THE TIMING OF SOUND IN THE OCEAN (*)

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ABSTRACT

The fundamental aspects of timing sound in the ocean are discussed from the point of view that the sound velocity for an elemental volume of the ocean is determined by its physical and chemical constitution. This velocity, nearly independent of frequency, can be measured automatically by modern techniques and be used to understand the phenomena exhibited by long-range sound transmission. The basic connections between the two are presented for a set of propagation experiments in which the sound projected and received has not interacted with the ocean's floor or surface. This embodies details in technique requisite to achieving eventually a precision of 10^{-5} in both local sound velocity and long-range sound pulse timing.

I. — INTRODUCTION

In recent years the growing interest in the complex properties of acoustic energy propagated through ocean regions has focussed attention on the physical parameter most basically involved, the velocity of sound in sea water. As a single variable dependent on all the heterogeneities of chemical composition, temperature, and pressure of the sea it permits a unified interpretation of acoustic properties in terms of its own variations and the boundaries of the sea medium.

It has been shown recently [1] that these velocity variations are significant when they are observed with respect to time and horizontal coordinates as well as depth.

(**) A large part of this paper was presented at the Sixty-fourth Meeting of the Acoustical Society of America, Seattle, Washington, November 7-10, 1962, in the Invited Talk "Use of Velocimeters in Deep-Ocean Water."

In the classic technique the thermodynamic variables in the sea were measured first. Samples of water were extracted from great depths to be chemically analyzed at the surface and thermocouples or thermistors were lowered to get a temperature-depth profile. The chemical analyses were interpreted as a single parameter, the salinity, to get a salinity-depth profile. This was based on collected data showing that mutual ratios of all but a negligible amount of the different [2] chemical compounds in sea water were invariant with respect to depth and locale while the total composition varied by 25 percent. Using laboratory measurements of the compressibility of standard sea water as a function of salinity, temperature, and pressure it is possible to construct a complete profile of the sea with salinity, temperature and pressure as a function of depth [3]. Then comparable relations [4-6] of sound velocity as a function of salinity, temperature, and pressure, give the desired velocity-depth profile.

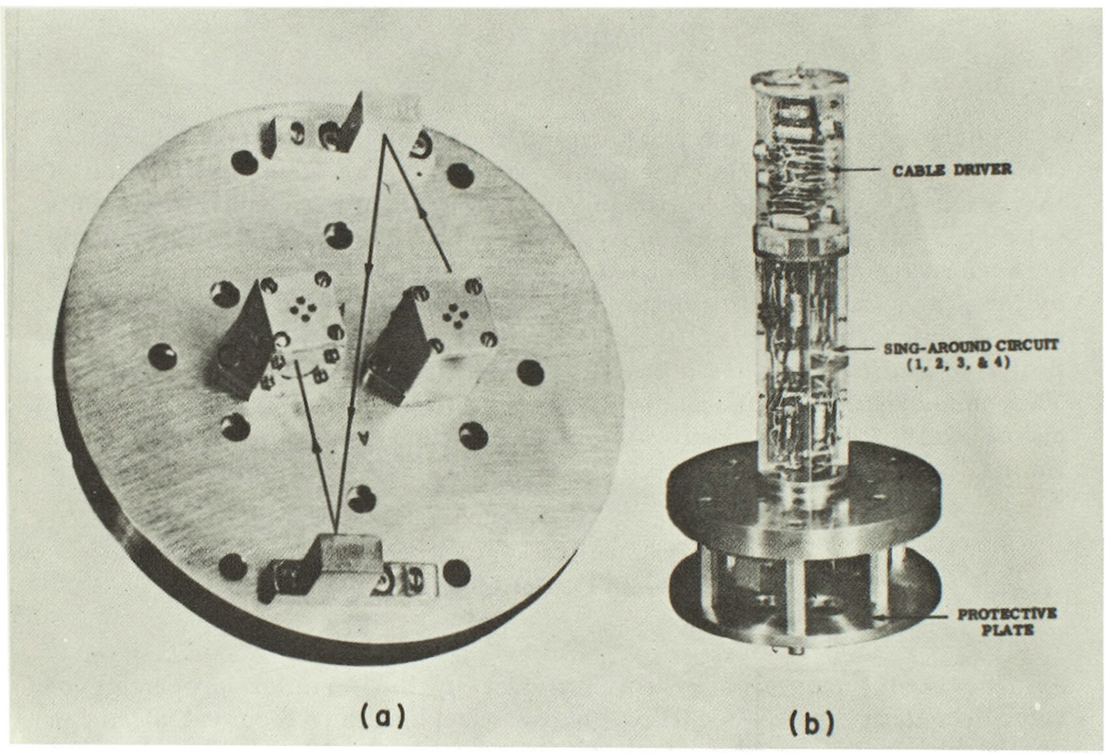


FIG. 1. — Velocimeter
a) Viewed from below without protective plate
b) Viewed without protective cylinder around circuit

In 1959 TCHIEGG and HAYS [7] introduced *in situ* measurements of sound velocity making use of the sing-around principle of automatic ultrasonic pulse timing developed by GREENSPAN and TCHIEGG [8] for pure water. Direct measurements of sound velocity, with suitable provisions for measuring depth precisely [9], at a particular ultrasonic frequency, obviate completely the need for the older work where the only interest is the interpretation of purely acoustic phenomena. The direct *in situ* measurements possible with the automatic velocimeter also provide some observational comment about the oceanographic determinations. In certain instances [10] where there are salinity and temperature data to correlate with them it is impossible to reconcile the two sets of data on the basis of the older procedure and modern laboratory determinations [11]. This is probably due to a combination of inaccurate old compressibility measurements [12, 13] and an unsuspected failure of the constant ratio hypothesis [10, 14].

II. — THE VELOCIMETER AND ITS CALIBRATION

The principal technical features of the automatic velocimeter are shown in the photograph of Fig. 1. The protective plate, left on only for the most casual use, is removed in the bottom view at the left. This particular meter shown is a few years old, but there has been no change in the basic design. Note particularly the construction on the bottom plate as seen here, where a short, high-frequency pulse traverses a folded path between a sending and receiving transducer. The signal at the receiving transducer, through amplifiers and a SCHMIDT trigger circuit, generates a new pulse at the sending transducer [8]. This process automatically iterates, providing a rate of pulse generation that is the inverse of the propagation time in the water path. Fully assembled the circuit is enclosed dry in a thick-walled cylinder (½ in.), which forms a water tight envelope and its components then feel only the temperature of the surrounding sea. The transducers on the plate are packed in oil and operate under deep-sea pressures hydrostatically applied through the oil packing.

For this NBS-designed meter used in most American studies the water path propagation time varies about an average of 136 μsec in use at sea. In effect, we are performing with great precision and frequency an ultrasonic propagation timing experiment using short pulses of 3.6-Mc sound and a fixed path 6 in. long. The rigid integrity of this path laid out on a steel plate presents another aspect in the use of the velocimeter as a precision ruler for measuring oceanic depths and ranges.

Let us consider the general salient factors in its calibration and precision. Figure 2 shows a fairly compact arrangement which can be set up on board ship as well as in a laboratory. It is possible to have the water sample here mixed well enough so that the spatial variations of temperature are less than 0.01 °C. For a few minutes it will be stable in time as well. Our sound transmission path, here, will be stable, affected only slightly by temperature. While several contemporary investigators disagree [8, 11, 15], we can look forward to knowing eventually the sound velocity of pure water as a function of temperature to better than 0.03 m/sec or about 1:50 000. Now, if this meter performs the same in sea water as in pure water, with care we can achieve the same precision in the ocean. This is clearly implied by the following considerations :

