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NETWORK TRANSFORMATION FROM LOCAL GEODETIC SYSTEM (Roma 40) TO WGS 84, FOR AN INTERNATIONAL MAP OF GEOCENTRIC ELLIPSOID

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Abstract

General algorithms are given to resolve the "GPS FIX", using vectorial procedures to transform coordinates from a local geodetic system to the WGS 84 system, with geodetic accuracy.

The theory is based on the "invariance", in different geodetic systems, of the spatial distance or slope distance, that GPS receivers give with centimetric precision. The following method employs rectangular ellipsoidic coordinates (X,Y,Z) and their respective variations (ΔX,ΔY and ΔZ).

1. VECTORIAL PROCEDURES OF GPS FIX

Two GPS receivers, located in two points A (known) and B (unknown), give the baseline vector AB. Its components are:

$$\Delta X = X_B - X_A$$
$$\Delta Y = Y_B - Y_A$$
$$\Delta Z = Z_B - Z_A$$

its module is:

$$AB = d = \left(\Delta X^2 + \Delta Y^2 + \Delta Z^2\right)^{1/2}$$

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In the national "Roma 40 system", the geodetic coordinates of point A are:

$$\varphi_A$$
$$\lambda_A$$
$$H_A$$

the "Hayford ellipsoid" parameters are:

a = 6378388 *(semi - major axis)*
e^2 = 0,00662267 *(eccentricity)*

$$N_A = a\left(1 - e^2 sin^2 \varphi_A\right)^{-1/2}$$

Through direct transformations, the rectangular co-ordinates of point A will be:

$$X_A = (N_A + H_A) \cdot cos\varphi_A \cdot cos\lambda_A$$
$$Y_A = (N_A + H_A) \cdot cos\varphi_A \cdot sin\lambda_A$$
$$Z_A = \left(N_A \cdot \left(1 - e^2\right) + H_A\right) \cdot sin\varphi_A$$

In the "WGS 84 system", GPS receivers give:

$$\Delta X_W$$
$$\Delta Y_W$$
$$\Delta Z_W$$

With almost geodetic precision, without significant errors, in the local "Roma 40 system", the rectangular coordinates of point B may be obtained:

$$X_B = X_A + \Delta X_W$$
$$Y_B = Y_A + \Delta Y_W$$
$$Z_B = Z_A + \Delta Z_w$$

The conversion of (X,Y,Z) into (φ,λ,H) is carried out using the formula given below:

$$r = \left(X_B^2 + Y_B^2\right)^{1/2}$$

$$\varphi_o = \tan^{-1}\left[\frac{Z_B}{r\cdot\left(1-e^2\right)}\right]$$

$$\varphi_1 = \tan^{-1}\left(\frac{Z_B + e^2\cdot N_o\cdot sin\varphi_o}{r}\right)$$

$$N_o = a\cdot\left(1-e^2\cdot sin^2\varphi_o\right)^{-1/2}$$

$$N_1 = a\cdot\left(1-e^2\cdot sin^2\varphi_1\right)^{-1/2}$$

$$\varphi_B = \tan^{-1}\left(\frac{Z_B + e^2\cdot N_1\cdot sin\varphi_1}{r_b}\right)$$

$$\lambda_B = \tan^{-1}\left(\frac{Y_B}{X_B}\right)$$

$$N_B = a\cdot\left(1-e^2\cdot sin^2\varphi_B\right)^{-1/2}$$

$$H_B = \frac{r}{cos\varphi_B} - N_B$$

This procedure enables to determine any double point from other previously known double points (Roma 40 – WGS 84).

2. DETERMINATION OF DOUBLE POINTS

Considering point A as a double point (Roma 40 and WGS 84), the aim is to obtain the coordinates in WGS 84 of a point B whose coordinates in Roma 40 are already known.

$$A \equiv \begin{pmatrix}\varphi_A\\ \lambda_A\\ H_A\end{pmatrix} \quad \text{geodetic coordinates in Roma 40 system}$$

$$A \equiv \begin{pmatrix} \varphi_{AW} \\ \lambda_{AW} \\ H_{AW} \end{pmatrix} \quad \text{geodetic coordinates in WGS 84 system}$$

$$B \equiv \begin{pmatrix} \varphi_{B} \\ \lambda_{B} \\ H_{B} \end{pmatrix} \quad \text{geodetic coordinates in Roma '40 system}$$

$$B \equiv \begin{pmatrix} \varphi_{BW} \\ \lambda_{BW} \\ H_{BW} \end{pmatrix} \quad \text{geodetic coordinates in WGS 84} \quad ? \quad \text{(unknown)}$$

