

TOWARDS ADEQUATE MULTIBEAM ECHOSOUNDERS FOR HYDROGRAPHY

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Abstract

As IHO's new standards for hydrographic surveys [1] are adopted by the Hydrographic Offices, a logical next step will be, given the order of a survey, to optimize the data acquisition. A very important parameter in this context is the design and capability of the multibeam echosounder and the enclosed software which extracts information from the pings during the survey. This article points out the advantage of homogeneous sea bed coverage, gives an example of useful information which is present in the swath, but not available to the surveyor today, and demonstrates how a perturbation of the sound velocity profile affects the depths measured by a multibeam echosounder, thereby opening up for the feasibility of designing on-the-spot warnings to alert the surveyor against excessive fluctuations in the velocity of sound during the survey. A spin-off of this analysis is explicit expressions for the contribution, from errors in the velocity of sound, to the error budget for multibeam surveys.

INTRODUCTION

Hydrographic surveying, being the art and science of providing quality assessed and blunder free observations of the depth of the sea bed, is in the foreseeable future going to be dominated by multibeam echosounders. Recognizing this fact the IHO recently has revised its former standards for hydrographic surveys [2] to include, among other developments, the potential of the multibeam echosounder [1]. A side effect of setting minimum standards is that the hydrographic community uses [1] as a departure point the purchase of surveying equipment and this affects the market, as every manufacturer has an interest in being able to claim that his product adheres to the most accurate IHO's standards. As matters now stand, it becomes

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important for the hydrographic community to clarify points in [1] which are obscure, as for instance the 100% bottom coverage, which is discussed in [3].

Of course, cost-efficiency is of prime concern when new equipment is purchased and a major drive in the transition from singlebeam towards multibeam surveys has been the wish to optimize the economy of hydrographic surveys. However, the economy of quality assessed depths is more subtle than the solution which today is offered by the manufacturers of multibeam equipment, namely a trend towards even wider swath widths, disregarding the fact that the coverage of the sea bed inside the swath consequently becomes still more uneven and putting the pressure on the surveyors knowledge of the spatial variation in the refraction during the survey.

ADEQUACY

The author of this paper has been engaged in the process of providing the Royal Danish Navy with two multibeam systems for use on small vessels operating in shallow waters. As a by-product from the evaluation of sample data sets provided by four of the multibeam manufacturers on the market, it gradually became plain that, while none of the systems are ideal for hydrographic surveying today, it might be feasible to combine features from the four systems and add a little extra something to make up a multibeam echosounder which would be more adequate for hydrographic surveying. Here the term "adequate" is used to characterize a multibeam echosounder which makes available to the surveyor the relevant information which is contained in its observation. Below, the author presents his reflections on

- sea bed coverage
- measurement redundancy, and
- surveying in the presence of spatial variation in the refraction

The last paragraph is treated in depth, as it follows from the investigation that it is feasible (at least theoretically) to design multibeam echosounders with a build-in robustness with respect to small changes in the velocity of sound. Alternatively 'intelligent' software should present an online warning to the hydrographer whenever the velocity of sound is disturbed by an amount which affects the measurements beyond a preset limit during the survey.

OPTIMAL SEA BED COVERAGE

As mentioned in the introduction, the subject of sea bed coverage is treated in [3]. However, granted 100% bottom coverage, how this coverage is achieved makes a difference from the economic point of view. Indeed, once the desired resolution for a survey is chosen, say as a depth dependent area on the sea floor, it is optimal to require that the sea floor is covered homogeneously with measurements of this size.

It appears that a survey of an area is never better than its worst part. Today most of the multibeam systems on the market are designed in a way which aims at using the same, fixed beam angle throughout the swath, irrespective of the direction of the beam. Consequently, the sea bed directly below the survey vessel is scanned with a much higher resolution than the one carried out with the outer beams. The real draw back in this design can be ascribed to the fact that any depth sampled by a multibeam echosounder represents a weighted mean of the depth of the sea floor inside the footprint of a beam, while the surveyor, having ship safety in mind, wants the depth at the top of the sea floor inside the said footprint.

In [4] this problem was illustrated by surveying a field stone with two different multibeam systems which both pointed out large differences in the size of the object as determined by several trial runs. The practical way for the surveyor to resolve this dilemma is to limit the gap between the depth requested and the depth requested by imposing an upper limit on the size of the footprint used in his survey. Today trend in multibeam echosounders towards extra swath width is therefore of limited value for hydrographic surveying as long as the resolution of the outer beams is not increased accordingly.

A desirable improvement would be to achieve a more homogeneous coverage of the sea bed by requiring that, for any fixed depth, each beam in the swath covers an equal part of the sea floor athwartships. The size of the beams can be determined as follows: Let $\theta_1, \theta_2, \dots, \theta_n$ be a list of the beam angles in a half swath width, with θ_1 the nadir beam and θ_n the outermost. Then

$$\theta_i = \text{Atan}(i \cdot \tan \theta_1) - \text{Atan}((i-1) \cdot \tan \theta_1) \quad i=2, \dots, n$$

For instance, let the footprint of the nadir beam be $3.0^\circ \times 1.5^\circ$, then a swath width of $[-60^\circ, 60^\circ]$ can be covered in this way by 66 beams, where the outermost beams have a footprint of $0.8^\circ \times 1.5^\circ$. Compared to present standard of $1.5^\circ \times 1.5^\circ$ the footprints of the beams in the example are smaller from 45° to 60° , i.e. on one half of the area covered.

