transmitters, more than 50 km apart on the coast road, adds extra difficulty to the work of the technician who alone supervises this pattern.

For the eastern pattern (Pattern II), since no site could be found between Antibes and the Italian frontier, we had to accept the idea of setting it up on Italian soil in spite of the great difficulties foreseeable from an

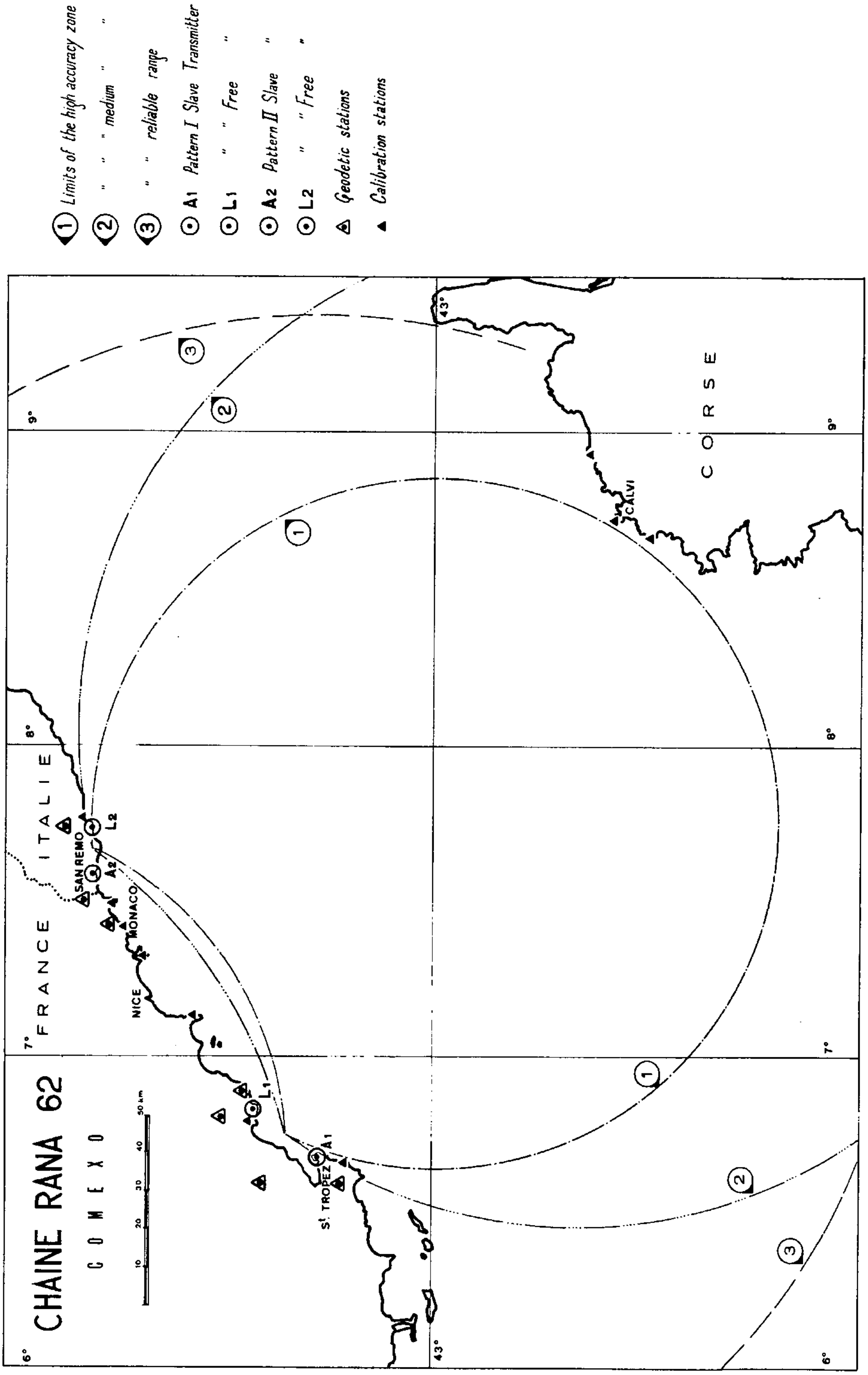
① Limits of the high accuracy zone
② " " " medium " "
③ " " reliable range
⊙ A1 Pattern I Slave Transmitter
⊙ L1 " " Free "
⊙ A2 Pattern II Slave "
⊙ L2 " " Free "
Geodetic stations
▲ Calibration stations
CHAINE RANA 62
COMEXO
10 20 30 40 50 km
FRANCE
ITALIE
NICE
MONACO
SAN REMO
St. TROPEZ
CALVI
CORSE
A1
L1
A2
L2
6°
7°
8°
9°
43°
1
2
3

Fig. 2

administrative and logistic point of view in operating a chain astride a frontier. Thanks to the invaluable help of the Italian *Consiglio nazionale delle ricerche,* all the necessary authorizations were quickly obtained.