With direct transformations, the rectangular coordinates of A and B in the Roma 40 system are obtained:

$$N_A = a\left(1 - e^2 sin^2 \varphi_A\right)^{-1/2}$$

$$X_A = \left(N_A + H_A\right) \cdot cos\varphi_A \cdot cos\lambda_A$$
$$Y_A = \left(N_A + H_A\right) \cdot cos\varphi_A \cdot sin\lambda_A$$
$$Z_A = \left(N_A \cdot \left(1 - e^2\right) + H_A\right) \cdot sin\varphi_A$$

$$N_B = a\left(1 - e^2 sin^2 \varphi_B\right)^{-1/2}$$

$$X_B = \left(N_B + H_B\right) \cdot cos\varphi_B \cdot cos\lambda_B$$
$$Y_B = \left(N_B + H_B\right) \cdot cos\varphi_B \cdot sin\lambda_B$$
$$Z_B = \left(N_B \cdot \left(1 - e^2\right) + H_B\right) \cdot sin\varphi_B$$

$$\Delta X = X_B - X_A$$
$$\Delta Y = Y_B - Y_A$$
$$\Delta Z = Z_B - Z_A$$

With direct transformations, the geodetic coordinates in WGS 84 of point A are converted into rectangular coordinates.

The geocentric ellipsoid parameters are:

$$a = 6378137 \text{ (semi - major axis)}$$
$$e^2 = 0{,}00669438 \text{ (eccentricity)}$$

$$N_{AW} = a\left(1 - e^2 sin^2 \varphi_{AW}\right)^{-1/2}$$

$$X_{AW} = (N_{AW} + H_{AW}) \cdot cos\varphi_{AW} \cdot cos\lambda_{AW}$$
$$Y_{AW} = (N_{AW} + H_{AW}) \cdot cos\varphi_{AW} \cdot sin\lambda_{AW}$$
$$Z_{AW} = \left(N_{AW} \cdot (1 - e^2) + H_{AW}\right) \cdot sin\varphi_{AW}$$

As explained in paragraph 1:

$$X_{BW} = X_{AW} + \Delta X$$
$$Y_{BW} = Y_{AW} + \Delta Y$$
$$Z_{BW} = Z_{AW} + \Delta Z$$

With inverse transformations, the geodetic coordinates of point B are obtained in WGS 84 system (see paragraph 1).

EXAMPLE

Geodetic surveys carried out in Sicily with GPS (Trimble 4000 SSE, L_1, L_2) enabled the establishment of a double point on CAPO MILAZZO LIGHTHOUSE (A):

$$A_{Roma40} \equiv \begin{cases} \varphi_{AR} = 38°16'11''.2817 \text{ N} \\ \lambda_{AR} = 15°13'51''.134 \text{ E} \\ \text{H}_{\text{AR}} = 83.835 \text{ m} \end{cases}$$

$$A_{WGS84} \equiv \begin{cases} \varphi_{AW} = 38°16'13''.7718 \text{ N} \\ \lambda_{AW} = 15°13'51''.1534 \text{ E} \\ H_{AW} = 137.756 \text{ m} \end{cases}$$

Considering now that point B is located in ANTENNAMARE, whose coordinates in Roma 40 are known:

$$B_{Roma40} \equiv \begin{cases} \varphi_{BR} = 38°09'32''.236 \text{ N} \\ \lambda_{BR} = 15°27'52''.602 \text{ E} \\ \text{H}_{\text{BR}} = 1123.1985 \text{ m} \end{cases}$$

The coordinates of point B in WGS 84 must be determined.

$$B_{WGS84} \equiv \begin{cases} \varphi_{BW} = ? \\ \lambda_{BW} = ? \\ H_{BW} = ? \end{cases}$$