Pitch compensation during the survey is a must, for the same reasons above given. Whenever the top of objects is the prime concern, a uniform scan of the sea floor in the direction of sailing will, other things being equal, yield the best result. The lack of pitch compensation during a survey is a contributory cause for the large variation in the outcome of the above mentioned trial runs on the field stone [4].

Also from the point of view of quality control it is desirable to cover the track homogeneously with measurements. Anomalies in the measurements caused by malfunctioning sensors are most easily discovered by inspecting sun illuminated plots of measurements adequately filtered to remove noise. The distribution and size of the footprints of the beams are decisive for the resolution to be achieved with this technique. In contrast, the shortcoming of sun illuminated plots of tracks sampled by multibeam echosounders with uniform beam width is well illustrated in [5].

MEASUREMENT REDUNDANCY

At present the return signal from the sea bed contains unexploited information which can be of utility for the surveyor. For instance, let's suppose that the time spanned by the return signal is measured for each beam. Then, from these measurements, the slope of the sea bed in the direction of the swath can be determined as follows. In Fig.1 a ping with pulse length ℓ and an opening angle ϕ is transmitted towards the sea bed at an angle ω with respect to the nadir. After the time t the ping touches the sea bed, so that $a=t\cdot\phi$, and starts to return back towards the transducer, where the front of the signal arrives after the time $2t$. The tail of the signal arrives at the transducer after the time $2t+2b+l$, which determines b and $\theta=Atan(b/a)$, so that the sea bed angle becomes $\theta-\omega$. In this derivation, allowance must be made for the case where 'the sign of b is negative', i.e. where the slope of the sea bed is steeper than ω . This ambiguity, however, can be resolved by inspecting the depth variation in a neighbourhood of the measurement. Conversely, the slopes enhance the feasibility of intelligent interpolation in the swath, thus allowing for a better spike detection.

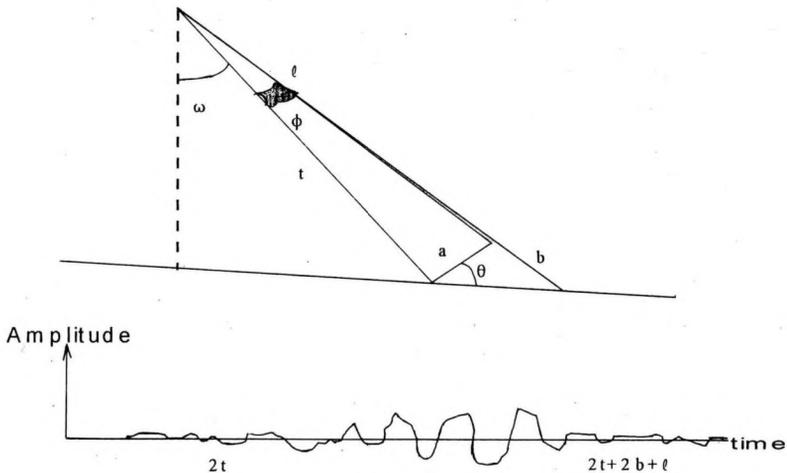


FIG. 1.

Furthermore, θ would in itself be of interest as a quality measure of the depth. For instance, for any fixed ping angle ω it appears that, other things being equal, smaller values of $|\theta|$ represent less space for errors present in the bottom detection algorithm. Finally, correlation between the amplitude of the return signal and the angle θ could be established and, if present, removed, thereby normalizing the amplitude returns for bottom slope.

Before the topic of strength can be addressed properly it is necessary to investigate the interplay between the pair (beam angle, travel time) - or equivalently (depth, position) - and small changes in the sound velocity profile. A spin-off from this investigation is the contributions to the error budget for multibeam surveys, which differ from those in [6].

Recently, the author's attention has been drawn to [7] in which the results that more or less comply with (14a,b) and (16a,b) below are presented but not derived. However, in order to be able to utilize approximations of this kind in the way suggested below, it is important to be able to form a correct estimation of when the approximation breaks down. Therefore, in the section below, some care has been taken to use closed expressions and to present limits for the remainder.

THE CHANGE IN DEPTH AND POSITION CAUSED BY SMALL VARIATIONS OF THE VELOCITY OF SOUND

A multibeam echosounder measures the travel time of a ping transmitted through the water at a specified angle. In order to transform this measurement into a depth, it is necessary to know the velocity of sound in the water along the ping's path. Supposing that the transducer is placed at the depth d below the surface of the water, and that the ping leaves the transducer at an angle $\omega(d)$. On its way through the water towards the sea bed, which is situated at the depth D , the ping follows a path governed by Snell's law, i.e. at any depth $z \in [d, D]$, the direction $\omega(z)$ of the ping can be found from the relation

$$\sin \omega(z) = p \cdot V(z) \quad (1)$$

where $V(z)$ is the velocity of sound in the water at the depth z . The constant p is found by replacing $z=d$ in (1), as the transmit angle $\omega(d)$ at the transducer is supposed to be known. At any time, the increment dt of the distance from the ping to the sea bed is reduced with dz ,

$$dz = V(z) \cdot \cos \omega(z) \cdot dt$$

while the ping removes itself the distance dx from the plumb line,

$$dx = \tan \omega(z) \cdot dz$$

and these relations can be integrated to find the travel time t for the ping from the moment it leaves the transducer until it arrives at the sea bed:

$$t = \int_d^D \frac{dz}{V \cdot \cos \omega} \quad (2)$$

and the distance x from the transducer's nadir point to the position where the ping arrives at the sea bed

$$x = \int_d^D \tan \omega \, dz \quad (3)$$

See for example [8]. The travel time t is a function of the starting angle $\omega(d)$ (or, equivalently, p) and the velocity of sound in the water $V(\bullet)$. This relationship can be made more clear by eliminating $\cos \omega$ in (2):

$$t(\omega(d), V) = t(p, V) = \int_d^D \frac{dz}{V \cdot \sqrt{1 - p^2 \cdot V^2}} \quad (4)$$