On this steep heavily populated coast, devoted to intensive flower culture, the choice was equally limited. It was possible to install the slave transmitter on a clear crest overlooking Ventimiglia and the free transmitter in a less favorable site on the heights of Capo Pino near San Remo. We had to have a telephone line shifted, and since it was impossible to bring a shelter to the site, we had to house the transmitter in a hut.

These sites define a baseline of 12 613 metres oriented to 91° true. They are near to the sea as the crow flies but the synchronization path between the two transmitters lies entirely over land and the locking antenna could not be placed in direct view of the free transmitter. During the first tests, the amount of modulation was found to be too poor to ensure an efficient enslavement and the synchronism of pattern failed whenever interference occurred. This disadvantage was remedied after triangulation by slightly shifting the site of the locking antenna to screen it completely from the slave transmitter, and by reducing slightly the transmitter's power. Under normal propagation conditions this power reduction had no effect on the pattern's effective range which later tests have proved to be more than adequate.

The centers of the patterns are about 90 km apart. Figure 2 shows the limits of the high accuracy zone within which the angle of intersection of the hyperbolae is greater than 30°. These limits have been plotted by substituting the asymptotes for their hyperbolae.

Because of the relatively favorable orientation of the two baselines, hyperbolic lanes within which readings are the most accurate cover almost the entire 8 000 square mile zone.

Principal characteristics of the patterns

	Pattern I	*Pattern II*
Baseline length	22 728.53 m	12 613.02 m
Distance from HF-head to slave transmitter	1 664.71 m	1 045.50 m
Characteristic (mean) frequency	3 782 kc/s	3 782 kc/s
Characteristic wave length, reduced to projection		
over sea water	79.213 249 m	79.226 799 m
over land	79.089 019 m	79.102 549 m
Number of fine lanes	573.92	318.90
Lane length along baseline	39.60 m	39.55 m
Pattern constant	1 345.02	2 235.71

We adopted propagation velocities of 299 690 km/s over sea water and of 299 220 km/s over land, based on the results obtained by the French Hydrographic Office during a Rana season in the Mediterranean.

Geodetic work

The absolute accuracy of the radioelectric fix depends directly on the accuracy of the geographical position of the transmitting antennae (foci of the hyperbolae) and of the locking antenna. The error must be less than a metre to obtain an accuracy consistent with that of the Rana system.

We were therefore led to determine the positions of the six antennae by connection to geodetic stations of the first order, which entailed taking long-range sights (average 20 km) with a Wild T 3 theodolite. Because of poor visibility, we attempted a night test using gas lamps. Since the results were inconclusive, we resorted to sun mirrors which were very easy to see at great distances in spite of a slight fog. Identification was facilitated by the use of radio communications.

At least one observation was made at all the geodetic stations and at all the six positions to be located, each observation calling for a minimum of six swings. About 1 500 sights were taken and the results computed in Lambert III coordinates, South Zone. The graphs were plotted at the scale of 1/10 for the French pattern and at 1/20 for the Italian points. Each position was thus defined by at least three geometric loci and usually more, the final accuracy being of about 50 cm.

All the coordinates were converted into the geographic coordinates of the adjusted European network, then into U.T.M. coordinates, international ellipsoid, using the tables of the U.S. Army Map Service.

Since one of the patterns is on foreign soil, difficulties could be expected during control surveys and could cause discrepancies between the determined positions of the two patterns, one in relation to the other. We profited by the fact that the sites chosen in Italy were visible from two French geodetic stations of the first order near the frontier and that the coordinates of the Italian station of the first order which we were to use had been computed in Lambert III coordinates for the France-Corsica connection in 1954.

Foreseeing the setting up of the monitor receiver, the positioning of the Oceanographic Museum's mast was included in the control survey.

