REDUCING METEOROLOGICAL EFFECTS ON MONTHLY MEAN SEA LEVELS

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Introduction

ROSSITER (1958, 1960) has considered the errors in daily, monthly and annual mean sea levels, due to incomplete elimination of the tide. The purpose of this note is to point out that meteorological effects with periods less than a month are not adequately reduced by the direct averaging presently used to compute monthly mean levels, and to suggest the adoption of an improved averaging procedure.

Monthly mean sea levels are being computed and published for several hundred ports, and the number is increasing each year. The main purpose in choosing an averaging period of one month is to reduce the effects of disturbances with periods less than one or two months, so that seasonal and secular changes can be studied. The shorter-period disturbances in sea level are usually of meteorological origin, and may be of greater amplitude than the seasonal oscillation.

It has become traditional to think of a " monthly mean " as the simple arithmetic mean computed from the beginning to the end of a month. Such simple averaging however is not always the best — for example, there are serious difficulties in the use of a simple average of 24 hourly readings as a " daily mean " sea level, since tidal effects are not adequately reduced. In computing daily mean sea level, it is usual to use a numerical filter (GROVES, 1955) specially designed to reduce the effects of tides as much as possible. These filters extend over more than 24 hours.

Numerical Filters

A general review of filtering has been given by HOLLOWAY (1958). Here we shall give, without proof, only the results needed for the present discussion. To fix ideas, we assume that daily mean sea levels x_i are available, and that $i = 0$ corresponds to the middle of each month. Then a monthly mean X will be defined as :

$$X = \sum_{i=-n}^{+n} x_i w_i$$

where w_i are " weights ", and $(2n + 1)$ is the total number of daily means used in forming the monthly mean. The weights w_i are symmetrical about the central value w_o, i.e. $w_{-i} = w_i$. They also satisfy the condition :

$$\sum_{i=-n}^{+n} w_i = w_o + 2 \sum_{i=1}^{n} w_i = 1$$

so that a constant input will not be altered by the filter. The usual monthly mean corresponds to equal weights, $w_i = 1/N$, where N is the number of days in the month.

In considering the effects of different filters (i.e. different sets of weights, w_i), we consider that the series x_i to be filtered is made up of many sinoidal terms of different frequencies, amplitudes and phases. The effect of the filter is then to leave the frequency and phase of each term unchanged, but to multiply the amplitude by a factor R(f), depending on the frequency. We can discuss the merits of different filters by plotting R(f) as a function of frequency, f.

