AN ANALYTIC ADJUSTMENT OF A SOUNDING LINE

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ABSTRACT

Horizontal data consisting of positional fixes, ship's heading, and an average constant speed of advance, may be utilized in the adjustment of a sounding line. The adjustment technique of variation of coordinates resolves the horizontal data into the most probable line determination. Proof for the quality of the adjustment is established by the comparison of depth differences (residuals) at line crossings.

The quality of each line is indicated by a standard error which reflects the consistency of the horizontal data comprising each line. The residuals at the line crossings indicate the quality of the entire chart. Translation of the depth residuals into horizontal shifts, with a subsequent determination of a standard error of these shifts, reflects the consistency of both the horizontal and vertical data comprising a chart.

A comparison of the standard graphic method of adjustment is made with this new adjustment method.

INTRODUCTION

During the past decade technical advances in the field of navigation have enabled the hydrographic surveyor to conduct survey operations at ever-increasing distances offshore within acceptable standards of accuracy.

A few of the technical advances that have been developed to span large distances are electronic surveying systems, inertial guidance systems, satellite positioning methods, and SOFAR (a hyperbolic system using sound transmissions in the ocean). While the shorter range electronic systems display a remarkable relative accuracy which is generally referred to as repeatability, the longer range systems (including the newer techniques) show a decrease in this relative accuracy. This loss of repeatability imposes the need to smooth each sounding line according to the conditions which the ship is known to have achieved. For example, if a ship should run a line while maintaining a steady speed and recording each course change, then it could be expected that the individual positioned fixes would conform to the navigational conditions. When the positional fixes contradict what is known to have happened, then these fixes are graphically adjusted to conform. A problem arises when the graphic adjustment is not satisfactory and it becomes necessary to readjust these lines.

The Analytic Line Adjustment Method presented in this article requires the use of a general-purpose electronic computer. The adjustment resolves simultaneously three types of horizontal data into the most probable position for each fix.

1. Positional data (obtained by an electronic positioning system and/ or any independent navigation system).
2. Ship's heading (the direction in which the ship is pointing).
3. Speed of advance (this is deduced from positional fixes).

The adjustment method is by variation of coordinates. The positional values are considered as the approximate coordinates to be corrected by the differential changes resulting from the observation equations.

A two-dimensional analysis results in the determination of the standard error, which indicates the quality of consistency of the horizontal data. The standard error is important, for it gives the hydrographer the means of quality control in the process of survey data portrayal.

ADJUSTMENT METHOD

A ship in the process of running a survey line attempts to maintain a steady course and speed. However, since the course registered by the ship's gyro compass is subject to various errors, the compass recordings will deviate from the course indicated by the positional fixes. The Analytic Line Adjustment Method utilizes the average of the deviations between plotted course (based on positional fixes) and compass heading, to remove the constant compass errors.

Errors in the compass heading may include (1) gyro error, (2) wind and current — a combination of wind and current may act upon the forward direction of the ship (see Figure 1).

The corrected compass headings constitute the means of adjusting the shape of the sounding line.

If a constant engine RPM is maintained by a survey vessel during the progress of a sounding line, the velocity of the ship will not change appreciably over short periods of time. This assumption is used in the Analytic Line Adjustment Method to produce an average velocity value for successive overlapping segments of the sounding line. If the line is short, a constant value of velocity may be assumed for the entire line.

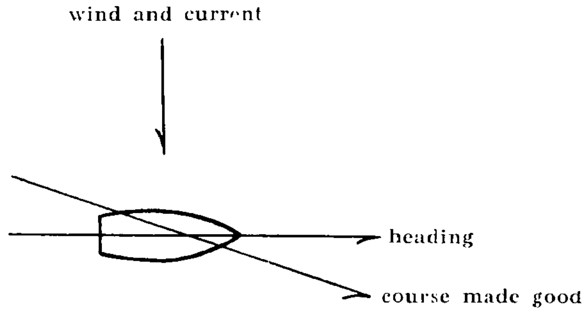

FIGURE 1

If the distance between fixes is established in conjunction with corrected headings, the size and shape of the sounding line is thereby determined. This constitutes, in essence, the method of the Analytic Line Adjustment.