System of coordinates and projections

The equiphase hyperbolic lattices were computed in U.T.M. coordinates, international ellipsoid. We adopted a special zone centered on meridian 6° East, thus astride the usual zones 31 and 32, in order to use the same system of coordinates later on for placements when the chain would be set up in the Gulf of Lions or along the western coast of Corsica (fig. 3). The scale factor along the Y axis passing through the zone's center is equal to 0.999 820.

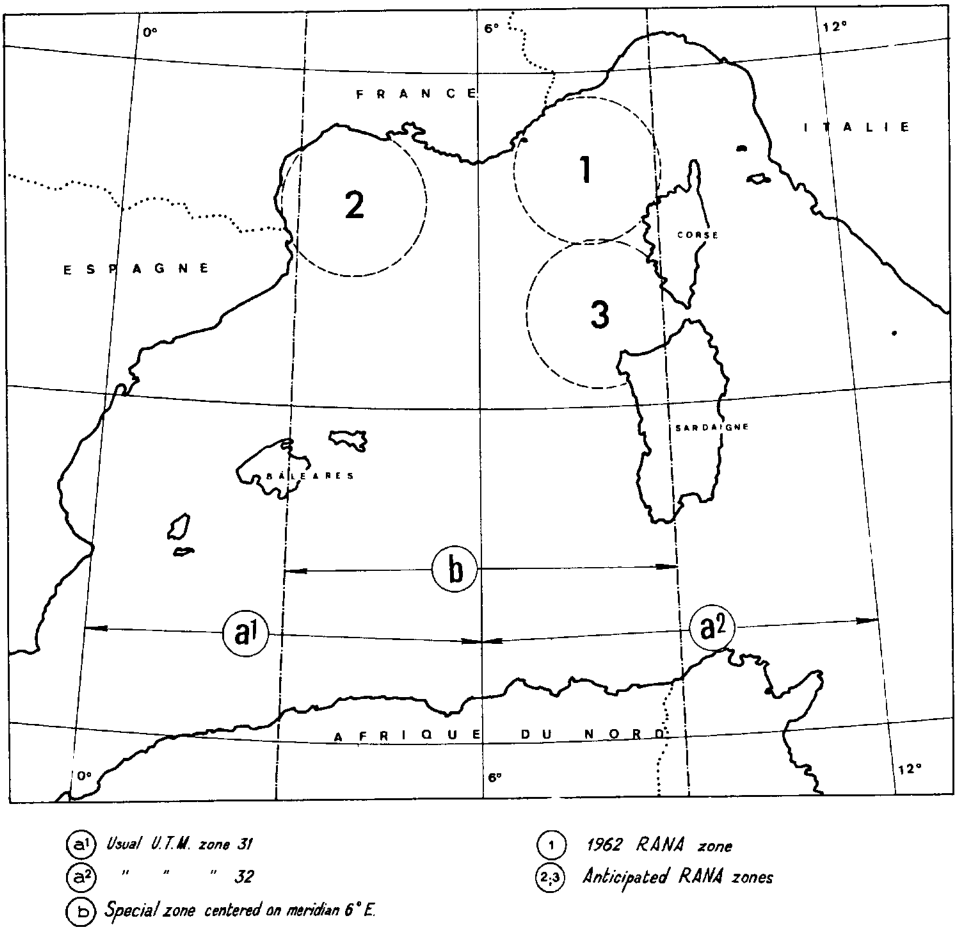

FIG. 3

The constant of each pattern (phase value along the bisector of the baseline) was determined by applying the mean value of the scale factor along the different synchronization paths to the wave lengths over land and over sea water.

For navigational use, planning of the surveys and fix plotting, hyperbolic lattices were drawn on six charts ·

— one *Grand Aigle* chart at 1/200 000, of the entire zone;

— three *Grand Aigle* charts at 1/100 000, of the NW, SW and SE parts of this zone;

— one *Grand Aigle* chart at 1/50 000, of the Saint Tropez - Cap Drammont region (baseline of Pattern I);

— one *Demi Aigle* chart at 1/50 000, of the Calvi region.

The hyperbolae were drawn by plotting the U.T.M. coordinates X and Y, each of which was computed for each value of φ (round number of hyperbola). The *Compagnie Bull* computed the 26 000 points with a Gamma 30 computer following a program developed by the *Compagnie des Compteurs*. It was possible to feed in three values for φ and three values for D, and two different intervals were chosen taking account of the scale and the curvature of the hyperbolae in the region under consideration. Complementary computations were also carried out on a CAB 500 computer.

The charts show the reference marks of the U.T.M. graticule and a grid of parallels and meridians as well as the simplified coastline (fig. 4).