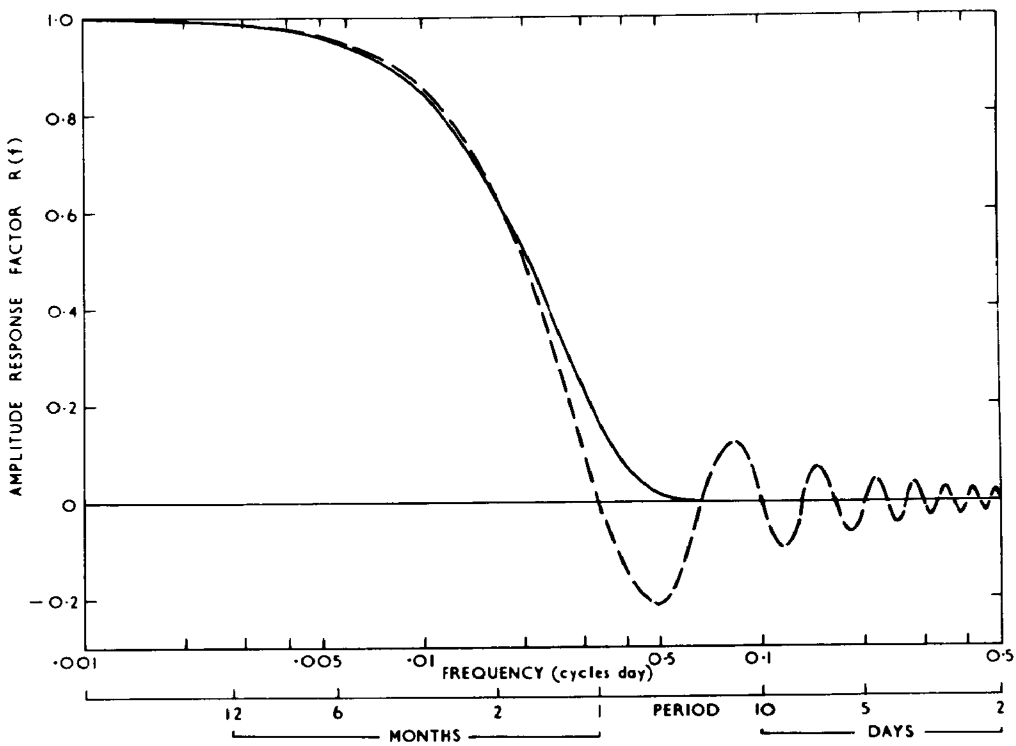

FIG. 1. — Amplitude response factors as functions of frequency.
Dashed curve : equal-weight filter (simple average).
Solid curve : probability curve filter (weights given in Table I).

The dashed curve in Figure 1 shows R(f) as a function of the logarithm of frequency, for the case of equal weights. (Negative values of R(f) simply mean a change of sign of the corresponding sinoidal terms; this is of no interest in the present discussion). It will be seen from Figure 1 that annual or semi-annual frequencies are only slightly reduced by forming monthly means. The main difficulty with the use of the equal-weight filter is the slow decay of the response factor R(f) for periods less than one month. To show the effect of this, consider the computation of monthly means for Port MacDonnell (lat. 38°0′ S, long. 140°40′ E). Figure 2 shows daily sea levels for this port for July 1958. Oscillations of large amplitude (~ 20 cm) and a period of the order of 10 days are clearly indicated. Reference to Figure 1 shows that forming a simple monthly mean may reduce the amplitude of these oscillations by a factor of only 0.1, so that the monthly mean may be " in error " by as much as 2 cm. This is large compared to typical errors to be expected from incomplete elimination of the tide, or to instrumental errors, each of which can be kept down to the order of a few millimetres.

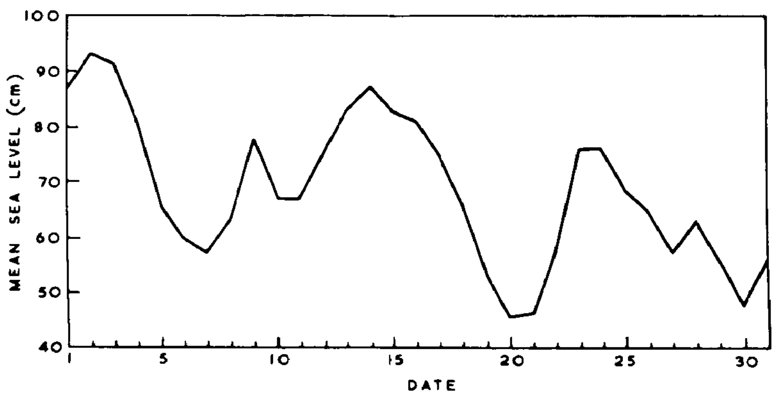

FIG. 2. — Daily mean sea levels at Port MacDonnell, July 1958.

The slowly-damped oscillation of the response factor R(f) for periods less than one month is due to the abrupt ends of the equal-weight filter. More acceptable response factors can be obtained by using weights whose values decay gradually on either side of the central weight. As an example,

TABLE I

Filter weights

i	w_i	i	w_i
0	0.044	14	0.013
1	0.044	15	0.011
2	0.043	16	0.009
3	0.042	17	0.008
4	0.040	18	0.006
5	0.038	19	0.005
6	0.035	20	0.004
7	0.033	21	0.003
8	0.030	22	0.002
9	0.027	23	0.002
10	0.024	24	0.001
11	0.021	25	0.001
12	0.018	26	0.001
13	0.016	27	0.001

Table I gives a set of weights based on the ordinates of the normal probability (HOLLOWAY, 1958, p. 359). The filter extends over a total of 55 days (27 on each side of the mid-month day). This extent has been chosen to make the filter response factor (solid curve, Fig. 1) equal to that of the equal-weight filter for a component of frequency .01667 cycle/day (period two months), which is the highest frequency that can be studied using monthly means. The two filters have equal responses, within less than one per cent, at all periods greater than two months, so that no difficulty should arise in seasonal or secular studies if results calculated by the two different filters are combined.