The ping transmitted from the transducer head in the direction $\omega(d)$ is generated by activating a set of transducer elements at an exact timed sequence, s_{ω} . If the velocity of sound is changed according to

$$V(z) \rightarrow V(z) + \Delta V(z) = V(z)[1 + \rho(z)], \quad z \in [d, D]$$

while the sequence s_{ω} remains the same, then the angle of the transmitted ping is changed by an amount $\Delta\omega(d) \neq 0$ unless its direction is perpendicular to the surface of the transducer. Referring to Fig.2, being θ the inclination of the transducer at the moment of transmission in accordance with Snell's law:

$$\frac{\sin(\omega(d) - \theta + \Delta\omega(d))}{V(d) \cdot [1 + \rho(d)]} = \frac{\sin(\omega(d) - \theta)}{V(d)}$$

or

$$\sin(\omega(d) - \theta) \cdot \cos(\Delta\omega(d)) + \cos(\omega(d) - \theta) \cdot \sin(\Delta\omega(d)) = \sin(\omega(d) - \theta) \cdot [1 + \rho(d)]$$

so that the change $\Delta\omega(d)$ in the angle $\omega(d)$ due to the relative change $\rho(d)$ in the velocity of sound at the transducer becomes

$$\sin(\Delta\omega(d)) = \tan(\omega(d) - \theta) \cdot \rho(d) \quad (5)$$

whenever $\Delta\omega(d)$ i.e. $\tan(\omega(d) - \theta) \cdot \rho(d)$ is so small that $\cos(\Delta\omega(d)) \approx 1$. Snell's constant for the perturbed case becomes

$$\frac{\sin(\omega(d) + \Delta\omega(d))}{V(d) \cdot [1 + \rho(d)]} \approx \frac{\sin(\omega(d)) + \cos(\omega(d)) \cdot \tan(\omega(d) - \theta) \cdot \rho(d)}{V(d) \cdot [1 + \rho(d)]}$$

which, using (1) with $z=d$, can be expressed by Snell's constant p for the unperturbed case, yielding the change

$$p \rightarrow p \cdot \frac{1 + \frac{\tan(\omega(d) - \theta)}{\tan \omega(d)} \cdot \rho(d)}{1 + \rho(d)} \quad (6)$$

in Snell's constant, caused by the relative change $\rho(d)$ in the velocity of sound at a transducer mounted at the angle θ .

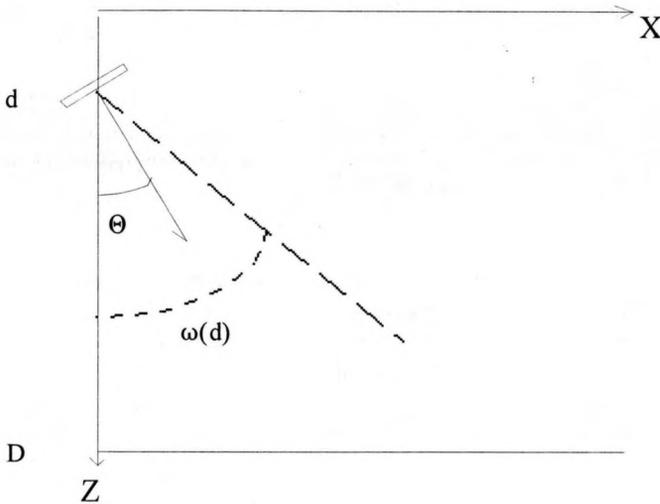


FIG.2.

As (1) governs the ping's direction at any depth, it follows that an error in Snell's constant affects the ping's path throughout the water column, as opposed to an isolated error in the velocity of sound at a layer situated at the depth $z \in]d, D[$, which only causes the ping to follow a path parallel to the correct one as soon as it enters into the neighbouring layer. In accordance with (6), being $\theta=0$, when the transducer is parallel to the layers of changing velocity at the moment it transmits the ping, ρ is unchanged, that is independent of $\rho(d)$, even though the ping by (5) is transmitted in the wrong direction. Consequently, when the velocity of sound is known throughout the remaining part of the water column, the error in the subsequent ray tracing will, as a rule, be negligible, as it only depends on the thickness of the layer around the transducer head and of the transmit angle. Conversely, for a curved transducer array, substitution of $\theta=\omega(d)$ in (5) and (6) shows that the angle of the ping at the transducer is unchanged, and therefore the ray tracing will be worked out with $\rho/(1+\rho(d))$ which, $\sin\omega$ being monotonous in the angular sector covered by most swath widths, it has the effect of accumulating the error in the ray tracing throughout the ping's path to the sea bed.