Calibration

With the cooperation of the *Bureau d'études océanographiques de la Marine* (Naval Office of Oceanographic Studies) and of the surveying vessel *Origny*, we made two calibrations of the patterns six months apart, one in December 1962 and the other in June and July 1963. The positions to be compared were obtained by Wild T 2 and T 3 theodolite sights set up near the geodetic stations. Fortunately there are many signal stations well spaced along the coast that are almost all geodetic stations of the second order. The theodolite position was easily connected to them. A monitor receiver was continually in use at the Cap Ferrat signal station during the calibration operations.

A first run, by day, 5 miles off Pattern I and at 5 knots, showed that, in order to calibrate this pattern suitably, night sights as distant as possible from the baseline would have to be taken. From nearby, it turned out, only 0.50 m of displacement caused a variation of 1/100 of a lane, and the least time lag gave rise to a large error in the phasemeter readings.

Thus night sights were taken on a powerful lamp at the vessel's mast top. An electric timer on board ship controlled flashing signals on which sights were taken from shore and at the same instant Rana readings were recorded. In the computations we took into consideration the distance between the mast and the receiving antenna by a correction according to the vessel's heading.

We made several of these runs 12 and 15 miles off the two patterns, and in addition a run following the bisector of Pattern I's baseline to the limit of visibility. The vessel progressed at reduced speed or stopped at regular intervals. In the same way we made similar calibrations along the coast of Corsica which allowed us to make at long distance interesting checks on the corrections near the patterns' range limits.

These operations furnished more than 800 points of comparison between the radioelectric and the visual positions. The computations were made on an IBM 1620 computer. The results, in tabular form, give for each fix and each pattern the divergence in hundredths of a lane between the theoretical and the phasemeter readings. When fixes were obtained from three bearings, we computed separately the correction for each apex of the triangle of error.

Obviously not all the measurements could be taken under identical conditions and the final results differ in accuracy. Due to the large number of points, there followed a rather wide scattering of gross divergences. The calibration curves were plotted, giving the corrections to be applied to the readings as functions of the hyperbolic numbers, by weighting each measurement according to factors having an influence on the accuracy of the computations or of the seaboard Rana readings (angle of bearing sights, lane widths, paths over land or tangent to the coast, equipment failure, etc.). It was easy to remove the aberrant values due to random errors without reducing the validity of the results.

For all the calibration fixes of Pattern I the wave paths lay entirely over sea water. The curve, which covers 97 % of the hyperbolic lanes of this pattern, is almost horizontal. We conclude from this that the propagation velocity adopted (299 690 km/s) is correct. The constant correction to be applied to seaboard readings is + 18/100 of a fine lane.

The calibration curve of Pattern II, almost horizontal between hyperbola 2 100 (which passes through the HF-head) and hyperbola 2 235 (which is the bisector of the pattern's baseline) and giving a constant correction of + 25/100 of a lane, afterward bends regularly to the other extremity of the pattern where the observed difference is not more than 4 or 5 hundredths of a lane. This is explained to a certain extent by the outline of the coast between the two transmitters, placed slightly inland, which gives rise to mixed or tangent paths along which propagation velocity is difficult to determine accurately.

The calibrations in Corsica have shown up no appreciable variation in the corrections along the hyperbolae no matter how great the distance to the transmitters. On the other hand we found on Pattern I a difference of about 5/100 of a lane between the December 1962 calibration results in Corsica and those of July 1963, a difference which might be explained by a change in the characteristics or adjustments of the transmitters' LF circuits.

Crossing the shadow lines (*)

For lack of time, the regions in the neighborhood of the shadow lines were not calibrated. We also felt that, since they were outside the Rana high accuracy zone, they were less interesting except from a theoretical point of view. However we found out a great deal about the functioning of the patterns, especially as to stability and repeatability, by cutting as often as possible across the baseline extensions at sea or on land, and sometimes across both simultaneously. In this way we were able to deduce the values for the corrections along the shadow lines.

(*) *IHB Note :* Shadow lines are extensions of baselines.