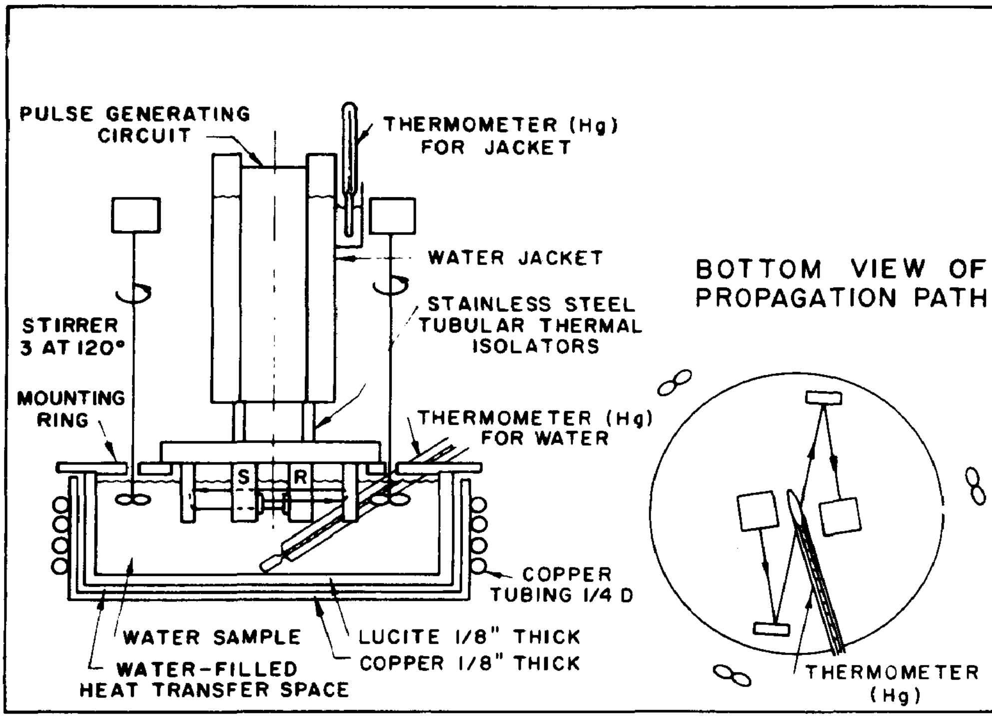


FIG. 2. — Calibration assembly for water sample test

1) The effective length of the sound transmission path from S to R remains the same. For the same speed range we have the same wavelengths and the same pattern of wavefronts. In pure water the temperature range 20° - 50° provides a wide range of sound velocity matching that which is found in sea water between the surface and depths down to several thousand meters.

2) The pulse regeneration time *after* reception at the receiving transducer, R, depends mostly on the electrical parameters of the transducers and amplifying circuits.

The period, T, of our pulse repetition rate, F, is the sum of three required times in sequence.

$\frac{1}{F} = T = \tau_w$	transmission time in water sample	134 - 139 μsec
$+ \tau_i$	rise time of first received wavefront before threshold of operation is reached	$\sim$ 0.04 μsec
$+ \tau_e$	electrical delay in amplifier and trigger circuits before pulse is regenerated at sender	$\sim$ 0.4 μsec

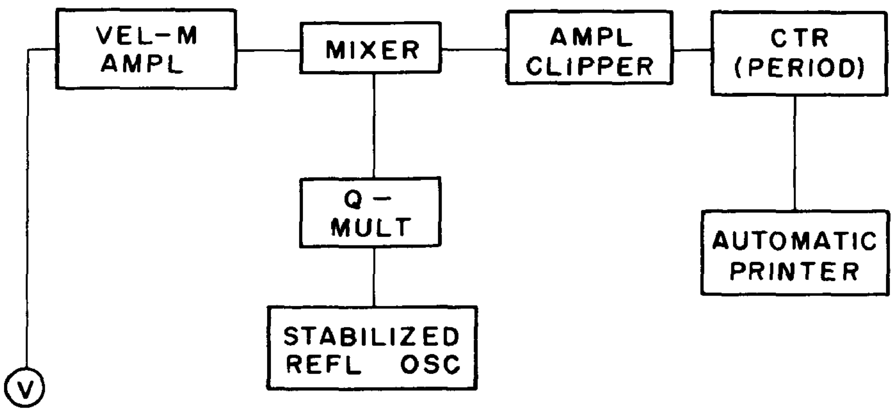
While τ_e is independent of the particular water sample, τ_i will be affected by different attenuations. However, a change of 10 percent, which can be monitored simply by comparing received wave amplitudes in pure and sea water, will affect T or F by only 1 part in 30 000.

The electrical delay τ_e will depend somewhat on the ambient temperature. If the whole meter were to be totally immersed in a large temperature-controlled water bath, we would face an unknown difference in τ_e, the electrical delay time, that occurs between the calibrating temperatures, averaging 34 °C, and a typical deep-sea operating temperature of 4 °C. A separate water jacket for a pulse generating circuit that is thermally isolated from the water path is shown in Fig. 2, temporarily in the place of the heavy cylindrical shield used at sea. When the shield is remounted, the thermal isolators remain. In this manner the velocimeter can be calibrated at any circuit operating temperature. Indeed, it was found that the error just mentioned may get to several parts in 10 000.

Thermal expansion of the bottom plate will expand the transmission path, but is is rather small and known; for certainty in this the water sample bathes the plate, fixing its temperature. The copper container is cooled through the coils or heated from below by a hot plate. In practice it is convenient to use a short pulse of heat and let the water temperature vary through a maximum. At such a steady state the net heat flow into or out of the water sample is minimal. A calibration procedure incorporating these ideas and taking account of the thermal expansion for the pure water temperature variation has recently been described [16] in detail. The important point is that the pulse repetition rate F can be linearly related to the temperature dependent pure water sound velocity through a log-log plot of constant slope. This permits a systematic least-squares fit of the calibration points. In this way the calibration sensitivity becomes better than the accuracy of our present knowledge of C(T) derived from the basic experiments [8, 11, 15]. Although this latter may be no better than 10^{-4} we can expect some improvement in the future. The chief difficulty at present is the guiding effect that occurs when we transmit acoustic pulses between two transducers in a confined medium such as a tube. The McSkimin [15] measurement, done at high frequencies, avoids this effect. With an accuracy and sensitivity of calibration both being around 10^{-5} one can, as we shall see later, obtain significant geodetic measurements through acoustic transmission.

To take full advantage of this eventually expected accuracy in C(T) for pure water, some simple improvements in the apparatus of Fig. 2 can be made. Three more stirring propellers would probably halve our water circulation time. The mercury thermometer can be replaced with a fast response, platinum resistance strip which should be calibrated against the very thermometer or thermocoupler used in the standard distilled water measurements. Sensitive differential thermopiles may be added to measure the gradients between the thermometer, a central point between S and R, and points along the rim. When we know in advance the salient operating temperature and sound-velocity range, the circuit temperature can be appropriate and the pure water sample used for a minimal time with comparative chemical and gas analyses done before and after. Finally the pulse rise time, τ_t, should be monitored by careful comparison of received wave amplitudes so as to account for differences of even a few percent between pure and sea waters.

As an aid in calibrating, as well as in modern use at sea, we require a sophisticated means of monitoring the velocimeter's operating frequency which is around 7.3 kc. On the best electronic counters simple frequency counting good to 1:70 000 takes 10 sec and can't be done more than once every 20 sec. However, there is no need to measure the whole frequency. In the course of calibration and at sea F varies between 7.2 and 7.4 kc. It is only the variation in this range that has physical importance. A good tuning fork oscillator of fundamental frequency 7.2 kc will have a stability of 10^{-6} pertinent even to cycle-to-cycle changes in its period. Mixing its output with the typical 7.3-kc velocimeter signal yields a 100-cycle beat frequency, whose period can be monitored to 10^{-4}. This measures the 7.3-kc signal to 1:730 000.



SENSITIVITY VS BEAT NOTE, ΔF FOR 100 KC COUNTER

ΔF	10 PER	LEAST COUNT	SENSITIVITY
50	200 MS	10 μS	2.5×10^{-3} CPS
100	100 MS	10 μS	1.0×10^{-2} CPS
200	50 MS	10 μS	0.4×10^{-1} CPS
316	31.6 MS	10 μS	1.0×10^{-1} CPS

FIG. 3. — One-Velocimeter data sampling system

Such a system was built and is shown in Fig. 3 with a few extra electronic embellishments. The Q-multiplier is a very sharp filter with a " Q " of 85 - 100 and eliminates the noise components of the oscillator's output. The oscillator used a crystal-stabilized fundamental of 1 kc multiplied seven times which necessitated this. Actually, cycle-to-cycle instability is averaged out in the several hundred oscillator cycles comprised in the ten-period time interval. The amplifier clipper gives the counter a sharper crossover point at the zero signal level. The rate of sampling here is limited by the mechanical automatic printer to four lines per second. Even so, in comparison with simple frequency counting, data point density has been increased eighty times and precision up to forty times with this elementary elaboration.

The system has been used at sea in two applications, discussed later. It has also been used with the unimproved calibration equipment to test the basic stability of the velocimeter as well as the mixing of the water bath. Visual observations of mechanical mixing using neutral floats revealed that they cycled from top to bottom and around the bath in a few seconds. As a temperature extremum is approached following a heat pulse of the water bath, the periodic fluctuation of velocimeter frequency gradually diminishes until we have left a small random fluctuation while the temperature maintains a steady state.