MATHEMATICAL STRUCTURE

With a ship operating at constant speed and heading, erratic control will produce a sounding line similar to the example in Figure 2.

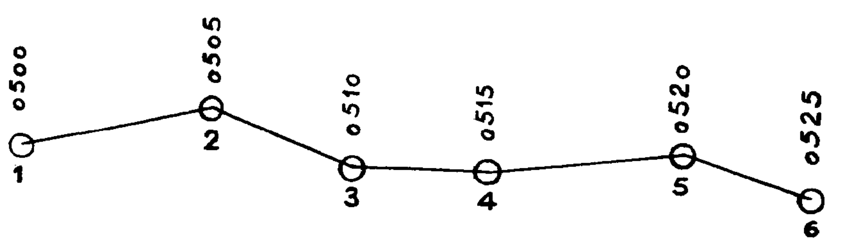

FIGURE 2

Positional information received by the survey vessel during the progress of a line may be in the form of either time differences, phase differences, or other observational values which can be converted into a desired coordinate system. In any case, the received information may be transferred to x and y coordinates on a grid system; for convenience the UTM grid has been designated.

1. Average Velocity

On relatively long sounding lines the average velocity (speed of advance) is quite likely to vary from one portion of the line to another. In this case an average velocity is determined for successive pre-determined overlapping segments of the line. If a sounding line, for example, consists of 100 positional fixes, the first 21 fixes may be used to determine an initial average velocity; this value of velocity is used in conjunction with heading information to determine the adjusted positions of the first 21 fixes. If the value of overlap was one-third, the successive segments would analyze fixes 14-35, 28-49, 42-63, 56-77, 70-91, and 84-100 respectively. Assuming a changing velocity, varying values of adjusted position would be obtained for all but the terminal fixes. The Analytic Line Adjustment Method utilizes an average of these adjusted coordinates to arrive at a unique value for each fix.

Referring to Figure 2 the distance between successive fixes will be designated as s :

$$s = [(N_n - N_{n-1})^2 + (E_n - E_{n-1})^2]^{1/2} \qquad (1.1)$$

where N and E represent the Northing and Easting respectively.

The average velocity ($\overline{v}$) then equals :

$$\overline{v} = \Sigma\, s \;/\; \Sigma\Delta T \qquad (1.2)$$

where ΔT represents the time interval between fixes.

2. Corrected Heading

The deduced heading (a_o) as indicated by successive positional fixes equals :

$$a_o = \text{arc tan}\ \frac{N_n - N_{n-1}}{E_n - E_{n-1}} \qquad (2.1)$$

If the heading indicated by the gyro compass is a_g, then the mean deviation ($\overline{d}$) between deduced and observed heading for $m - 1$ segments is :

$$\overline{d} = \Sigma\,(a_o - a_g) \;/\; m - 1 \qquad (2.2)$$

where m equals the number of fixes.

The corrected heading (a_c) for each segment is then :

$$a_c = a_g + \overline{d} \qquad (2.3)$$

The gyro heading used should be the average for the segment.

3. Formation of Observation Equations

Two types of observation equations are used in the Analytic Line Adjustment Method. The first concerns speed of advance and is in the general form :

$$aN_n - aN_{n-1} + bE_n - bE_{n-1} + L = 0 \qquad (3.1)$$

where

$$a = \frac{dN}{ds} = \frac{N_n - N_{n-1}}{s} \quad (3.2)$$

$$b = \frac{dE}{ds} = \frac{E_n - E_{n-1}}{s} \quad (3.3)$$

and E and N are the differential corrections to the observed positions. The residual of the observed velocity and average velocity ($\bar{v}$) between successive fixes is represented by L.

The second type of observation equation concerns heading and is the same form as equation (3.1).