Pattern I's baseline, for example, was crossed fifteen times at sea on the slave side at various times and at various distances from the transmitter. The extreme values observed at the phase minimum, only 20 % of the observations, do not vary by more than 4/100 of a lane. We adopted the figure 1 057.98 corresponding to the theoretical phase minimum which is 1 058.06. The baseline on the side of the free transmitter was cut four times at sea and twice on land. We found a 1 631.62 value four times and 1 631.98 twice, the theoretical phase maximum being 1 631.98. The differences of + 8/100 of a lane on one side and of + 38/100 on the other, thus determined, do not agree with the overall results of the calibrations, even after taking into account variations in propagation velocity due to wave penetration into the ground over the mixed land/sea paths. This led us to admit the existence of disturbing phase shifts due to phenomena of reflection or diffusion over land masses. During a limited calibration in November 1964 we verified the existence of these anomalies by studying the variations of difference between the visual and Rana fixes from the ship *Winnaretta Singer* coming in from the open sea to the port of Monaco with her receiver in constant operation.

Ambiguity elimination

The method for eliminating ambiguity constitutes an essential advantage of the system in permitting identification of the lanes in open sea and out of sight of land in the absence of other accurate means of determining position. The method consists in comparing “ coarse ” and “ medium ” values with corresponding values deduced from the fine measurements by reduction. The differences measured, in the form of error voltage, represent the “ divergence ”. Theoretically the divergence is nil for a correct setting. In practice this is rarely the case because of propagation irregularities and since induction phenomena around the antennae cause disturbing phase shifts which are not the same for each frequency transmitted. The observed differences can be partially cancelled by using a potentiometer to vary the LF locking of each frequency which permits phase shifting by 5/100 to either side of the mean position.

Before the chain started operating, we proceeded in the following manner. From a ship sailing near to shore and in the open sea we noted the values of divergences and their variations, and instructions about adjustments were given by radiophony to the transmitting stations so that the divergence became nil or near zero. This is a long procedure, but the later effectiveness of eliminating ambiguity depends largely on the great care used in the process. It is necessary to check the settings in view of shore and to be sure that the hooking device works without failure during the entire period of adjustments.

There still remain local and momentary propagation disturbances. They appear in the shape of fluctuations in the divergence value which, particularly near the coast, may be rather high, the minimal value not necessarily corresponding to the correct setting. Sometimes we find, at a given position, equal values but with contrary signs when there was only one pointer revolution between the fine readings on the phasemeter.

In consequence lane identification is often difficult and requires long training of the operating personnel.

However we have noticed that the divergences are relatively stable offshore all along the hyperbolae and it seems likely that at sea they do not vary with time.

The solution of the difficulties in eliminating ambiguity thus consists in empirically establishing curves or a chart of divergences by lane count from checked fixes using hooking procedure.

Observations on equipment performance

The statistical results which follow take no account of incidental failures in the equipment which in most cases are easy to spot. From these results we can judge the capabilities of the Rana H system under normal conditions of operation.

a) *Stability*

A monitor receiver was installed at the Cap Ferrat signal station, 60 km from Pattern I and 30 km from Pattern II, and later for convenience at the Monaco Oceanographic Museum, a somewhat less favorable site. In both cases, however, we found reception conditions for the two patterns to be excellent; the amplitude of the disturbing oscillations rarely exceeding 2/100 of a revolution.

Having monitored the stability over long periods, we must consider it very good. 90 % of the time the phasemeter readings varied by no more than 2/100 of a lane. The rest of the time we observed slow variations and oscillations, amplitude of which rarely exceeded 10/100, probably due either to changes in propagation conditions or to the transmitters' servo-mechanisms reaching their full range (synchronization remaining however assured).

Atmospherics rarely disturbed reception. The radioelectric transmissions of the signal station with an antenna very near the Rana H antenna and at a close frequency caused no interference.

b) *Repeatability*

For two and a half years numerous resettings were made by the *Winnaretta Singer* when setting out and returning to her mooring in the port of Monaco. They differ by no more than 3/100 of a revolution in 85 % of the cases for Pattern I nor by more than 4/100 in 70 % of the cases for Pattern II, extreme differences occasionally reaching 10/100 of a revolution for Pattern II. Conditions of reception and low accuracy of the patterns at this spot make this check not very significant. More valid results are listed in the table shown. They were all obtained at the same mooring in the port of Calvi, 185 km from Pattern I and 160 km from Pattern II. Displacements of 6.60 m and 13.50 m correspond to 1/100 of a revolution. The table shows very satisfactory repeatability.