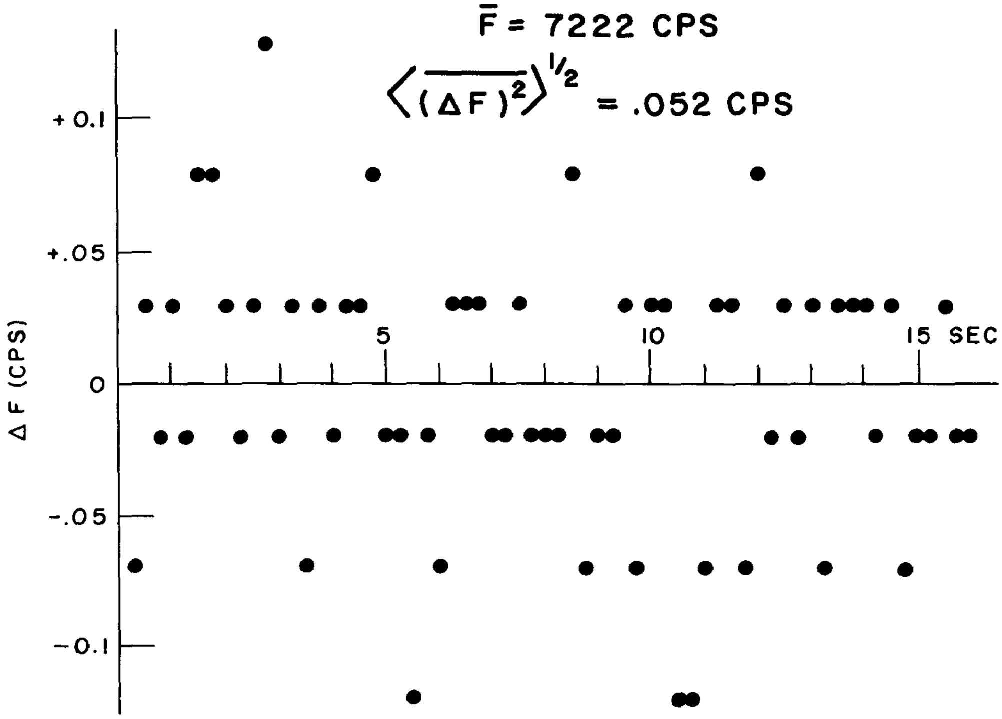


Fig. 4. — Velocimeter fluctuation in distilled H_2O at 23.6° C

Figure 4 is one case among many which exhibited the same statistics, even though $\bar{F}$ varied and different sea water samples were used. The reference oscillator in this test is 7 000 cps, and the least count in measuring the period of the 222-cps beat signal gave rise to the easily evident discrete spacing of the ordinates. The frequency fluctuation of 0.052 cps is most probably due to fluctuation of the average temperature of the transmission path amounting to 0.004 °C. As a figure for the residual instability of the velocimeter it represents 1:140 000. The contribution of the reference oscillator to this value was negligible according to a separate check. The signals from two oscillators of 7 200 and 7 000 cps were filtered through separate Q-multipliers and mixed together. The issuing 200-cycle beat note was stable to within 0.014 cycles or at least 1:500 000.

III. — SMALL EFFECTS AT SEA

There are several effects in the normal use at sea of the velocimeter which are too small to be effectively monitored in a laboratory but by this same virtue can be compensated or calculated well enough that our final precision is not seriously affected.

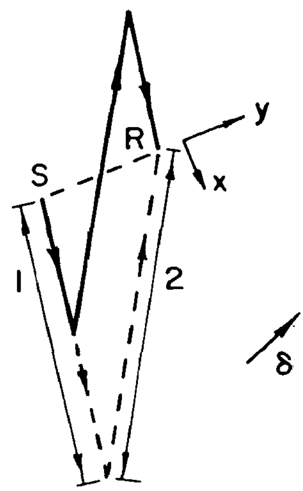


Fig. 5. — Velocimeter sound path in presence of water motion. δ is virtual relative displacement of transducers

A. Water Flow across Transmission Path

The mass movement of water affects sound transmission between two points by virtually displacing them with respect to each other. For a sound pulse the transit time is affected by a small amount, the ratio of the component of water flow in the path direction to the pulse propagation speed. The velocimeter path (Figs. 2 and 5) is partially folded to minimize this effect but it still becomes appreciable for even the moderate flow rates such as the 1 m/sec we encounter in towing the meter (Fig. 13). The path is shown schematically in Fig. 5. The z-direction which contains no components of this path is perpendicular to the plane shown.

Let us consider the effect of a small water movement, δ, which occurs during the 136 μsec required to transmit one pulse. It is very small and can be regarded as equivalent in effect to a small displacement of either the sender or receiver. The transmission path is not folded completely, the distance $S \rightarrow R$ being quite sizeable. Assume that the water flow is constant during the single transmission so that δ can be divided proportionately between the two linear path sections (1) and (2). For any straight sound path, r

$$\frac{\delta}{r} = \frac{V}{C} = \frac{\text{Water flow rate}}{\text{Speed of sound}} \tag{1}$$

with similar equations in terms of the components. The section (1) is, of course, the sum of two equal, parallel parts of the actual path and the same is true of (2). For no water flow, i.e., $\delta = 0$, we have an undisturbed path with the scalar sum

$$\langle r \rangle = \langle r_1 \rangle + \langle r_2 \rangle .$$

With water flow, on the other hand, we have a disturbed path and, using proportionate division, for each section we add the appropriate δ and compute its new total distance. For example,

$$r_1 = [r_{1x}^2 + 2r_{1x}\delta_{1x} + \delta_{1x}^2 + r_{1y}^2 + 2r_{1y}\delta_{1y} + \delta_{1y}^2 + \delta_{1z}^2]^{\frac{1}{2}}.$$

The scalar sum of these new distances, r_1 and r_2, approximating the square roots is

$$r = r_1 + r_2 = \langle r_1 \rangle + \langle r_2 \rangle + \frac{r_{1x}\delta_{1x}}{\langle r_1 \rangle} + \frac{r_{2x}\delta_{2x}}{\langle r_2 \rangle} + \frac{r_{1y}\delta_{1y}}{\langle r_1 \rangle} +$$
$$+ \frac{r_{2y}\delta_{2y}}{\langle r_2 \rangle} + \frac{1}{2}\left(\frac{\delta_1^2}{\langle r_1 \rangle} + \frac{\delta_2^2}{\langle r_2 \rangle} + \text{small terms in } \delta^2\right).$$

Since the δ and r components are proportional, that is,

$$\frac{\delta_1}{\langle r_1 \rangle} = \frac{\delta_2}{\langle r_2 \rangle} = \frac{\delta}{\langle r \rangle}; \; \frac{\delta_{1x}}{\langle r_1 \rangle} = \frac{\delta_{2x}}{\langle r_2 \rangle} = \frac{\delta_x}{\langle r \rangle}; \; \frac{\delta_{1y}}{\langle r_1 \rangle} = \frac{\delta_{2y}}{\langle r_2 \rangle} = \frac{\delta_y}{\langle r \rangle}$$

and since the path is folded

$$r_{1x} = - r_{2x} ,$$

we have, finally, that

$$\frac{r}{\langle r \rangle} = 1 + \frac{r_{1y} + r_{2y}}{\langle r \rangle}\,\frac{\mathrm{V}_y}{\mathrm{C}} + \frac{1}{2}\left[\left(\frac{\mathrm{V}}{\mathrm{C}}\right)^2 + \text{small}\left(\frac{\mathrm{V}}{\mathrm{C}}\right)^2 \text{term}\right] \quad (2)$$

the ratio $\dfrac{r}{\langle r \rangle}$ giving the fractional change in path, or transmission time, due to water flow. Only the water flow component parallel to S — R or y will contribute to the first order term. For the NBS velocimeter

$$\frac{r_{1y} + r_{2y}}{\langle r \rangle} \cong 0.19$$

A water movement of 1 m/sec in this $\hat{y}$ - direction will introduce an error of 1:8 000. Clearly, in all applications where the velocimeter moves appreciably, the direction S - R should be kept perpendicular to the movement, if possible the transmission path plane itself, when only the negligible second order term matters.

B. Pressure-induced Path Length Changes

At depths of several thousand meters even the $\frac{1}{2}$ -in. thick stainless steel of the bottom plate and cylinder experiences deformation. Reference to Fig. 1 shows that the pressure outside the cylinder containing the circuitry will tend to buckle the lower plate, tilting the reflector plates inward and bending the transmission path out of its plane. If the bottom guard ring is left off, the buckling effect for a depth of a thousand meters may be over 1.10^{-4}. This is the figure for a straightforward calculation by Willig [17]. However, the $\frac{1}{2}$ -in. annular bearing surface between plate and cylinder will provide extra rigidity and the 3.6-Mc wavefronts will be incident on the receiving transducer at a new angle. One of the commercial manufacturers [17] of the instrument will soon be testing the effect using double-thick plates and making comparisons. Finally the effect will be eliminated in future models. Eventually only one transducer will be used for both receiving and sending a pulse which travels from a thick base of the cylinder to a reflector suspended below on the axis by a rigid connecting frame. Of course this eliminates all water flow effects as well.