In this case :

$$a = -k\ dE = -k(E_n - E_{n-1}) \quad (3.4)$$

$$b = k\ dN = k(N_n - N_{n-1}) \quad (3.5)$$

where

$$k = \rho \ / \ s^2 \quad (3.6)$$

The residual between the deduced heading (a_o) and the corrected gyro heading (a_c) is represented by L.

The total number of observation equations will be two less than twice the number of fixes contained in the sounding line, with the initial point held fixed. Solution by the method of variation of coordinates presents a number of unknowns which in this case are also equal to the number of observation equations, thereby resulting in a unique solution for the computation.

4. Solution of Observation Equations

As the initial position was held fixed, the adjusted line should be shifted so as to determine the most probable average of all positional data. This is accomplished by shifting each coordinate an additional distance based on the average of all computed adjustments.

The adjusted coordinates (N, E) for each fix then equal :

$$N = N_o + \Delta N - \overline{\Delta N} \quad (4.1)$$

$$E = E_o + \Delta E - \overline{\Delta E} \quad (4.2)$$

where N_o and E_o refer to the observed coordinates, ΔN and ΔE are the computed adjustments, and $\overline{\Delta N}$ and $\overline{\Delta E}$ are the mean value of all the adjustments.

If the observation equations are in the form

$$a_1 N_n - a_1 N_{n-1} + b_1 E_n - b_1 E_{n-1} + L_1 = 0 \quad (4.3)$$

$$a_2 N_n - a_2 N_{n-1} + b_2 E_n - b_2 E_{n-1} + L_2 = 0 \quad (4.4)$$

then :

$$\Delta E_n = \frac{a_{1_n}(L_{2_n} - a_{2_n} N_{n-1} - b_{2_n} E_{n-1}) - a_{2_n}(L_{1_n} - a_{1_n} N_{n-1} - b_{1_n} E_{n-1})}{b_{1_n} a_{2_n} - a_{1_n} b_{2_n}} \quad (4.5)$$

$$\Delta N_n = \frac{(L_{1_n} - a_{1_n} N_{n-1} - b_{1_n} E_{n-1})}{-a_{1_n}} - \frac{b_{1_n} E_n}{a_{1_n}} \quad (4.6)$$

When the initial position is fixed, the first N and E can be determined and the results utilized in the succeeding computations with formulas (4.5) and (4.6).

The above analysis will yield an adjusted pair of coordinates for each fix in a particular line segment. In the case where overlapping line segments are utilized, the final adjusted position is an average of the adjusted values for the individual fixes.

5) **Standard Error of Control Repeatability**

An excellent measure of the control repeatability is the standard deviation of the observed and adjusted positions.

The standard deviation (σ) of each survey line equals :

$$\sigma = \left[\frac{\Sigma (\Delta N)^2}{m - 1} + \frac{\Sigma (\Delta E)^2}{m - 1} \right]^{\frac{1}{2}} \qquad (5.1)$$

where m equals the number of fixes and ΔN and ΔE are the residuals between the observed and final adjusted positions.

EVALUATION METHOD

To evaluate the results of the computer solution, it was necessary to hypothesize that the system of coordinates giving the lowest summation of depth residuals at line intercepts is probably the result of the best adjustment.

This evaluation was accomplished by plotting information from several survey sites. Each site was adjusted by the conventional graphic method and then replotted and adjusted by the Analytic Line Method. The results of the two methods were then compared in detail. A three-dimensional analysis was performed in this comparison.