How large is this effect, expressed as changes $\Delta z, \Delta x$ in depth and distance? First, it can be observed that (1), for the changed sound velocity profile, becomes :

$$\sin(\omega(z) + \Delta\omega(z)) = \rho \cdot V(z) \cdot \frac{[1 + \frac{\tan(\omega(d) - \theta) \cdot \rho(d)}{\tan(\omega(d))}] \cdot [1 + \rho(z)]}{1 + \rho(d)} = \rho \cdot V(z) \cdot S \quad (1a)$$

where S is defined by (1a). By replacing (1a) in (4), the change Δt in travel time being a function of ρ , becomes

$$\Delta t = \int_d^D \left(\frac{1}{V(1+\rho) \cdot \sqrt{1-\rho^2 \cdot V^2 \cdot S^2}} - \frac{1}{V \cdot \sqrt{1-\rho^2 \cdot V^2}} \right) dz \quad (7)$$

The integral in (7) can be written as a series as follows: the quotient Q_z being defined by

$$Q_z = \frac{V\sqrt{1-p^2V^2}}{V(1+\rho)\sqrt{1-p^2V^2S^2}} = \frac{1}{(1+\rho)\sqrt{1+\frac{(1-S^2)p^2V^2}{1-p^2V^2}}} = \frac{1}{(1+\rho)\sqrt{1+(1-S^2)\tan^2\omega}}$$

and being $(1-S^2)\tan^2\omega=y$. Then, for $|y|<1$ the square root can be expressed by the absolutely convergent binomial series

$$\frac{1}{\sqrt{1+y}} = 1 - \frac{y}{2} + \frac{1 \cdot 3 \cdot y^2}{2^2 \cdot 2!} - \frac{1 \cdot 3 \cdot 5 \cdot y^3}{2^3 \cdot 3!} + \dots = 1 - \frac{y}{2} + R_2 \quad (8)$$

where the remainder R_2 is bounded by

$$R_2 = \frac{3y^2}{8} \cdot \left(1 - \frac{5y}{3!} + \frac{5 \cdot 7 \cdot y^2}{2 \cdot 4!} - \dots\right) < \frac{3y^2}{8} \cdot (1 + \nu y + y^2 + \nu y^3 + \dots) = \frac{3y^2}{8 \cdot (1 - \nu y)} \quad (9)$$

Neglecting R_2 , the quotient Q_z becomes

$$Q_z \approx \frac{1 - \frac{1}{2}(1-S^2)\tan^2\omega}{1+\rho} = 1 - \frac{\rho}{1+\rho} - \frac{\frac{1}{2}(1-S^2)\tan^2\omega}{1+\rho}$$

and then a division by the numerator of Q_z on both sides of the approximation sign finally yields an approximation of the integral in (7)

$$\Delta t \approx \int_d^D \frac{-\rho - \frac{1}{2}(1-S^2)\tan^2\omega}{V(1+\rho) \cdot \sqrt{1-p^2 \cdot V^2}} dz = \int_d^D \frac{-\rho - \frac{1}{2}(1-S^2)\tan^2\omega}{V(1+\rho)\cos\omega} dz \quad (10)$$

Up to this point, the error in the approximation is small and in most cases below the round off error for values of ω and ρ relevant to multibeam surveying in shallow waters. However, when terms of the second and higher orders in ρ are neglected, one finds that

$$1 - S^2 = 2 \cdot \left[\left(1 - \frac{\tan(\omega(d) - \theta)}{\tan(\omega(d))}\right) \cdot \rho(d) - \rho(z) \right] \quad (11)$$

which strongly suggests that the term depending on $\omega(z)$ in (10) should be replaced by its value at the transducer. The error made in doing this results from Snell's formula:

$$\frac{\sin(\omega(z) + \Delta\omega(z))}{V(z)} = \frac{\sin(\omega(d))}{V(d)}$$

which, going through a derivation similar to (5) above yields $\Delta\omega = \tan\omega(d)(V(z) - V(d))/V(d)$, this time depending on the variation of the sound velocity profile $V(\cdot)$ itself, that is on the intrinsic relative change, as opposed to ρ , the extrinsic relative change. As the transducer usually is mounted near the surface and $V(d)$ is likely to attain an extreme value, a breakdown may appear in the truncated Taylor series below:

$$\frac{\tan^2\omega(z)}{\cos\omega(z)} \approx \frac{\tan^2\omega(d)}{\cos\omega(d)} + (2 + \tan^2\omega(d)) \cdot \tan^2\omega(d) \cdot \frac{V(z) - V(d)}{V(d)} \quad (12)$$

when $\omega(d)$ grows beyond 50° .

Combining (10), (11) and (12) yields

$$\Delta t \approx - \frac{\tan(\omega(d)) [\tan\omega(d) - \tan(\omega(d) - \theta)] \cdot \rho(d)}{\cos\omega(d)} \int_d^D \frac{dz}{V} - \frac{1 - \tan^2\omega(d)}{\cos\omega(d)} \int_d^D \frac{\rho}{V} dz \quad (13)$$

As $\Delta z = \Delta t V(D) \cos\omega(D)$, the change in depth Δz , due to the relative change ρ in $V(\cdot)$ is finally:

$$\Delta z \approx - [D - d] \cdot [\tan\omega(d) - \tan(\omega(d) - \theta)] \cdot \tan\omega(d) \cdot \rho(d) - [D - d] \cdot [1 - \tan^2\omega(d)] \cdot \bar{\rho} \quad (14)$$

where the average relative change in $V(\cdot)$ is defined by

$$\bar{\rho} = \frac{1}{D - d} \int_d^D \rho dz$$

The change Δx in position is found from (3) in a way similar to the derivation of Δt from (2), except that the change in depth (14) has to be taken into account as follows:

$$\Delta x = \int_d^{(D-\Delta z)} \left(\frac{\rho \cdot S \cdot V}{\sqrt{1 - \rho^2 S^2 V^2}} - \frac{\rho \cdot V}{\sqrt{1 - \rho^2 V^2}} \right) dz + \int_{(D-\Delta z)}^D \tan\omega dz$$