C. Frequency Dependence of Sound Velocity

Acoustic propagation experiments, particularly those carried out over extended ranges, are done with low-frequency waves between 0.1 and 100 kcps. The propagation of the 3.6 Mc wavefronts is slightly faster than those of low frequency due to the absence of certain molecular vibrational effects, and this change must be known for sea water.

If the sea water medium were a continuous fluid whose bulk elastic modulus were independent of the time rate of pressure changes, there would be no dispersive effect for sound waves. However, in real sea water the pressure fluctuation in the passage of our wavefront must be felt by a composite of the translational or external degrees of freedom of the constituent molecules and the internal degrees of freedom of these molecules. As the sound wave energy progresses, it is imparted first to the external degrees of freedom. A delay in exchange between the external and internal degrees of freedom is a relaxation phenomenon that becomes important as the frequency of the sound wave is raised. In sea water this occurs in the range 150 to 200 kc depending on its temperature. After the frequency is high enough so that there is no exchange, we have a new

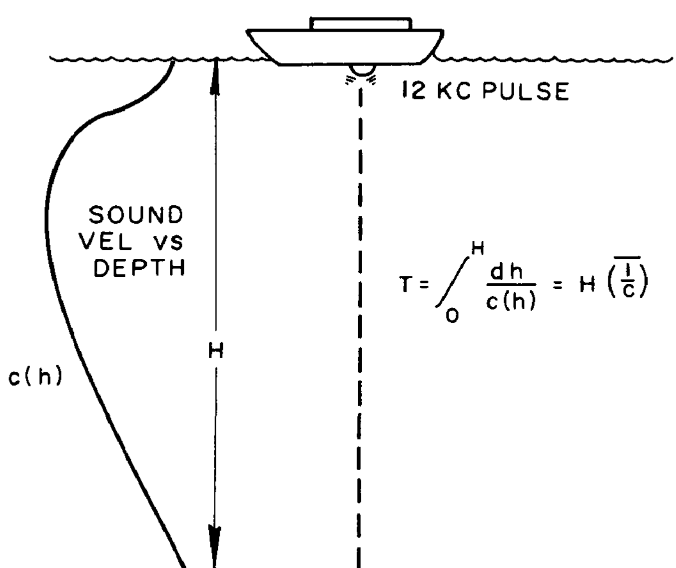


Fig. 6. — Echo-sounding for ocean depth measurement

sound velocity somewhat higher than at low frequencies. Using absorption measurements on $MgSO_4$, apparently the major cause of the effect in sea water, the change is $+ 1:60\,000$ [18]. Because of the difficulty of a direct measurement the uncertainty in even this small effect needs careful study in any work requiring such high precision.

IV. — PRECISION DEEP-SEA BATHYMETRY

An extremely important use of the velocimeter can be easily forgotten in the casual exercise of oceanographic routine. When we want to measure the ocean depth below a ship in deep water, in modern practice an echo sounder mounted on the bottom of the ship's hull transmits a short pulse of 12-kc sound and shortly thereafter receives the pulse reflected from the bottom, automatically recording the elapsed time. By itself this means nothing, but if we know the effective sound *velocity* to use, we can calculate the depth of the ocean according to the simple equations of Fig. 6. Note that it is the reciprocal of velocity which must be averaged. We can use a velocimeter to determine $c(h)$. Essentially in this work it is a standard of length, as we compare the echo-sounding transmission time, T, to the velocimeter path propagation time, $\frac{1}{F}$. Furthermore, the units of little h used in measuring $c(h)$ do not have to be known absolutely. Just so long as our variation of c is known from top to bottom in some proportional units we can calculate $\left(\overline{\frac{1}{c}}\right)$ over the whole depth HOW. and then infer large H from T. Hence, we can lower a velocimeter and use the wire measurement for h, suspending below it a weighted switch which closes upon reaching the ocean bottom. Our arbitrary h scale is easily corrected in terms of H.

This procedure cannot be recommended for rapid large-scale surveys. It is suited to special cases, such as a deep-sea anchored ship which is the prime operator in an underwater acoustic experiment. The limits of echo sounding that were discussed by Gabler [19] were due primarily to the impossibility of knowing $\left(\overline{\frac{1}{c}}\right)$ for a large region. This difficulty is transcended in our tightly localized situation. However, there are still unavoidable errors. If the ship is anchored, $c(h)$ and T can be measured at the same time. However, while the T measurement takes a few seconds, $c(h)$ may require hours. How stable in time at a fixed position is $c(h)$?

We have both the experience of hydrographers in water sampling encountering apparent temporal temperature and salinity variation in deep water and the recent observations of Hudson Laboratories using the velocimeter. If the velocity data at widely different depths are obtained at different times, even only an hour apart, they are not related closely enough to establish the physical reality of a continuous curve drawn through these data points. This holds true for most of the world's oceanic regions. The

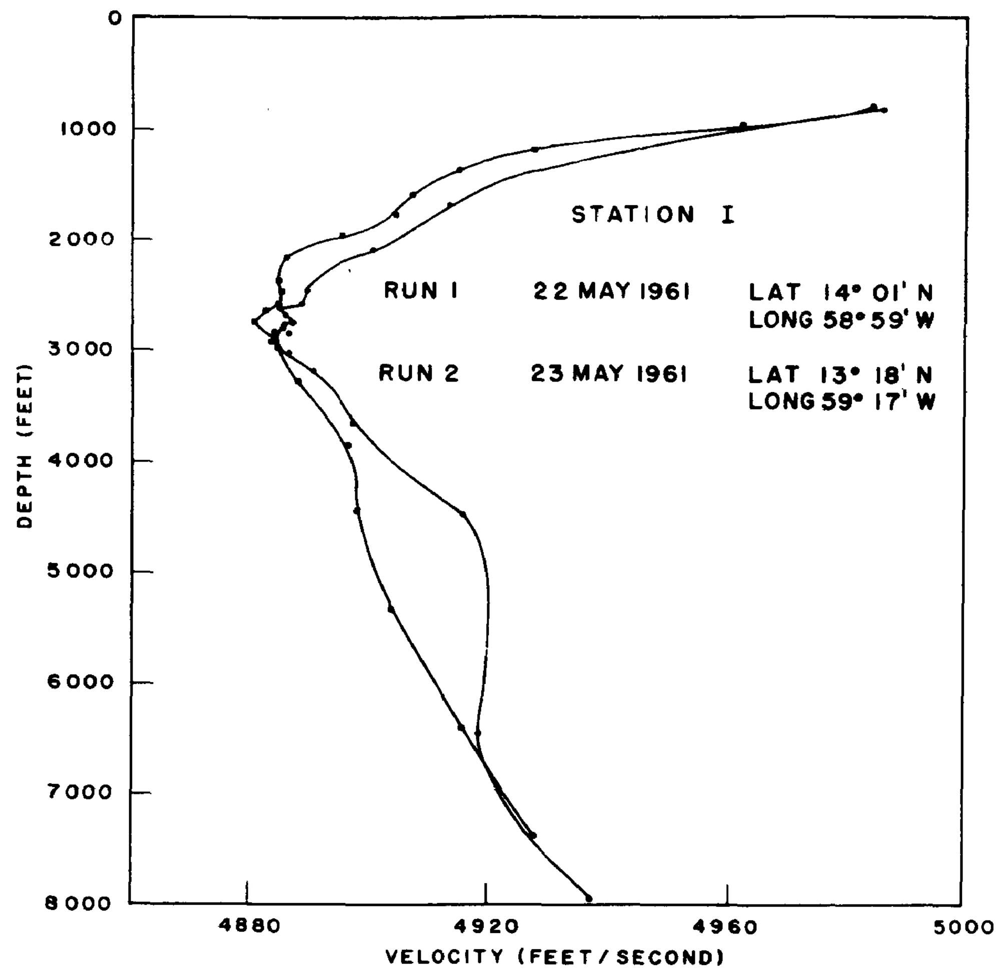


Fig. 7. — Velocity-Depth profile near Barbados

temporal sound velocity variation destroys any basis for the presentation of such a curve considering that it implies the existence of a specific velocity-depth variation as instantaneously true for a specific time. Consider the following two examples, shown in Figs. 7 and 8.

