DISTURBANCE OF RANA RECEIVER READINGS BY IONOSPHERE-REFLECTED WAVES

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In the course of Rana operations during the 1961 and 1962 survey seasons, it became apparent that phasemeter readings for long distances showed larger fluctuations during the first part of the night — up to midnight in the Alboran Sea and often until 1 a.m. near Belle Ile. Sub-lieutenant Cassou of the Argentine Navy noticed that the disappearance of the fluctuations was nearly coincident with the time of the appearance of the ionospheric E-layer reported by David (*Cours de radioélectricité générale*).

Moreover in its publicity documentation the *Compagnie des Compteurs* notes a "mean effective range", i.e. the distance from the transmitters at which reception of the direct wave predominates. This mean effective range depends on the frequency and is not the same by day and by night. For example :

Frequency	*Day*	*Night*
100 khz	700 km	300 km
300 —	500 —	230 —
2 000 —	300 —	130 —

With the French Hydrographic Office's chain, used it is true exclusively over sea and having a frequency about 1 600 khz, the receiver was readily employed at distances greater than 150 km from the transmitters during the day as well as at night.

The fluctuations noted, which appear as a widening of the recording trace on either side of a mean value, occasionally seem to modify this mean value which then shows regular periodic oscillations although the vessel remains on a constant heading.

It appeared to be of interest to discuss mathematically the influence on Rana receiver readings of the wave reflected by the ionospheric layers.

To return to the basic theory of Rana : at instant t the phases of the transmitted waves are :

$$\omega_0 \; t$$
$$\omega_1 \; t$$

for the free transmitter, and :

$$(\omega_0 + \Delta\omega)\, t + \varphi_0$$
$$(\omega_1 - \Delta\omega)\, t + \varphi_1$$

for the slave transmitter.

At the same instant, owing to the propagation at velocity v, the phases of the waves received by the receiver are :

$$\omega_0 \left(t - \frac{D_L}{v}\right) \qquad (\omega_0 + \Delta\omega)\left(t - \frac{D_A}{v}\right) + \varphi_0$$
$$\omega_1 \left(t - \frac{D_L}{v}\right) \qquad (\omega_1 - \Delta\omega)\left(t - \frac{D_A}{v}\right) + \varphi_1$$

By beating between waves having pulsations ω_0 and $\omega_0 + \Delta\omega$ on the one hand and waves having pulsations ω_1 and $\omega_1 + \Delta\omega$ on the other, oscillations are obtained having pulsation $\Delta\omega$ and phases :

$$\Delta\omega \left(t - \frac{D_A}{v}\right) + \omega_0 \frac{D_L - D_A}{v} + \varphi_0$$
$$\Delta\omega \left(t - \frac{D_A}{v}\right) - \omega_1 \frac{D_L - D_A}{v} - \varphi_1$$

which may be compared to one another. Their phase difference is :

$$(\omega_0 + \omega_1)\; \frac{D_L - D_A}{v} + \varphi_0 + \varphi_1$$

At the locking antenna this phase difference is zero :

$$\varphi_0 + \varphi_1 = - \frac{\omega_0 + \omega_1}{v}\; (d_L - d_A)$$

Putting :

$$\frac{\dfrac{\omega_0 + \omega_1}{2}}{v} = \frac{2\,\pi}{\lambda_m}$$

we obtain the Rana formula (which gives the hyperbola number in the fine pattern) :

$$n_f = \frac{D_L - D_A - (d_L - d_A)}{\dfrac{\lambda_m}{2}}$$

The waves reaching the receiver after reflection by an ionospheric layer at altitude h have covered a path D' and $D'^2 = D^2 + 4h^2$. If we neglect a possible phase change π due to the reflection, their phases at the receiver are :

$$\omega_0 \left(t - \frac{D'_L}{v}\right) \qquad (\omega_0 + \Delta\omega)\left(t - \frac{D'_A}{v}\right) + \varphi_0$$
$$\omega_1 \left(t - \frac{D'_L}{v}\right) \qquad (\omega_1 - \Delta\omega)\left(t - \frac{D'_A}{v}\right) + \varphi_1$$