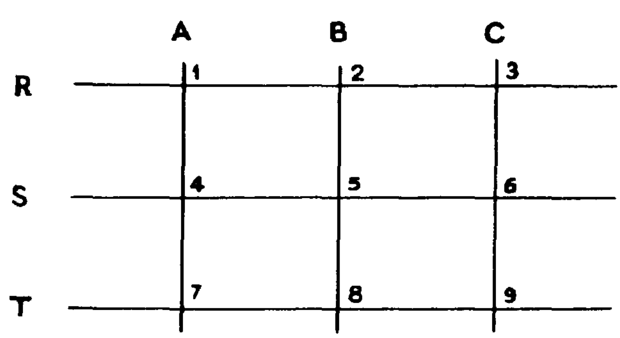

FIGURE 3
Sounding Line Intercepts

Referring to Figure 3, it is apparent that six sounding lines, three in a north-south direction, and three in an east-west direction will produce nine intersection comparisons. Line R will intersect line A at a specific time T_1 and line A will intersect line R at a specific time T_2. On the depth recordings for lines A and R, the depths Z_1 and Z_2, corresponding to times T_1 and T_2 can be determined and compared.

The residual $(Z_2 - Z_1)$ is the result of an error in x, y (horizontal) and Z (depth) coordinates. For convenience it was assumed in this evaluation that the depth measurements Z are errorless. This assumption is not strictly true, but by disregarding bottom gradients of less than three fathoms per 500 metres, the error in depth measurement is negligible, as precision depth recorders were used in determining depth.

The depth residual when related to the bottom gradient at the intersection point constitutes a means of determining the horizontal shift necessary to satisfy the residual.

With this data it was possible to compute a standard error for both systems of intercept coordinates over the entire survey area.

EVALUATION RESULTS

Five actual survey sites were selected and the intercept comparisons of each site evaluated. An average velocity was assumed for the entire line rather than to analyze successive overlapping segments of the line. It was convenient to assume a constant velocity because the lines were of

INTERCEPT COMPARISONS

	Site	Number of Intercepts	Summation of Residuals (in fathoms)	Mean Shift (in metres)	Standard Error (in metres)
Graphic Analytic	I	25	27 15	158 94	207 159
Graphic Analytic	II	25	37 24	184 82	292 154
Graphic Analytic	III	144	125 118	114 103	198 177
Graphic Analytic	IV	25	33 25	163 134	207 170
Graphic Analytic	V	16	18 14	101 70	159 99
Graphic Analytic	II *	12	19 16	85 61	130 76

FIGURE 4

(*) Site II was recomputed after rejecting several lines because of a large standard error. (The standard error for line rejection refers to formula (5.1). Each line is an entity and is evaluated in terms of two-dimensional analysis. The standard errors listed in the last column of Figure 4 are for line intercept comparisons and they refer to the three-dimensional analysis).

relatively short length. Figure 4 shows the results of this evaluation in tabular form. The graphic values refer to the conventional office graphic adjustment and the analytic values to the method outlined in the paper.

In all five cases the Analytic Line Adjustment Method produces improvements in the depth residuals, the mean shift, and the standard error.

CONCLUSION

The results of this study indicate that the quality of certain types of hydrographic surveying control can be improved by utilizing the Analytic Line Adjustment Method. This provides a consistency to the results often lacking with other methods. In addition, with the advent of the high speed electronic computer aboard survey vessels, the line adjustment can be performed while the survey is in progress and data may be rejected which does not meet a statistical standard.

In an automated hydrographic data processing system a coordinatograph plotter can be used in processing systems which are either ship- or shore-based. In either case the Analytic Line Adjustment Method is a necessary link in an automated data processing system; the data must be adjusted and evaluated prior to automatic plotting.

Use of the Analytic Method eliminates the need for office smooth plotting of field data. In the shipboard system, the electronic computer precludes the necessity for advance preparation of hyperbolic or ranging grid sheets as well as performing adjustment and evaluation of data.

FUTURE STUDY

Since the analysis described above is concerned only with individual line adjustment, a shift in datum during the progress of a survey will remain uncorrected. A shift in datum may be caused by equipment failure, change in propagation or transmitter conditions during the course of a prolonged electronic survey, or a shift subsequent to a power failure.

By utilizing the depth residuals and bottom gradients at the intersections, it is possible to shift the individual lines to conform to the configuration that produces a minimum of intercept depth residuals.

Preliminary programming has been undertaken at the U. S. Naval Oceanographic Office on small problems of this type and have proved to be adaptable to a large computer.