The location of Fig. 7 is a few hundred miles northeast of Barbados. It's the most exaggerated case of velocity profile instability I know, amounting to much more than a part in 500 for the time difference of a day. The raising and lowering each took 12 hours. Obviously their separate collections of points joined by separate curves do not merit any implication of separate physical integrity. Figure 8, on the other hand, is a more honest representation.

The location was very close to Bermuda, and we see as horizontal bars the velocity variation that occurred within an hour while the velocimeter depth remained stationary. The biggest variation is several parts in 5 000. This is probably a more typical situation on which to base an echo-sounding procedure. While it is true that in averaging over a sequence of

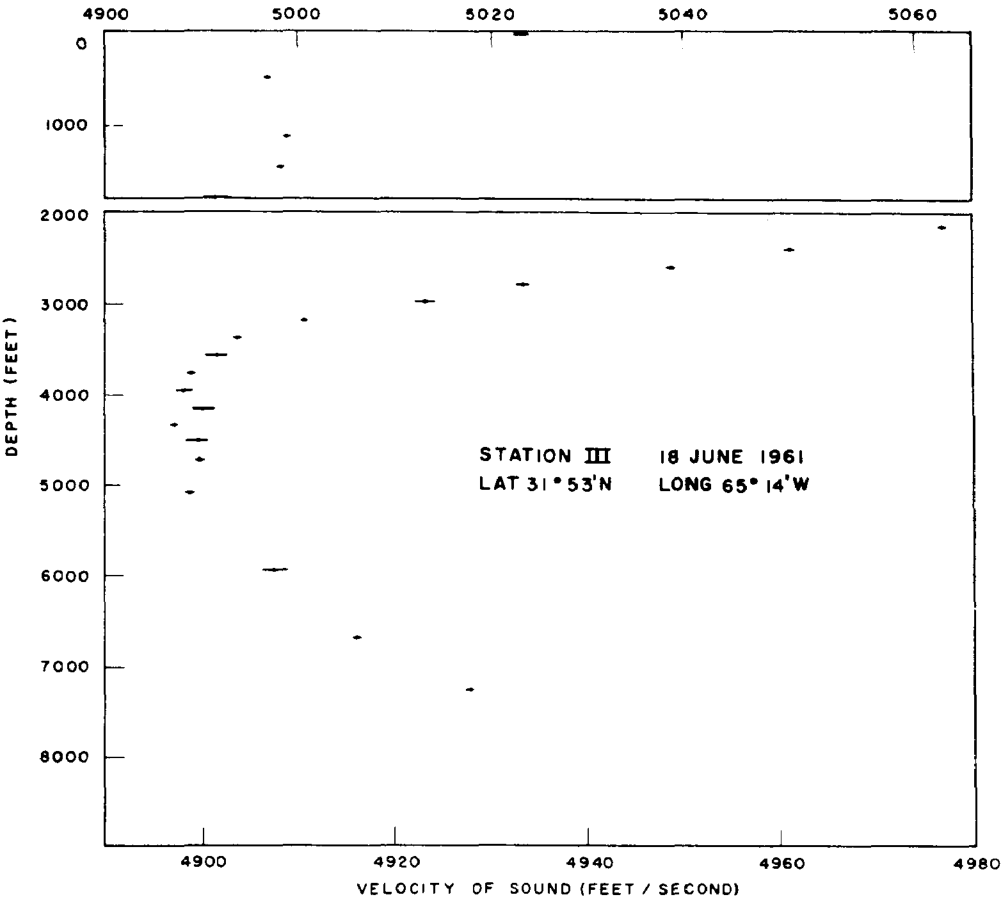


FIG. 8. — Velocity fluctuation vs depth near Bermuda

depths whose velocity variation is uncorrelated [20] the variation may be minimized somewhat [21], it seems wiser to trade precision for rapidity of sampling.

Accordingly, for precision echo-sounding work, one should aim at an accuracy of 1:5 000, calibrate the velocimeter very simply on board the ship, and lower it rapidly between depth stations, recording its operating frequency continuously at each depth. For high speed lowering using the rapid sampling technique to get a near-instantaneous profile, the water flow error is treated either as a known quantity (S-R vertical) or a null contribution (velocimeter vertical with opened bottom plate). At a series of depth stations simple electronic frequency counting achieves the calibration accuracy. For total depths within 2 500 m the error in depth measurement should be less than 1 m. Unquestionably this accuracy can be exceeded with, for example, a vertical array of well-calibrated velocimeters and basically we'll be limited only by the instability of vertical transmission [21].

V. — HORIZONTAL AND NEAR-HORIZONTAL SOUND-RANGING

In acoustic propagation studies the effects of velocity fluctuations are very similar, but the purposes behind the studies shift our area of concern from the velocity-depth profile to a sharp emphasis on the velocity fluctuations themselves.

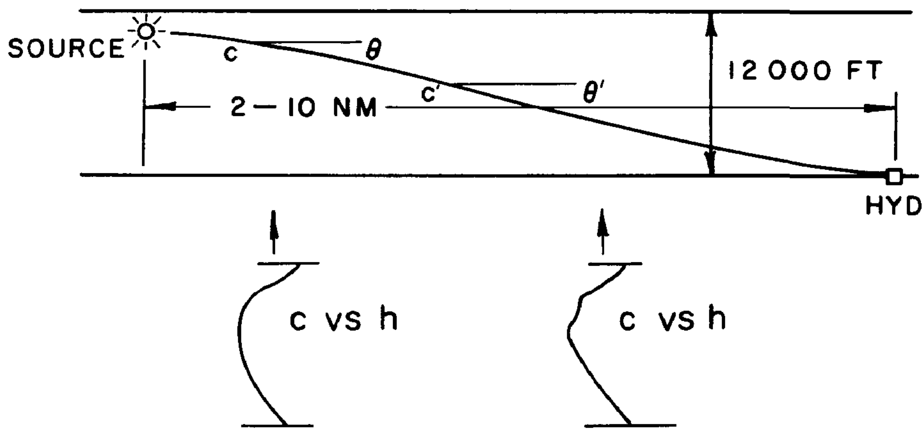


Fig. 9. — Acoustic ranging at close distances

A. Intermediate Ranges; Slanted Directions

For horizontal ranges up to 10 miles or so, it is possible to follow sound pulses along the specific paths we call rays. With a known sound velocity-depth profile and some given initial angle and depth, we can calculate the path of our ray to some other depth as it follows Snell's law of refraction :

$$\frac{c}{c'} = \frac{\cos\,\theta}{\cos\,\theta'}\,.$$

Such a typical experiment is shown in Fig. 9.

Only a direct path from source to detector is shown. In addition, we have present the many other paths which require reflection at the surface or bottom or both. While we have the same desire as in echo sounding for an instantaneous profile of c vs h, there is the added complexity of the profile depending on the geodetic coordinates. Its variation over a few miles is probably as great as its time variation at a fixed point. Plainly we cannot now resolve these questions without a tremendously extensive velocimeter survey. For a precision of 1 m in range the general procedure recommended for echo-sounding seems best, bearing in mind some points of difference.

If we were to take in sequence a number of velocity profiles at the same place or at the same time at a number of neighboring locations, we might construct a composite profile, as in Fig. 10. The envelope of this set