For this purpose, the geodetic coordinates of A and B in Roma 40 are transformed into rectangular coordinates as follows:

a = 6378388
e^2 = 0.00662267
N_{AR} = 6386629.57426
N_{BR} = 6386588.09808

$$X_{AR} = (N_A + H_A)\cdot cos\varphi_A \cdot cos\lambda_A = 4838102.39354$$
$$Y_{AR} = (N_A + H_A)\cdot cos\varphi_A \cdot sin\lambda_A = 1317282.89547$$
$$Z_{AR} = \left(N_A \cdot \left(1-e^2\right) + H_A\right)\cdot sin\varphi_A = 3929115.50744$$

$$X_{BR} = (N_B + H_B)\cdot cos\varphi_B \cdot cos\lambda_B = 4840814.66919$$
$$Y_{BR} = (N_B + H_B)\cdot cos\varphi_B \cdot sin\lambda_B = 1339257.35592$$
$$Z_{BR} = \left(N_B \cdot \left(1-e^2\right) + H_B\right)\cdot sin\varphi_B = 3920092.11856$$

The differences between the rectangular coordinates of points A and B are calculated:

$$\Delta X_R = X_{BR} - X_{AR} = 2709.61294$$
$$\Delta Y_R = Y_B - Y_{AR} = 21973.73592$$
$$\Delta Z_R = Z_{BR} - Z_{AR} = -9025.56595$$

from which the spatial distance between A and B is obtained:

$$d_{Roma40} = \left(\Delta X^2 + \Delta Y^2 + \Delta Z^2\right)^{1/2} = 23909.147\,m$$

The geodetic coordinates of point A are then transformed into rectangular coordinates (WGS 84):

a = 6378137
e^2 = 0.00669438

$$X_{AW} = (N_{AW} + H_{AW}) \cdot cos\varphi_{AW} \cdot cos\lambda_{AW} = 4837880.52726$$
$$Y_{AW} = (N_{AW} + H_{AW}) \cdot cos\varphi_{AW} \cdot sin\lambda_{AW} = 1317222.9761$$
$$Z_{AW} = (N_{AW} \cdot (1 - e^2) + H_{AW}) \cdot sin\varphi_{AW} = 3929145.09621$$

The rectangular coordinates of point B result as follows:

$$X_{BW} = X_{AW} + \Delta X_R = 4840590.1402$$
$$Y_{BW} = Y_{AW} + \Delta Y_R = 1339196.71202$$
$$Z_{BW} = Z_{AW} + \Delta Z_R = 3920119.53026$$

The rectangular coordinates of point B are then transformed into geodetic coordinates, using inverse formulas:

a = 6378137
e^2 = 0.00669438

$$\lambda_{BW} = \tan^{-1}\left(\frac{Y_{BW}}{X_{BW}}\right) = 15°27'52''.57517$$

$$r_{BW} = \frac{Y_{BW}}{sin\lambda_{BW}} = 5022425.78231$$

$$\varphi_{OW} = \tan^{-1}\left(\frac{Z_{BW}}{r_{BW}(1 - e^2)}\right) = 38°.1596430179$$

$$\varphi_{BW} = \tan^{-1}\left(\frac{Z_{BW} + e^2 \cdot N_{BW} \cdot sin\varphi_{OW}}{r_{BW}}\right) = 38°\,09'34''.71435$$

$$H_{BW} = \frac{r_{BW}}{\cos\varphi_{BW}} - N_{BW} = 1176.973\,\mathrm{m}$$

Finally, the geodetic coordinates of point B are obtained in WGS 84:

$$B_{WGS84} \equiv \begin{cases} \varphi_{BW} = 38°09'34''.71435\,\mathrm{N} \\ \lambda_{BW} = 15°27'52''.57517\,\mathrm{E} \\ \mathrm{H}_{BW} = 1176.973\,\mathrm{m} \end{cases} \quad (1)$$

GPS surveys (with Trimble 4000 SSE, L_1, L_2 receivers) between points A and B provided the following slope distance:

$$d_W = 23909.150 \text{ m} \equiv d_{Roma\,40} = 23909.147 \text{ m}$$

which confirms the invariance of distances, while the coordinates of point B (ANTENNAMARE) obtained with GPS are:

$$B_{WGS84} \equiv \begin{cases} \varphi_{BW} = 38°09'34''.7149 \text{ N} \\ \lambda_{BW} = 15°27'52''.6572 \text{ E} \\ \text{H}_{BW} = 1177.118 \text{ m} \end{cases} \quad (2)$$

The coordinates of B in (1) and (2) are pratically identical, therefore the validity of this method is confirmed.

References

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