TABLE

Date	Ship	Pattern I	Pattern II
2-10-1962	*Winnaretta*	1 395.76	2 326.70
21-10-1962	*Calypso*	1 395.78	2 326.76
13-12-1962	*Origny*	1 395.77	2 326.75
6- 3-1963	*Winnaretta*	1 395.76	2 326.73
11- 3-1963	*Winnaretta*	1 395.75	2 326.73
3- 3-1964	*Calypso*	1 395.70	2 326.74

Absolute accuracy

Within the normal range limit, in the absence of sky waves, the absolute exactness depends essentially on the accuracy of the calibrations and on that of the patterns. In present actual use and whatever the circumstances, we believe we can count on a final accuracy of about ± 4/100 of a fine lane. In the vicinity of the bisector of the baseline, this accuracy corresponds to a position error equal to ± 1/10 000 or ± 2/10 000 of the distance to the transmitters for Pattern I and to ± 2/10 000 or ± 3/10 000 for Pattern II.

Range

The power radiated is 10 watts for each of the frequencies. It was proved that up to a distance of 210 km from the transmitters the night effect and sky wave interference do not distort the phase measure but cause an instability in the phasemeters which reduces somewhat the accuracy of the readings. The recorders help a great deal in determining the mean value. At 210 km the amplitude of the disturbing oscillations reaches 10/100 to 15/100 of a lane. It is only 3/100 to 4/100 at 160 km from the transmitters. During the day, accuracy is very satisfactory within ranges up to 250 km.

Conclusion

Our chain has now been in operation for a total of 8 000 hours. It has been used mainly for bathymetric and seismic soundings and for current measurements. It helped to solve all positioning problems during the towing and anchoring of the Laboratory Buoy, a 230-ton oceanographic and meteorological floating station.

During these different operations we encountered many difficulties for which, however, the fundamental Rana principle and process were never held responsible. Most of them could be ascribed to failures of electronic components. The large shifts in the quartz oscillators have almost always been at the root of the synchronization troubles.

Our set already dates back a few years. Equipment incorporating recent electronic advances, now in process of construction, will allow surer use of the Rana system which has already proved its excellent capacity for stability and accuracy.

This identification method is unquestionably valuable in spite of the practical precautions to be taken. However the use of a great number of frequencies is required. Although these are in close range, it is sometimes difficult to obtain the allocation desired.