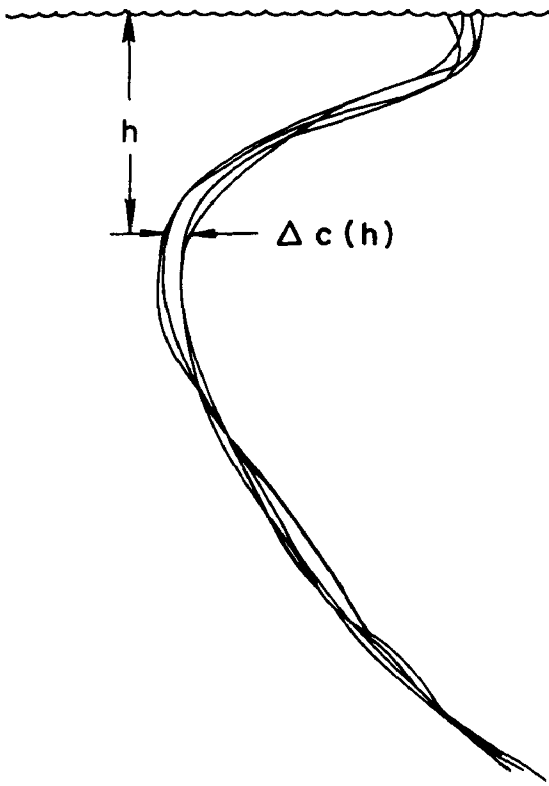


FIG. 10. — Hypothetical superposition of instantaneous velocity-depth profiles

of profiles will have a width at each depth that we may regard as the variation in sound velocity. However, since the envelope is of only a few profiles, we have only a few data points, namely, the intersections with each depth level. For acoustic ranging at distances of a few miles we wish to be able to calculate the kinds of angular dispersion of these rays that will be due to the velocity fluctuation inherent in the sea. For such calculations we want continuous data at a set of discretely spaced depth levels rather than a few data points haphazardly obtained for a whole continuum of depth levels. In this class of acoustic experiments it's best to lower the velocimeter to a series of depth levels in sequence, staying at each level for an hour and not taking data at all between levels. Our recording instrumentation remains the same. As before, with a commercial electronic counter we record the 7-kc velocimeter signal. Usually there is a 10-sec gate time so that a 7-kc signal can be frequency monitored to 1:70 000 every 20 sec. This is fast enough for most deep-sea velocity fluctuation. If the meter is suspended from an anchored ship, the effects on depth of the swinging cable will be far slower than the velocity fluctuations. However, because of this swing and also deep-water currents, we face the effect of water flow across the velocimeter's transmission path. This assumes added importance since we are trying to follow continuously the velocity fluctuation with the added monitoring sensitivity. Using an anchored ship a horizontal mounting of the meter with S — R vertical will normally be safest. Swinging cable and deep current effects will be minimized. The heaving of the ship will affect it, but this is well damped at the deep suspension levels. It's best, generally, to forget about doing this sort of work in the deep ocean at levels shallower than 500 ft.

B. Acoustic Geodesy

So far we have presented the use of our velocimeter as a standard of length in two short-range sound propagation experiments, with vertical and slanted paths. Let us project this application to its ultimate as a geodetic measure. Long-range horizontal propagation experiments are beginning to show us how stable transmission times can be over ranges such as 1 000 miles. For example the work of BERMAN and STERNBERG [22] offers promise that some day we may be able to time low frequency sound that has traveled this far over a precisely defined path, namely, the locus of ocean depths where sound velocity is minimal. Their work indicates that the travel time instability is no greater than 10^{-5} for sound paths which, in the vertical plane, undulate across this locus. If such a precision can be matched in sea water sound velocity, we could accomplish our geodetic measurement to better than 50 ft. This may be the only application which demands the highest absolute precision of the knowledge of sound velocity as distinct from sensitivity of relative measurements. However, it is so important that its bare possibility merits some detailed exploration.

Naturally the required precision would have to be obtained in the basic pure water measurements, and all the refinements detailed in Section II carried out. We can presume that the effects of Section III will be taken into account by changes in the velocimeter structure and careful work on the velocity dispersion effects of relaxation processes in sea water. This leaves the question of sampling the long ranges of a geodetic experiment with a small number of meters. Specifically, a sound pulse requires 1 200 sec or 20 minutes to traverse a range of 1 000 nautical miles. To be certain about the measurement the velocity should be measured every point along the way at the precise instant of the sound pulse's passage. However, the period of stability [22] of the whole propagation time is over 1 hour. Consider then a set of measurements sufficiently dense and equally spaced along the path to give demonstrably a value for the average reciprocal of velocity (like $\left(\overline{\dfrac{1}{c}}\right)$ in echo sounding [see Fig. 6]) that is stable to 1:100 000 for periods of 1 hour. We then relate transmission time, T, to the average $\left(\overline{\dfrac{1}{c}}\right)$, both as a function of time, taking care to calculate the latter with appropriate delays between the points along the path, and find the geodetic distance as

$$\mathrm{R}(t) = \frac{\mathrm{T}(t)}{\left(\overline{\dfrac{1}{c}}\right)(t)} \quad . \tag{3}$$

From velocimeter surveys in the Atlantic discussed below in Section V-C we know that within a few minutes, sound velocity varies as much as

1:5 000. However, using moving averages over ½-hour intervals, we smooth out this fluctuation to no more than 1:100 000. The remaining unknown is the variation over horizontal distances greater than a few miles. Until we have this knowledge we will not know whether to use 10 or 100 deep-suspended velocimeters.

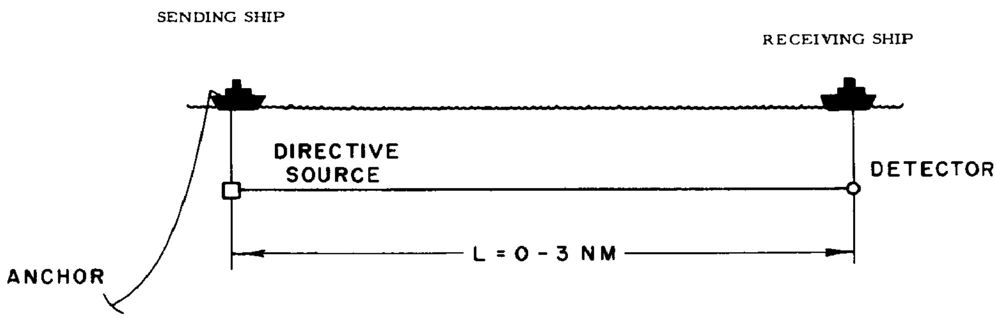


FIG. 11a. — Propagation from a directive source at constant depth

C. Direct Horizontal Propagation

We may understand more clearly the relation between the values $\left(\frac{T}{c}\right)$ obtained by the local measurements with a velocimeter and transmission times, T, obtained by timing sound pulses over great distances by considering a logical starting point in a class of sound propagation experiment that requires primarily the sensitivity of the velocimeter, being rather indifferent to questions of absolute precision. Sound is projected from a directional source and received at hypothetically fixed ranges of zero to 6 nautical miles. This has been done recently in the Caribbean using ships with a slowly varying relative distance, the objective being a precision measurement of the fluctuation in transmission time. For short distances the directionality eliminates from relevant detection all indirect propagation paths, and all sound received and timed has experienced the same continuous set of c-values along the path. The ship arrangement is shown in Fig. 11a. The parameters of the statistics of variation in c are formulated as μ, the relative variation in c, averaged for a time series as $\bar{c}$, and the correlation function $N(\tau)$ for the series defined as follows :

$\delta(t) = c - \bar{c} =$ Deviation in velocity from the average

$$\mu = \frac{\delta(t)}{\bar{c}} = \text{Relative variation of } C(t) \qquad (4)$$

$$N(\tau) = \frac{\int_0^{T-\tau} \mu(t) \qquad \mu(t+\tau)\, dt}{\left(\int_0^{T-\tau} \mu^2(t)\, dt\right)^{\frac{1}{2}} \left(\int_0^{T-\tau} \mu^2(t+\tau)\, dt\right)^{\frac{1}{2}}} = \qquad (5)$$

= Approximately normalized correlation function of μ for internal τ.

The independent variable t should be regarded merely as a running index for a time series. Its actual unit depends on the method of data taking. If c is measured along the points of a horizontal path, t will be distance along the path and τ can be a short space interval. To consider the time, T, for a hypothetical pulse propagated only along this path we imagine such data existing in terms of a and α, the distance and interval renamed.

Starting from the same relation used in echo sounding

$$T = \int_0^L \frac{da}{c(a)} = L\left(\overline{\frac{1}{c}}\right).$$

We can relate the variation, Δt, in T, due to variation, δ, in $c(a)$ along the path L, using the statistics of $\left(\overline{\frac{1}{c}}\right)$.

$$\langle(\Delta t)^2\rangle = \frac{2\,\langle\mu^2\rangle\,L}{c^2} \int_0^L N(\alpha)\, d\alpha\,; \tag{6}$$

$$\frac{\langle(\Delta t)^2\rangle^{\frac{1}{2}}}{\overline{T}} = \langle\mu^2\rangle^{\frac{1}{2}} \left[\frac{2}{L}\int_0^L N(\alpha)\, d\alpha\right]^{\frac{1}{2}}. \tag{7}$$

Since $N(\alpha)$ or $N(\tau)$ is normalized its integral is a scale in the α or τ dimension. The most important point to note here is that the relative variation of T is proportional to the square root of the ratio of the scale of sound velocity variations in space to the path length. It is this scale of variation, $\int_0^L N(a)\, da$, as well as the root mean square of μ that can be explicitly measured with the velocimeter and compared to results on real transmission times which, because of diffraction effect and path perturbations will depart from Eq. (7) as L increases.