As above a quotient Q_x is formed from the entries in the first integral

$$Q_x = \frac{S}{\sqrt{1 + (1 - S^2) \cdot \tan^2\omega}}$$

and the results from (8), (9) and (11) carry over unchanged to yield an approximation of Δx

$$\Delta x \approx \int_d^{(D-\Delta z)} \left(\left[1 - \frac{\tan(\omega(d) - \theta)}{\tan\omega(d)} \right] \cdot \rho(d) - \rho \right) \cdot (1 + \tan^2\omega) \cdot \tan\omega \cdot dz + \int_{(D-\Delta z)}^D \tan\omega \cdot dz \quad (15)$$

As above, the value at the transducer is replaced by the angle in (15), yielding an expression for Δx which, following the degree of approximation achieved, corresponds to the expression for Δz in (14)

$$\Delta x \approx (D - d) \cdot ([\tan \omega(d) - \tan(\omega(d) - \theta)] \cdot \rho(d) - 2\bar{\rho} \cdot \tan \omega(d)) \quad (16)$$

THE CONTRIBUTION TO THE ERROR BUDGET FOR MULTIBEAM ECHOSOUNDERS

The change in depth caused by errors in the velocity of sound (14) can be broken down into the influence from the error at the transducer

$$- [D - d] \cdot \rho(d) \cdot \tan \omega(d) \cdot [\tan \omega(d) - \tan(\omega(d) - \theta)] \quad (14a)$$

and that due to the error in the remaining part of the sound velocity profile

$$- [D - d] \cdot \bar{\rho} \cdot [1 - \tan^2 \omega(d)] \quad (14b)$$

A corresponding classification of (16) yields

$$[D - d] \cdot \rho(d) \cdot [\tan \omega(d) - \tan(\omega(d) - \theta)] \quad (16a)$$

and

$$- 2[D - d] \cdot \bar{\rho} \cdot \tan \omega(d) \quad (16b)$$

respectively. These four expressions are included in the error budget for a multibeam survey once the standard deviations of $\rho(d)$ and of the average of ρ are estimated. As regards the former, the expressions (14a) and (16a) depend on the type of transducer and on its mounting angle. Consequently, different measures have to be applied to different systems in order to achieve a desired accuracy.

The contribution from the spatial variation in the velocity of sound and that from the error committed when the profile is measured are different.

PROFILE MEASUREMENTS

As a matter of fact, the expressions (14a) and (14b) can be used to specify the accuracy required of instruments which measure the velocity of sound, in order that some requested resolution in the depths can be met. In the Royal Danish Administration of Navigation and Hydrography (RDANH), one of the methods employed to determine the velocity of sound in the water is to lower a probe down through the water column while it measures the temperature and the conductivity at regular depth

intervals, which of course also are subject to measurements, this time by a pressure sensor. For the part of the profile below the transducer, (14b) shows that the entity which determines the accuracy is the average of p . As the depth intervals of the probe's measurements are equidistant, the integral can be approximated by a sum of n measurements, p_i with expectation $\bar{E}(p_i) = \bar{p}$ and uniform variance σ^2 . Provided that the measurements are unbiased and independent, the central limit theorem states that the average of the measurements for all practical purposes enjoys a normal distribution centered at the average \bar{p} of p_i with variance σ^2/n . As the probe only presents one measurement at each depth interval, it follows that the standard deviation of $p(d)$ is \sqrt{n} times larger than the standard deviation of the average of p provided that the measurements are unbiased and independent and the distance from \bar{p} to the average of p can be ignored.

diff. in cm.

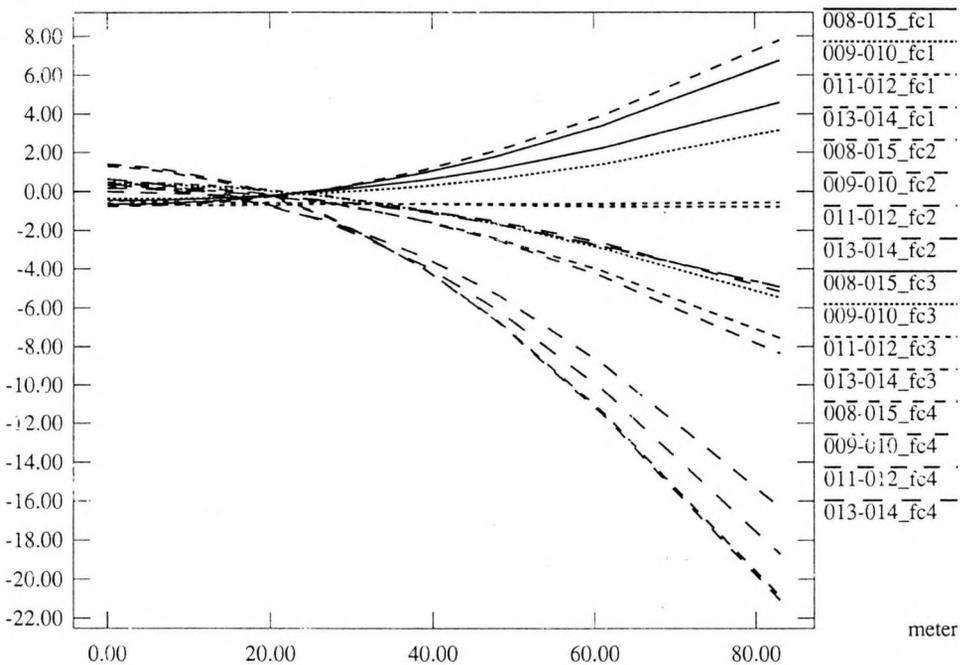


FIG. 3.- SeaBird versus GMI SVEP 5001 at 20 m for ELAC BCC.