Each of these phases is mixed with the wave having the same pulsation. If we call Δ the path difference $D' - D$, A and a the amplitudes of the components of the direct and reflected waves, the resultant will have, in comparison with the direct wave, a phase lag α given by :

$$\tan \alpha = \frac{a \sin \frac{\omega}{v} \Delta}{A + a \cos \frac{\omega}{v} \Delta}$$

Finally the phases of the received mixed waves will be :

$$\omega_0 \left(t - \frac{D_L}{v} \right) - \alpha_{0_L} \qquad (\omega_0 + \Delta\omega) \left(t - \frac{D_A}{v} \right) + \varphi_0 - \alpha_{0_A}$$

$$\omega_1 \left(t - \frac{D_L}{v} \right) - \alpha_{1_L} \qquad (\omega_1 - \Delta\omega) \left(t - \frac{D_A}{v} \right) + \varphi_1 - \alpha_{1_A}$$

Those of the beatings will be :

$$\Delta\omega \left(t - \frac{D_A}{v} \right) + \omega_0 \frac{D_L - D_A}{v} + \varphi_0 - \alpha_{0_A} + \alpha_{0_L}$$

$$\Delta\omega \left(t - \frac{D_A}{v} \right) - \omega_1 \frac{D_L - D_A}{v} - \varphi_1 + \alpha_{1_A} - \alpha_{1_L}$$

Therefore the final phase difference will differ from that resulting from direct waves alone by the quantity :

$$\varepsilon = \alpha_{1_L} - \alpha_{1_A} + \alpha_{0_L} - \alpha_{0_A}$$

If $\frac{a}{A}$ is small, we may write :

$$\alpha = \frac{a}{A} \sin \frac{\omega}{v} \Delta$$

and :

$$\varepsilon = \frac{a_L}{A_L} \left(\sin \frac{\omega_1}{v} \Delta_L + \sin \frac{\omega_0}{v} \Delta_L \right) - \frac{a_A}{A_A} \left(\sin \frac{\omega_1 - \Delta\omega}{v} \Delta_A + \sin \frac{\omega_0 + \Delta\omega}{v} \Delta_A \right)$$

$$= 2 \frac{a_L}{A_L} \cos \frac{\omega_1 - \omega_0}{2v} \Delta_L \sin \frac{\omega_0 + \omega_1}{2v} \Delta_L - 2 \frac{a_A}{A_A} \cos \frac{\omega_1 - \omega_0 - 2\Delta\omega}{2v} \Delta_A \cdot \sin \frac{\omega_0 + \omega_1}{2v} \Delta_A$$

$\frac{a}{A}$ is a function of Δ if propagation conditions are fixed :

$$\frac{2a}{A} = K(\Delta)$$

Furthermore :

$$\frac{\omega_1 + \omega_0}{2v} = \frac{2\pi}{\lambda_m}$$

and putting :

$$\frac{\omega_1 - \omega_0}{2v} = \frac{2\pi}{\Lambda}$$

$$\frac{\omega_1 - \omega_0 - 2\Delta\omega}{2v} = \frac{2\pi}{\Lambda'}$$

then :

$$\varepsilon = K\,(\Delta_L)\cos 2\pi \frac{\Delta_L}{\Lambda} \sin 2\pi \frac{\Delta_L}{\lambda_m} - K\,(\Delta_A)\cos 2\pi \frac{\Delta_A}{\Lambda'} \sin 2\pi \frac{\Delta_A}{\lambda_m}$$

The error is the difference between two semi-periodic functions of Δ_L and Δ_A of wave length λ_m and modulated at frequencies corresponding respectively to wave lengths Λ and Λ'.

For the French Hydrographic Office's chain :

$$\lambda_m \# 185 \text{ metres}$$
$$\omega_1 - \omega_0 = 3 \text{ khz}$$
$$\Delta\omega = 40 \text{ hz}$$
$$\Lambda \# \Lambda' \# 200 \text{ km}$$

The error was computed by assuming $\frac{a}{A}$ to be small, but the phenomenon will remain qualitatively the same if this condition is not satisfied. If the receiver moves along a circle centred on one station (for instance the free station), then D_L and Δ_L will be constants; starting from a point where the error is maximum, a maximum having the same sign will be found when Δ_A will have varied by λ_m, and since :

$$\frac{d\Delta}{dD} = \frac{-1}{1 + \frac{D}{\Delta}}$$

D_A will have varied by $\left(1 + \frac{D}{\Delta}\right) \lambda_m$, i.e. $2\left(1 + \frac{D}{\Delta}\right)$ hyperbolae will have been crossed.