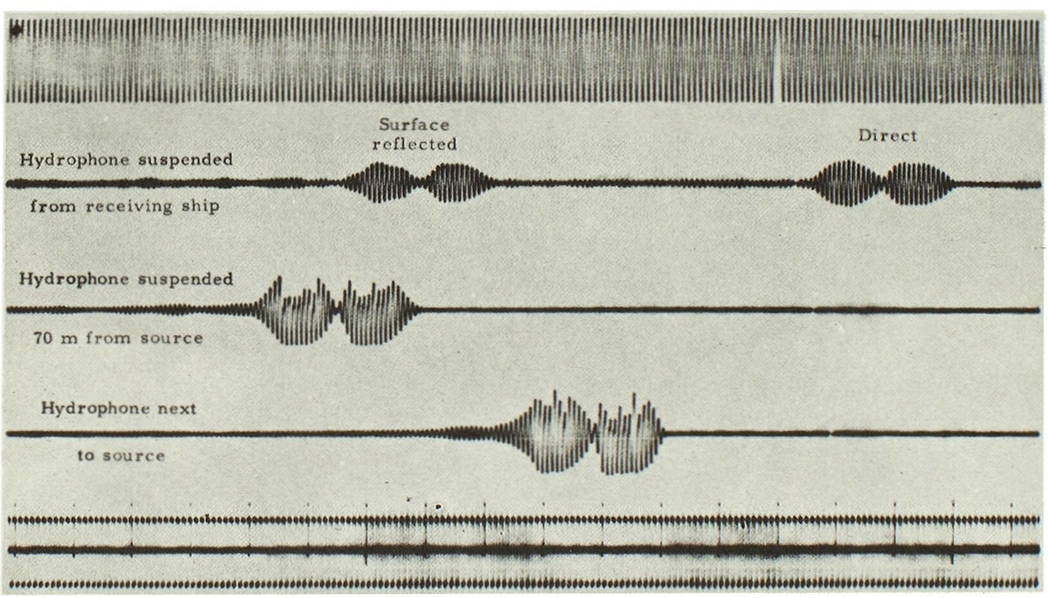


FIG. 11b. — Sound pulses received at series of suspended hydrophones. Receiving ship radios back the top signal

To show the sensitivity possible for real transmission times we consider the actual experiment, as shown in Fig. 11*a*. Short bursts of 1.2 kc sound were projected regularly at intervals of 0.6 sec. The pulse was 20 msec long, and in the middle its phase was abruptly reversed. The sound was received on three hydrophones, one next to the directional source, a second suspended from the source ship at a distance of 200 ft, and a third hung from the receiving ship. These received signals are shown in Fig. 11b reading in sequence from the bottom up. The two near signals have been distorted by overamplification in this optical recording taken from magnetic tape. The direct signal received on the distant ship and displayed on the top channel is quite distinct in its phase reversal point. It is possible to measure the time elapsed to this point to better than 0.1 msec. In this particular case the range is almost 6 nautical miles, the pulse received on the top channel corresponding to one that is ten pulses back on the lower channels. Hence, at this distance of 33 000 ft we can measure fluctuation in T to 1:66 000.

The fluctuations, $\delta(a)$, of sound velocity are measured as a function of distance along a path when the velocimeter is moved uniformly through the water at a known geodetic speed. Moreover, at a geodetically fixed point the water mass is moving by, and most probably this movement of an inhomogeneous medium is the generator of $\delta(t)$ since both temperature and chemical diffusion are too slow either to create observed fluctuations or to appreciably influence them even with slow movement.

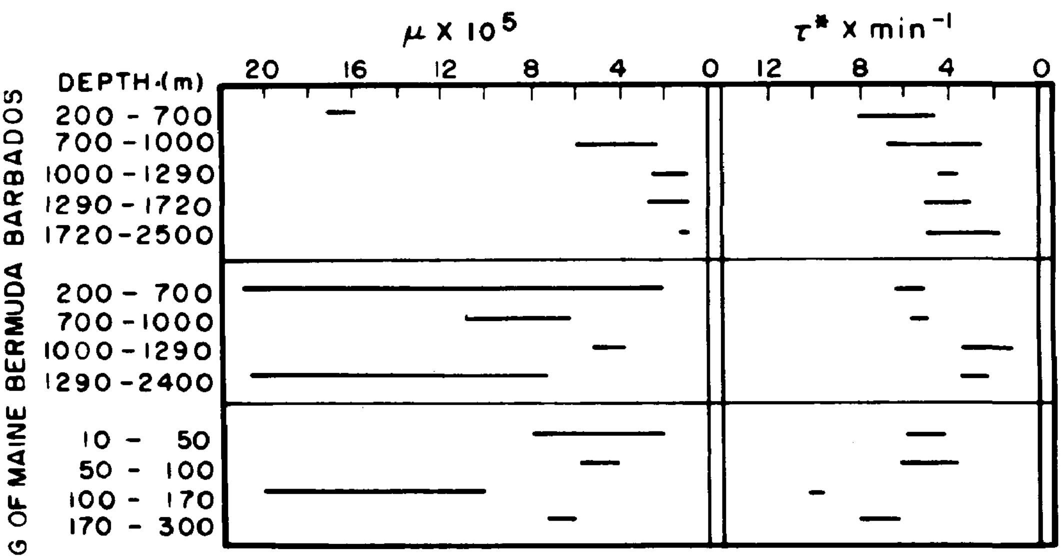


FIG. 12. — Bars show ranges of μ and τ^* which contain approximately half of the values pertaining to the depth range

Measurements at a fixed point with the ship anchored in a region of steady surface currents and winds using direct frequency counting of a single velocimeter were taken a year ago in several different Atlantic areas. Consider the results now which are shown in Fig. 12 as an example of the size of the parameters that can be encountered.

The total range of μ shown is from zero to 2×10^{-4}. The area called Barbados is a thousand miles northeast of the island and shows a drop in μ with great depth that is not clearly present for the other areas. The quantity τ^* here is the first value of τ where $N(\tau)$ equals zero. For this data $N(\tau)$ was oscillatory but converging to zero for large τ. The integration times used in the averaging for $\overline{c}$ were only 30 minutes or so. More recent data of the same type have shown that for very long averaging times of several hours μ approaches 10^{-3}, and τ^* for this size of variation will be over half an hour.

The high-density velocimeter sampling system permits a proper alternative to the relatively slow and haphazard study with a fixed velocimeter in that attention can be focussed directly on the spatial fluctuation pertinent to acoustic propagation. It has been used in two kinds of arrangements designed to measure the correlation functions for horizontal and vertical coordinates.

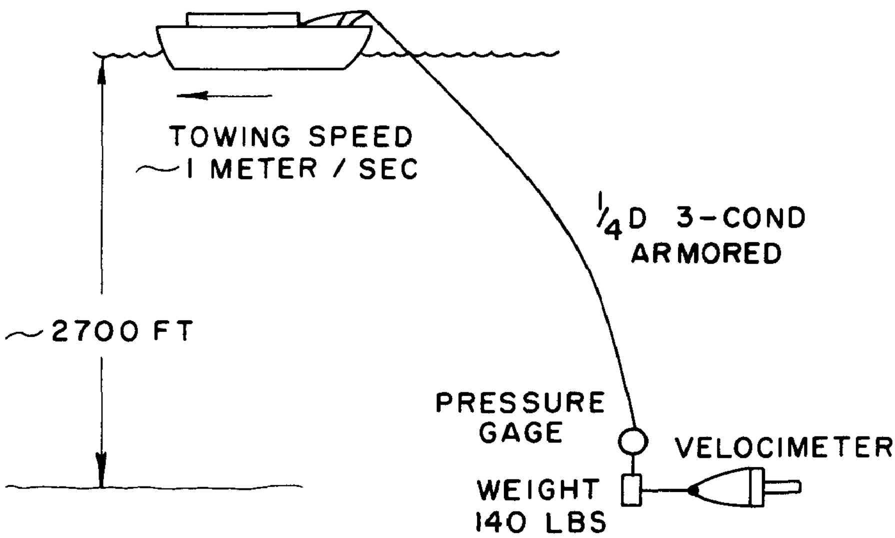


Fig. 13. — Velocity measurement with respect to horizontal coordinate

For the horizontal case a velocimeter was towed at the depth of the propagation experiment with the depth monitored in terms of a pressure gauge (see Fig. 13). While for absolute accuracy we must know a great deal about the density of the overlying water, the depth variation is plainly indicated. The best Bourdon tube available was used here to monitor depth in discrete steps of 8 ft. Vibrating reed devices have been developed recently to a much higher sensitivity. The figure shows an ideal velocimeter orientation that is effective only when the bottom plate has been cut out or altered in such a way that water flows freely across the propagation path. In the tests actually made, the meter was clamped to the main cable, whose twist and orientation vary much more slowly than did the sound velocity. Geodetic speed was measured using the fine lines of a Loran grid. These are fairly good in the Caribbean where the tests took place. For the correlation computations from the data, we take the formulae (4) and (5) and interpret the quantities T and τ as lengths " L " and " α " using our geodetic speed and the velocimeter sampling rate. This latter is 4 times per second, giving a data point every 25 cm. Compare this to the fixed point technique where we can expect typically a deep current of 0.1 m/sec and we see that in 1 hour of towing we have sampled as much water as in 10 hours of the earlier technique. In addition, we measure directly N(α) instead of converting N(τ) through an uncertain estimate of water currents.