In August 1997 a series of tests were carried out in order to investigate the accuracy of the sound velocity profiles measured by the probe SVEP 5001 which is manufactured by Geological & Marine Instrumentation Aps. Four of these probes, which are standard equipment at RDANH, were tested against a Sea-Bird 911 CTD manufactured by Sea-Bird Electronics, Inc in the waters south of Korsør harbour at a depth of 30 m. The test consisted of five trial runs, where the probes in each run were lined up with a separation of one meter between neighbouring instruments, positioned at the surface of the water for more than one minute and finally lowered simultaneously down through the water column at approximately the same speed. For each trial run,

the sound velocity profile measured by the Sea-Bird has been used to calculate the travel times between a curved array transducer and a fictitious sea bed situated 20 m below the transducer, for a fixed set of beam angles situated in the interval $[0^\circ, 75^\circ]$. For each pair of corresponding travel time and beam angle the difference between 20 m and the depth obtained by using each of the remaining sound velocities in the same run is depicted in Fig.3.

(14a) For a curved array, the contribution to Δz is found by replacing $\theta = \omega(d)$ in

$$-[D - d] \cdot \rho(d) \cdot \tan^2 \omega(d) \quad (14c)$$

Being $\omega(d)=0$ and $\omega(d)=45$ in the expressions (14c) and (14b) it follows that $[D-d]$ times $\rho(d)$ and $[D-d]$ times the average of ρ is depicted in Figure 3 at $x=20\text{m}$ and $x=0\text{m}$ respectively. Consequently, the variation of the graphs in Figure 3 at 0m . and 20m reveals that the theoretical derivation above breaks down as it is, for the case at hand with one measurement each $\frac{1}{2}$ m., predicts that the standard deviation of $\rho(d)$ should be about 6 times superior to the standard deviation of the average of ρ at this depth. Further inspection of the SVEP 5001 profiles reveals that the measurements are biased and that the variation of the measurements within the group also exceeds what is to be expected from the accuracy found during the annual calibration. As this calibration only involves static measurements of the performance of the sensors in the probe, while the trial runs in contrast are dynamic, one lesson which can be inferred from this experiment is that another method of calibration has to be employed.

Another lesson to be learnt relates to the calibration for roll of a multibeam echosounder with the twin transducers. Any error in the velocity of sound, either from spatial variation or from measurement of the profile, migrates into the transducer angle θ . When a multibeam system of this type is calibrated for roll, common sense dictates that the calibration should only include a subset of the beams and that several sound velocity profiles should be measured during the calibration in order to provide a realistic estimate of the standard deviation of the calibration.

SURVEYING IN THE PRESENCE OF SPATIAL VARIATION IN THE VELOCITY OF SOUND IN WATER

Four multibeam transducers have been selected, covering a wide range of the systems available in multibeam surveying today. For each of these transducers, the change in depth 10m below the transducer caused by a 1m/s error in the velocity of sound at the transducer, is depicted in Figure 4. Notice that the slope of the graph for EM3000 differs from zero. As explained above, this is caused by the thickness of the layer which is changed by 1m/s. In Table 1, the same values - in units of centimeters - are presented for a subset of selected beam angles in the interval $[45^\circ, 75^\circ]$, together with the interval, in brackets, which contains Δz for a simulated $\pm 10^\circ$ roll. The sound velocity profile seabird_fc1 sampled during the above-mentioned test and depicted in Figure 6 was used for the computation.

Table 1
 Δz in units of cm at 10 m caused by a 1m/s change
 in the soundvelocity at the transducer.

Name	SIMRAD EM3000	ELAC BCC	ATLAS FANSWEEP 20	RESON SeaBat 8101
mounting angle	0°	30°	60°	curved array
45°	0 [-0.2, 0.2]	0.5 [0.4, 0.6]	0.8 [0.7, 1.0]	0.7
50°	0 [-0.3, 0.3]	0.6 [0.5, 0.8]	1.1 [0.9, 1.2]	0.9
55°	0 [-0.4, 0.4]	0.9 [0.7, 1.1]	1.4 [1.2, 1.6]	1.3
60°	0 [-0.6, 0.6]	1.3 [1.0, 1.5]	1.9 [1.7, 2.1]	1.9
65°	0 [-1.0, 1.0]	2.0 [1.6, 2.3]	2.9 [2.6, 3.1]	3.0
70°	0 [-1.8, 1.8]	3.4 [2.7, 3.9]	4.6 [4.2, 4.9]	4.9
75°	0 [-3.8, 3.8]	6.6 [5.6, 7.4]	8.4 [7.9, 8.9]	9.1

The curved array transducer obviously is the most sensitive to errors in the velocity of the sound at the transducer. This is also shown in (14c). For this transducer type, the total change in depth of the sea bed caused by $\rho(\cdot)$ becomes

$$\Delta z = -[D - d] \cdot \bar{\rho} - [D - d] \cdot [\rho(d) - \bar{\rho}] \cdot \tan^2 \omega(d)$$

which proves that the absence of a 'smiling' or 'sour' sea bed is not a guarantee for a correct sound velocity.