It should be noted that ε will be zero if Δ_L and Δ_A are both multiples of $\frac{\lambda_m}{2}$, which will give in the plane the points of intersection of circles having corresponding radius D.

In the case of the French Hydrographic Office's chain, as Λ and Λ' are equal to 200 km, the amplitude of the error will be practically maximum when Δ is a multiple of 100 km and zero when Δ is an odd multiple of 50 km.

For the F-layer ($h = 300$ km, according to DAVID), the amplitude is maximum for :

$\Delta = 500$	$D = 110$
$\Delta = 400$	$D = 250$
$\Delta = 300$	$D = 450$

and zero for :

$\Delta = 550$	$D = 52.3$
$\Delta = 450$	$D = 175$
$\Delta = 350$	$D = 339.3$

while for the E-layer (h = 120 km) the amplitude of the error is maximum for :

$$\Delta = 200 \qquad D = 44$$
$$\Delta = 100 \qquad D = 238$$

and zero for :

$$\Delta = 150 \qquad D = 117$$
$$\Delta = 50 \qquad D = 551$$

We note that at a distance of 110 to 120 km the F-layer produces a large eror while the effect of the E-layer is negligible. This explains the observed phenomenon, as in the second part of the night the E-layer's presence prevents waves from reaching the F-layer.

Remark

For the rough measurement (on a pattern for instance 20 times less close than the fine pattern), obtained from waves having pulsation ω_2 at the free transmitter and $\omega_2 + \Delta\omega$ at the slave transmitter, in the same way as for the fine measurement we obtain an error :

$$\varepsilon_g = \alpha_{2_L} - \alpha_{0_L} - \alpha_{2_A} + \alpha_{0_A}$$

$$\varepsilon_g = 2 \frac{a_L}{A_L} \cos \frac{\omega_2 + \omega_0}{2v} \Delta_L \sin \frac{\omega_2 - \omega_0}{2v} \Delta_L - 2 \frac{a_A}{A_A} \cos \frac{\omega_2 + \omega_0 + 2\Delta\omega}{2v} \Delta_A \cdot \sin \frac{\omega_2 - \omega_0}{2v} \Delta_A$$

Putting :

$$\frac{\omega_2 + \omega_0}{2v} = \frac{2\pi}{\lambda} \qquad \frac{\omega_2 + \omega_0 + 2\Delta\omega}{2v} = \frac{2\pi}{\lambda'}$$

as :

$$\omega_2 - \omega_0 = \frac{\omega_1 + \omega_0}{20} \quad \text{and} \quad \omega_1 = \omega_0 + 3 \text{ khz}$$

λ and λ' are close to λ_m.

$$\varepsilon_g = K(\Delta_L) \cos 2\pi \frac{\Delta_L}{\lambda} \sin 2\pi \frac{\Delta_L}{20 \lambda_m} - K(\Delta_A) \cos 2\pi \frac{\Delta_A}{\lambda'} \sin 2\pi \frac{\Delta_A}{20 \lambda_m}.$$

If we consider the quantity $20 n_g - n_f$ used for the identification of hyperbolae, which must be zero when the phasemeter indicates the correct hyperbola number, as a result of the reflections this quantity will be involved with an error :

$$20 \varepsilon_g - \varepsilon_f$$

If now, instead of an ionosphere-reflected wave, we consider a wave following a path different from the direct wave and longer by Δ than the direct path, the expression of the error remains valid. Still assuming $\frac{a}{A}$, i.e. K, to be small, and for Δ small before λ_m, we see that the error $20 \varepsilon_g - \varepsilon_f$, if not exactly zero, is nevertheless infinitely small. However, if Δ reaches the value $\frac{\lambda_m}{4}$, $20 \varepsilon_g$ will be infinitely small while ε_f will be close to K. The quantity $20n_g - n_f$ will never be zero even if the phasemeter shift is correct.

We thus see the so-called "divergence", which expresses the fact that in practice the fine and rough pattern cannot everywhere be exactly superposed.

Finally reflections in the vicinity of the antennas themselves may also cause this " divergence ".