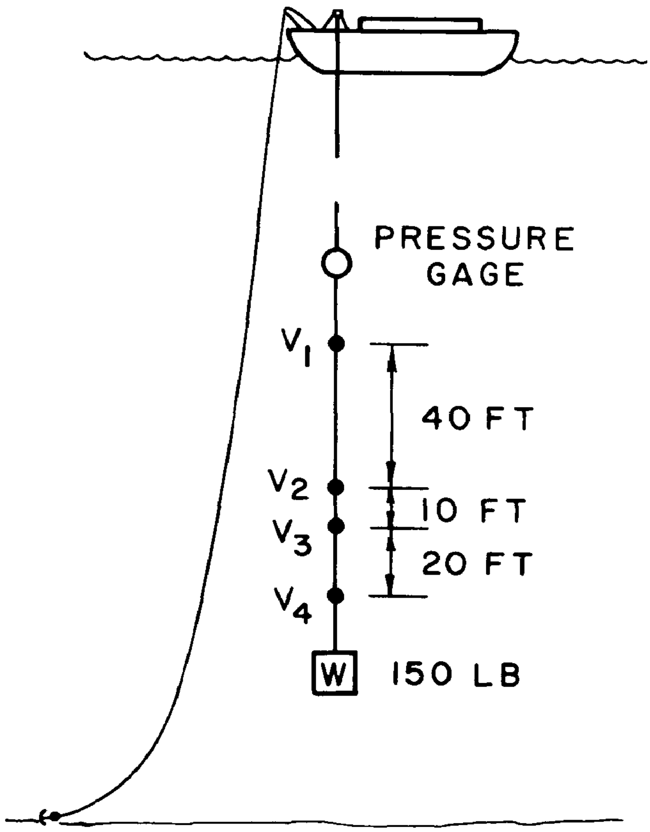


FIG. 14. — Velocity measurement with respect to vertical coordinate

To get vertical correlation statistics we use cross correlation of the data from two velocimeters suspended from an anchored ship with the following formula for the correlation function:

$$N(12) = \frac{\int_0^T \mu 1\,(t) \qquad \mu 2\,(t)\,dt}{\left(\int_0^T \mu 1^2\,(t)\,dt\right)^{\frac{1}{2}} \left(\int_0^T \mu 2^2\,(t)\,dt\right)^{\frac{1}{2}}}\,.$$

Here for interpretation of N(12) as N(a), a will be the distance between meters 1 and 2. Without a multitude of velocimeters one can't hope for a complete N(a) curve in the vertical direction. However, a trial of the arrangement in Fig. 14 has made a start.

With these four velocimeters we have the six vertical cross-correlation distances of 10, 20, 30, 40, 50, and 70 ft. Their midpoints fall within a 35-ft range. The very practical problem of getting synchronized time scales for the readings of these velocimeters is very easily solved using the beat frequency technique.

We have four channels like the top one in Fig. 3 of velocimeter - velocimeter amplifier - mixer - amplifier clipper - to counter. The reference oscillator output is fed through the Q-multiplier to all r mixers. These outputs are then counted separately. Before going to any printers a switching circuit is interposed. This circuit holds the counts in each counter for about 8 sec and then prints them out in sequence as a group of 4 on an automatic printer. It takes only a second, and then the system is ready for the next count. This gives us a group of 4 every 9 sec.

The data from these systems, being rather voluminous, and requiring a large computer has not been analyzed enough to be discussed in any complete way. However, a very rough non-computer comparison illustrates how this kind of work begins to yield in a reliable way fundamental oceanographic data. We can look at the fluctuating velocity in two time series, a velocimeter towed at a meter per second and a velocimeter suspended at the same depth at a fixed position. Each time series shows the sequence of fluctuations as random in period but with a wide difference in apparent mean of this period. Let us recognize now that whatever the detailed results in terms of correlation function, the scale of variation in time will be related as the ratio of deep water current to towing speed, following the above remark that the inhomogeneous velocity structure of deep water maintains itself for a long time compared with observed fluctuations. Comparing the average periods of these fluctuations we find that the deep current must be about 35 times slower than the towing speed, or less than 3 cm/sec. This is for the depth of 2 900 ft and for a protected basin such as the Caribbean a reasonable value. I know of no other Caribbean measurements with which to check it.

VI. — CRITIQUE

The basic concepts, apparatus, and experiments centered on the problem of precise timing of sound in the ocean have been discussed in terms of a set of acoustic propagation experiments. Echo sounding and acoustic geodesy are length measurements whose precision depends fundamentally on the observable stabilities in transmission time of low-frequency sound. Slant-range and direct propagation experiments aim to understand the phase fluctuation and interference properties of the general class of low-frequency acoustic propagation in the ocean. The automatic timing of high-frequency sound possible over a short, known, fixed path length measures a common sound speed in a way that is not affected by the geometric situation and refractional phenomena so important for the longer range low-frequency waves, yet does accurately reflect the oceanographic variables of chemistry, temperature, and pressure. It was considered as an intermediate level in a sequence of causal relationships. However, the speed of sound must not be neglected in its role as a basic physical parameter linked with and capable of identifying specific elemental water volumes. It is this role which can be used to measure water movements as shown in Section IV-C. Moreover, in large scale oceanographic surveys relating it to salinity and temperature its great potential precision will provide added insight on the dynamics of their variation.

ACKNOWLEDGMENTS

The various experimental techniques discussed in this paper could not have been developed without the collaboration of the supporting technical staffs of Hudson Laboratories. Notably, the electronic instrumentation and mechanical equipment for the velocimeters and directional source owe everything to Thomas PAPPAS and Peter WEBER. Also, I am particularly indebted to Drs. Robert FROSCH and Theodore POCHAPSKY for several very illuminating discussions and to Dr. Alan BERMAN for graciously providing all the pertinent details of his long-range experiment.

This paper is Hudson Laboratories, Columbia University Contribution No. 163.

FOOTNOTES

[1] N. W. LORD, " Fluctuating acoustic refraction in deep ocean water ". Hudson Laboratories, Columbia University Informal Documentation No. 2, presented at the Fourth International Congress on Acoustics (Paper K25) in Copenhagen, Denmark, August 21-28, 1962.

[2] W. DITTMAR, " Report on researches into the composition of ocean-water collected by H.M.S. Challenger during the years 1873-76 ", in Great Britain. Challenger Office. *Report on the Scientific Results of the Voyage of H.M.S. Challenger during the Years 1873-76* ([Edinburgh], 1884). *Physics and Chemistry*. Vol. I, No. 1, 251 p.

[3] With salinity and temperature as a function of depth, D, the density, ρ, is thereby known and the pressure must be calculated as an integral

$$P = \int_0^D g\,(D)\,\rho\,(D)\;dD\,,$$

where g is the acceleration of gravity with $\rho(D)$ corrected for the compressibility effects of pressure by iteration.

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[9] The most dependable measure is the pressure, which now must be used in the relation of Footnote [3] to determine the depth D by numerical solution of the integral equation, regarding D as unknown. A systematic routine for this is described in Hudson Laboratories, Columbia University Technical Report No. 102 by N. W. LORD.

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[17] D. F. WILLIG, Private communication. The calculation done at the request of the author applies to the velocimeter constructed at ACF Industries, Paramus, New Jersey.

[18] V. A. DEL GROSSO, " Velocity of sound in sea water at zero depth ", NRL Report 4002, June 11, 1952.

[19] H. M. GABLER, *Int. Hydrographic Rev. 38*, 7 (1961).

[20] The statistics of $\left(\frac{\overline{1}}{c}\right)$ are set down in section IV-C using equations that are a generalization of those originally set forth by CHERNOV (see Footnote [21], who considered the timing of real wavefronts in inhomogeneous media.

[21] L. A. CHERNOV, *Wave Propagation in a Random Medium* (McGraw-Hill Book Co., New York, 1960).

[22] A. BERMAN and D. STERNBERG, " Travel-time fluctuations in sound channel propagation ". Hudson Laboratories, Columbia University Informal Documentation No. 8, presented at the Sixty-fourth Meeting of the Acoustical Society of America, Seattle, Washington, November 7-10, 1962.