Once a year, the Danish Oceanographic Office samples the velocity of sound in the water at several stations at Fyllas Banke off the west coast of Greenland. From the 1997 campaign, the profiles sampled at the stations 4 and 5 have been chosen to illustrate the characteristics of the expressions above at the other end of the depth spectrum. Figure 5 depicts the result of the following procedure. For the profile at station 4, the travel times from a curved array transducer to a fictitious sea bed at the depth 800m. was calculated for a set of fixed beam angles. Then the measurements at the transducer depth were interchanged for the two profiles, and the travel times were used together with these new profiles to determine the depth changes due to (14c) and (14b). For the x-axis parallel to that in Figure 2, the substitution $x=[D-d] \cdot \tan \omega$ demonstrates that both graphs in Figure 5 should be polynomials of the second degree in order to comply with the theoretical expressions and (14b) shows that the graph depending on the average of ρ should pass through [0,800], as $\tan \omega$ at that point attains the value 1.

dif. in cm

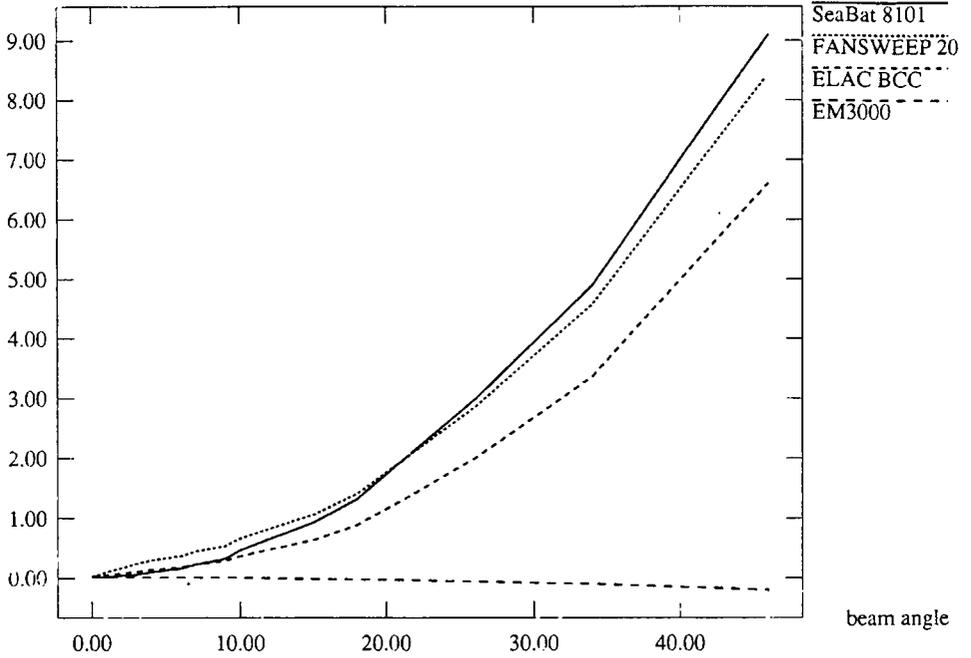


FIG. 4.- Effect at 10 m. of 1m/s change at transducer.

centimeter

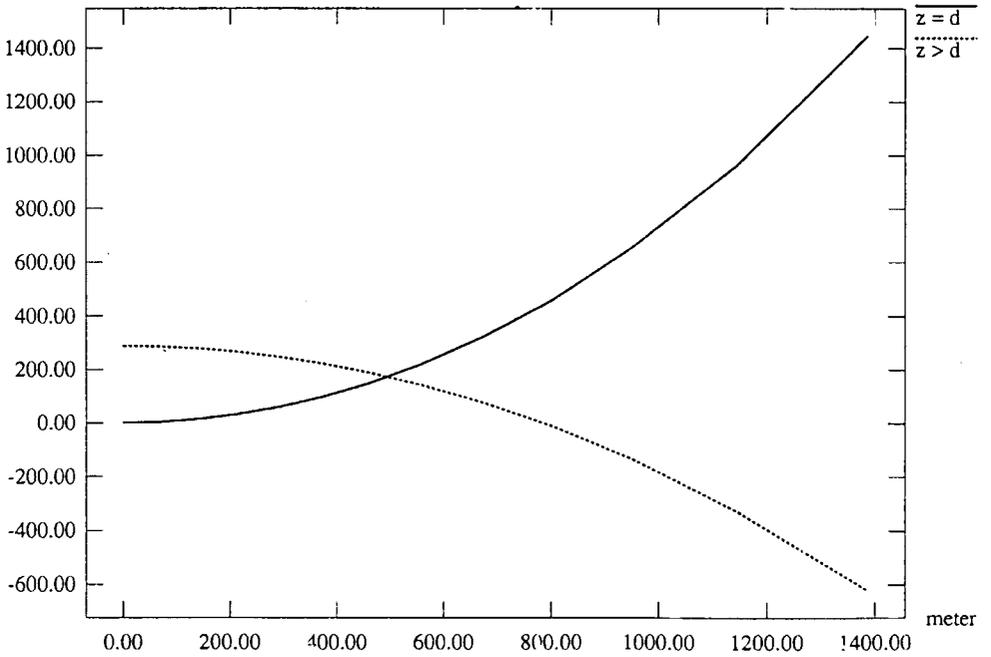


FIG. 5.- Depth differences at 800 m.