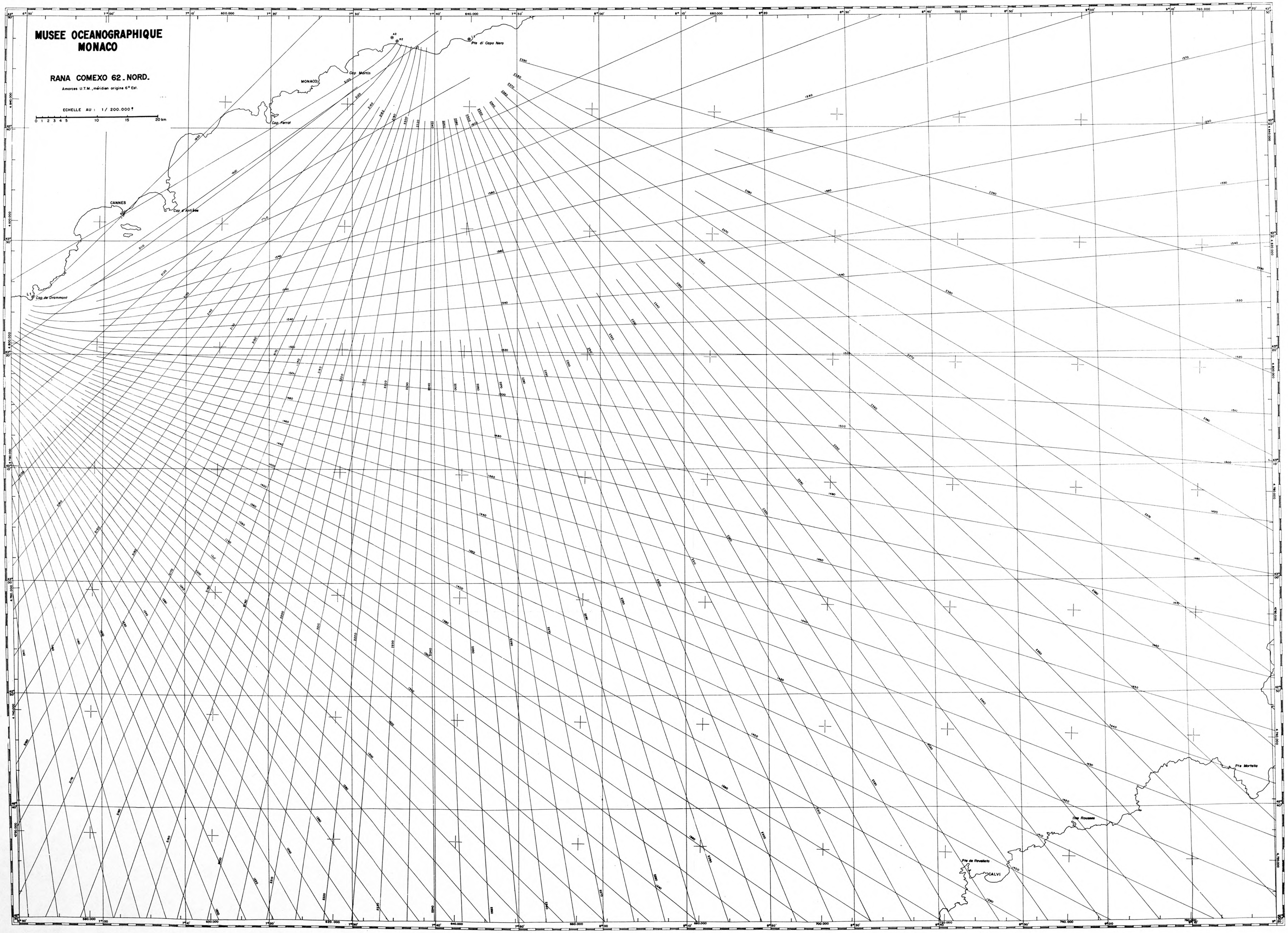
MUSEE OCEANOGRAPHIQUE
MONACO
RANA COMEXO 62 - NORD.
Amorces U.T.M., méridien origine 6° Est.
ECHELLE AU : 1/ 200.000e
0 1 2 3 4 5 10 15 20 km
6° 50'
7° 00'
7° 10'
7° 20'
7° 30'
7° 40'
7° 50'
8° 00'
8° 10'
8° 20'
8° 30'
8° 40'
8° 50'
9° 00'
9° 10'
9° 20'
43° 50'
43° 40'
43° 30'
43° 20'
43° 10'
43° 00'
42° 50'
42° 40'
600.000
620.000
640.000
660.000
680.000
700.000
720.000
740.000
760.000
4.840.000
4.820.000
4.800.000
4.780.000
4.760.000
4.740.000
4.720.000
A2
B2
L1
Pte di Capo Nero
Cap Martin
MONACO
Cap Ferrat
CANNES
Cap d'Antibes
Cap de Drammont
Pte Mortella
Ile Rousse
Pte de Revellata
CALVI

Fig. 4

S.S. : N.N | AREA : FARNDEEPS | RUN n° : 13

LEAST SQUARE ADJUSTMENT

Date : 31 May 1961 | time : $20^{h}45$ GMT (around sunset)

n°	stopwatch t	X	v_X	Y	v_Y	x	y	x'	y'	X'	Y'
1	02^m27^s	14 324	0	34 570	- 14	0	0	0	- 2 700	+ 1 500	- 1 349
2	57	14 174	- 17	34 727	- 19	0	300	0	- 2 400	+ 1 350	- 1 192
3	03 27	14 038	- 48	34 873	- 14	0	600	0	- 2 100	+ 1 214	- 1 046
4	57	13 844	- 20	35 016	- 3	0	900	0	- 1 800	+ 1 020	- 903
5	04 27	13 650	+ 7	35 159	+ 3	0	1 200	0	- 1 500	+ 826	- 760
6	57	13 483	+ 7	35 304	+ 9	0	1 500	0	- 1 200	+ 659	- 615
7	05 27	13 329	- 5	35 458	+ 7	0	1 800	0	- 900	+ 505	- 461
8	57	13 132	+ 25	35 601	+ 15	0	2 100	0	- 600	+ 308	- 318
9	06 27	12 949	+ 42	35 754	+ 14	0	2 400	0	- 300	+ 125	- 165
10	57	12 797	+ 27	35 909	+ 10	0	2 700	0	0	- 27	- 10
11	07 27	12 644	+ 13	36 064	+ 6	0	3 000	0	+ 300	- 180	+ 145
12	57	12 449	+ 42	36 205	+ 17	0	3 300	0	+ 600	- 375	+ 286
13	08 27	12 296	+ 28	36 360	+ 13	0	3 600	0	+ 900	- 528	+ 441
14	57	12 180	- 22	36 526	- 1	0	3 900	0	+ 1 200	- 644	+ 607
15	09 27	12 025	- 34	36 681	- 5	0	4 200	0	+ 1 500	- 799	+ 762
16	57	11 840	- 16	36 833	- 6	0	4 300	0	+ 1 800	- 984	+ 914
17	10 27	11 654	+ 4	36 985	- 6	0	4 800	0	+ 2 100	- 1 170	+ 1 066
18	57	11 490	+ 1	37 137	- 7	0	5 100	0	+ 2 400	- 1 334	+ 1 218
19	11 27	11 359	- 35	37 304	- 22	0	5 400	0	+ 2 700	- 1 465	+ 1 385
Z		12 824		35 919		0	2 700	0	0		
Σ			- 1		- 3					+ 1	+ 5
		$Z_X = \frac{[X]}{n}$		$Z_Y = \frac{[Y]}{n}$						$X' = X - Z_X$	$Y' = Y - Z_Y$