The response factor of the probability curve filter does not exceed 0.01 for frequencies in the range 0.06 to 0.5 cycle/day. This is too small to be shown in Figure 1. For the example already discussed, the error in the monthly mean due to meteorological effects would be reduced to the order of 0.1 cm by using the probability curve filter.

Other sets of smoothly-decaying filter weights can of course be used. If some weights are negative, the response factor can be brought nearer the ideal (unit response for periods greater than two months, and zero response for periods less than two months), but the number of weights that must be used becomes larger. It is doubtful if there would be any practical advantage in the use of such long filters.

Table II gives the monthly mean sea levels for Port MacDonnell, computed by the two filters that have been discussed. The differences are of the expected order.

TABLE II

Monthly mean sea levels, Port MacDonnell

Month	Equal-Weight Mean (cm)	Mean with Weights given in Table I (cm)	Difference (cm)
1957 Aug.	58.8	57.3	+1.5
Sept.	62.2	62.4	-0.2
Oct.	56.1	58.6	-2.5
Nov.	63.9	63.6	+0.3
Dec.	61.1	59.6	+1.5
1958 Janv.	51.5	52.9	-1.4
Feb.	50.9	51.2	-0.3
Mar.	58.6	55.6	+3.0
Apr.	57.2	59.1	-1.9
May.	82.6	81.9	+0.7
June	64.1	64.1	0.0
July	69.9	69.1	+0.8
Aug.	55.9	58.4	-2.5
Sept.	47.5	48.2	-0.7
Oct.	61.9	61.6	+0.3
Nov.	57.9	57.9	0.0

It is suggested that a filter with smoothly-decaying weights should be used for computing monthly mean sea levels at ports (generally in mid and high latitudes) where there is appreciable meteorological disturbance of sea level. The computations are lengthier, but will in future be carried out by computers in an increasing number of cases. The same filter can be used for all months; the central value could, for the sake of definiteness, be applied to the 15th day of the month when the month has 28, 29 or 30 days, otherwise to the 16th day.

Missing Values

Allowance can be made for a few missing daily means in two ways. The more time-consuming (and more subjective) method is to plot the daily means and interpolate the missing values by eye. The easier method is to take the missing values to be zero, and correct the final monthly mean by multiplying by $1/(1 - Sw_i)$ where Sw_i is the sum of the weights corresponding to the missing values. This method could easily be included in a computer programme.

Choice of Filter for Daily Means

Daily means computed by any of the methods given by ROSSITER (1958) may be used with the probability curve filter. The maximum contribution from a particular tidal constituent to a monthly mean can be found as the product of the appropriate factor in ROSSITER's (1958) Table I, and the corresponding factor in Table III (below). This table corresponds to ROSSITER's Table 3, but there is no longer a dependence on the number of days in the month, since the probability curve filter is of fixed length.

TABLE III

Factors by which contributions to daily mean sea level values are reduced when the probability curve filter (Table I) is used

Contribution from	Factor
K_1	+0.9878
O_1	-0.0011
M_2	-0.0010
N_2	+0.0012
M_3	+0.0003
M_4	-0.0030
M_6	-0.0028
Msf	-0.0010

Annual Mean

If the probability curve filter is used, annual means should be calculated as simple averages of the twelve monthly means. Since maximum contributions from tidal constituents O_1, M_2, N_2, M_3, M_4, M_6, and Msf are already small in the monthly means (Table III), we need consider only the contribution to the annual mean due to K_1. Calculation shows that the reduction factor for this component when computing annual means is .0062 for a year of 365 days, and .0053 for a year of 366 days, if the convention given above for the dates of mid-month days is adopted.

References

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