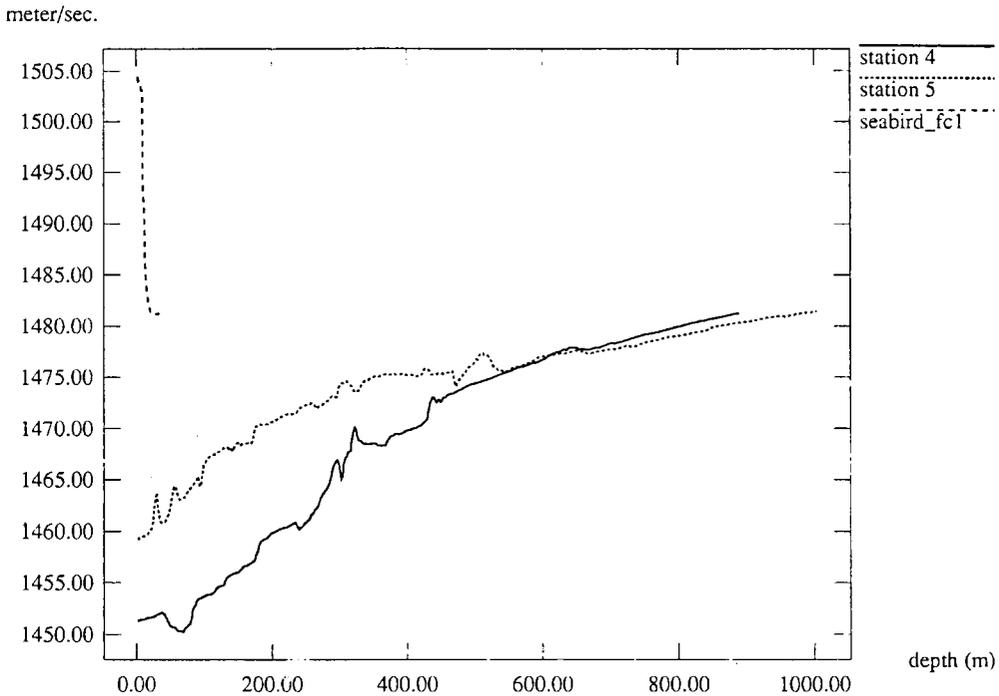


FIG. 6.- Sound velocity profiles.

It is supposed that the velocity of sound is measured at the transducer during the survey, i.e. that the influence from (14a) is eliminated - and that the depths of the pings are evaluated by combining one full sound velocity profile measured once and for all sometime during the survey with the measurements at the transducer. Then, the small variations in the velocity of sound during the survey will, if they have any effect, transform the sea bed below the survey vessel as the polynomial $c_0 + c_2x^2$. Furthermore, (14b) shows that the depths of the sea bed sampled during the survey at the beam angle $\omega = \pm\pi/4$ are correct, at least up to an approximation of the first order in ρ .

These characteristics make it feasible on the fly to localize featureless patches of the sea bed automatically and then, possibly subsequent to averaging, fit a polynomial of the second degree $c_0 + c_1x + c_2x^2$ to the depths. While the coefficient to x in this polynomial takes into consideration the slope of the sea bed athwartships, a statistical test can work out the probability that $|c_2|$ exceeds some preset limit and c_0 , which is the estimated depth at the center of the swath, can be subtracted from the average of the depths at $\pm\pi/4$ to yield an estimate of

$$[D - d] \cdot \bar{\rho} = [D - d]^2 \cdot c_2 \quad (17)$$

A more accurate way to estimate (17) is accomplished by the following change in the design of the multibeam echosounder: It is supposed that, during the survey, a beam pointing forward at the angle $\pi/4$ with nadir samples the depth of the

sea bed orthogonal to the swath. Then, the intersection of the measurements by the mentioned beam and by those in the swath can be integrated along smooth patches of the sea bed and a statistical test can work out the probability that the difference between these two series of measurements exceed a preset limit.

A stroke at the root of this matter would be to use the dual to the design above, i.e. a design where the swath is transmitted along the surface of a half-cone from a transducer situated at its apex. When the angle between the cone's side and axis is $\pi/4$ and its axis vertical, then, in accordance with (14b) the depths sampled with a multibeam echosounder of this design would react strongly to changes in the velocity of sound beneath the transducer. Of course, it would be necessary again to complement the measurements with those from a nadir beam in order to get an estimate of the average of p , which by (16b) is needed to calculate Δx .

Conclusion

Provided with a uniform beam size throughout the swath width, this multibeam echosounder would cover the sea bed below the survey vessel homogeneously within its swath width of $[-\pi/4, \pi/4]$ and furthermore react strongly to changes in the velocity of sound in water, provided this entity is monitored at the transducer. This is not always possible and the results could still be acceptable. Some items on a surveyor's list have been already discussed and their feasibility has been commented by the author; namely on-the-spot warning whenever the sound velocity profile needs to be updated, homogeneous sea bed coverage and estimates of the slope of the sea bed. A survey vessel with these capabilities would lay the groundwork for the sampling of quality assessed depths while practising economy.

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