$$\sigma_a^2 = \frac{\sigma_Y^2}{[1]} = \frac{141}{51.3} \times 10^{-6} = 2.76 \times 10^{-6}$$

$$\sigma_b^2 = \frac{\sigma_X^2}{[1]} = \frac{671}{51.3} \times 10^{-6} = 13.09 \times 10^{-6}$$

$$\sigma_\lambda^2 = \frac{1}{\lambda^2}\ (a^2.\sigma_a^2 + b^2.\sigma_b^2) = 1.7757\ (0.25 \times 2.76 + 0.31 \times 13.09) = 8.43 \times 10^{-6}$$

$$\sigma_\lambda = 2.90 \times 10^{-3}$$

$$\sigma_\alpha^2 = \frac{1}{\lambda^4}\ (a^2.\sigma_b^2 + b^2.\sigma_a^2) = 3.1715\ (0.25 \times 13.09 + 0.31 \times 2.76) = 13.09 \times 10^{-6}$$

$$\sigma_\alpha = 3.62 \times 10^{-3} \text{ radians}$$

$n = 19$ $\Delta t = t_{last} - t_{first} = 540$ seconds

$$S = \frac{3600(y'_l - y'_f)}{\Delta t}\ .\ \lambda = 3600\ \frac{5400}{540}\ .\ \lambda = 27\ 016 \text{ m./h.}$$

$$\sigma_S = \frac{S}{\lambda}\ .\ \sigma_\lambda = \frac{27\ 016}{0.7504}\ .\ 2.9.10^{-3} = \pm\ 78 \text{ m./h.}$$

$$\sigma_\alpha = 206\ 265''.\sigma_\alpha(\text{rad.}) = 725'' = \pm\ 12'.1$$

$$\text{knot} = \frac{\text{m./h.}}{1\ 852}$$

$[y'y'] = +\ 51\ 300\ 000 = [1]$

$[y'Y'] = +\ 25\ 888\ 200 = [2]$

$[y'X'] = -\ 28\ 492\ 800 = [3]$

$$\underline{a} = \frac{[2]}{[1]} = \underline{+\ 0.504\ 463} \qquad \underline{a^2 = 0.25}$$

$$\underline{b} = \frac{[3]}{[1]} = \underline{-\ 0.555\ 415} \qquad \underline{b^2 = 0.31}$$

$$\lambda^2 = a^2 + b^2 = 0.563\ 150$$

$$\boxed{\lambda = 0.750\ 433}$$

$$\text{tg}\ \alpha = \frac{b}{a} = -\ 1.100\ 610$$

$$\boxed{\alpha = 312°15'}$$

note : [ ] = sum of squares

$v_X = b.y' - X'$

$$[v_X v_X] = 12\ 089 \qquad \sigma_X^2 = \frac{[v_X v_X]}{n - 1} = 671 \qquad \sigma_X = \pm\ 25.9 \text{ m}$$

$v_Y = a.y' - Y'$

$$[v_Y v_Y] = 2\ 547 \qquad \sigma_Y^2 = \frac{[v_Y v_Y]}{n - 1} = 141 \qquad \sigma_Y = \pm\ 11.9 \text{ m}$$

Observations with either v_X or v_Y larger than $2\frac{1}{2}$ times σ_X or σ_Y must be rejected from the run ; the run must then be readjusted.

S = 14.587 knots ; st.e. = ± 0.042 = ± 0.29 % of S

α = 312°15' ; st.e. = ± 12'.1