A GENERAL METHOD FOR THE ANALYSIS OF HOURLY HEIGHTS OF TIDE

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Introduction

A number of methods have been devised (references 1-4) for the analysis of a sequence of observations of hourly heights of tide in terms of a suitable theoretical model, using a digital electronic computer.

These methods all contain restrictions or approximations of one form or another. These are sometimes due to the use on a computer of a method originally designed so that it would be a feasible calculation using desk calculating-machines; sometimes the restrictions applied are due to the limited size or speed of the computer used. The use of large modern computers (such machines as Ferranti ATLAS, IBM 7090, English Electric KDF9 come to mind) means that most, if not all, of these restrictions are no longer necessary, and that accurate analyses can be economically produced using quite general and unrestricted methods.

It is the author's intention to discuss in this paper the characteristics required of a flexible scheme of analysis, the methods by which the results might be obtained, and the way in which the resulting computer program would be used.

Characteristics

Analyses of tidal observations are normally made so that accurate predictions of future tides can be prepared. For some purposes, such as navigation in areas where the tides are regular, predictions of the times and heights of high and low water are useful. Predictions of hourly heights of tide may be required in other instances, such as for navigation in regions of mixed or otherwise irregular tides, or for storm-warning purposes, where a constant watch must be kept for any meteorologically induced changes in sea-level. It is necessary that tidal analysis should be carried out in terms of a suitable theoretical model, so that both these forms of prediction can be prepared.

In the same way, both the theoretical model and the method of analysis must also cater for the various forms of tide encountered. There is firstly the predominantly regular semidiurnal form of tide which is so widely encountered, and whose range may be anything up to fifty feet, and there is also, although much more rarely, the predominantly regular diurnal form of tide. There are also regions where the tides are of a mixed nature, being sometimes diurnal and sometimes semi-diurnal. Ports which are situated in shallow water may have distorted tidal profiles, and this distortion may take many forms. In some cases, the distortion takes the form of a short period of rising tide and a long period of ebb, and at some places this can take the extreme form of a bore, particularly at spring tides. At other ports, the shallow water effects may cause double high or low waters, or perhaps a stand of tide lasting several hours; again the effect can vary considerably between spring and neap tides.

The author knows of no completely satisfactory theoretical model for describing *all* these forms of tide, but believes that the harmonic representation of tidal height (described in the next section), supplemented in some cases by shallow water corrections of the form devised by DOODSON (reference [5]), is the best model at present available.

Another restriction which may affect the model, and which certainly affects the method of analysis, is the form of the observations. Ideally, the data would consist of a long period of actual tide-gauge records, but the data available often falls far short of this ideal. The analysis of the times and heights of high and low water (which is sometimes the only form of data available) will not be discussed in this paper : methods of analysing this type of data are at present being investigated by the author. More frequently, data is available in the form of tide-gauge charts, or tabulated hourly heights, covering a limited period. This period may be anything from a few days to several years, and the records may be unbroken or may contain gaps. The gaps may be brief and irregular (due, perhaps, to an unreliable tide-gauge); they may be very long, as when observations covering a period of a few days are made every few months; they may be very frequent, as when observations are made only during daylight hours. The method of analysis should be able to accommodate all these types of data.

Sometimes data may consist of more than just tide-gauge observations. Frequently observations (10-50 days) are made for a brief period at some point, when not far away there is a site where much more extensive observations have already been made; it is possible that plausible assumptions may be made about the relations holding between members of the sets of constituents at the two sites. Alternatively, a short period (say 3 months) of observations may have been made at some place where mean sea level observations are already available. It may even be decided that the values of z_0, S_a and S_{sa} obtained from the analysis of a year's observations are not sufficiently reliable, and that results obtained from mean sea level records extending over a number of years will be more accurate. The method of analysis must be able to accommodate the insertion of data of this sort.

Once an analysis has been carried out, it is desirable to check the results in some way. Although the data will presumably have been checked before the program is run, and this is discussed in the later section on the use of the program, data errors can still occur. There is also the fallibility of the computer and its peripheral equipment to be considered. Errors can be made in this way, and are so made; computers generally have a remarkable reputation for freedom from errors in their answers, but the author wonders whether some of this reputation is not due to the neglect of computer users to check the results they have obtained. It is convenient to use the results of the analysis to predict the period whose observations were analysed. The differences between the observations and the predictions can be computed, and then output in histogram form. This checks both the numerical accuracy of the analysis, and the goodness of fit of the theoretical model used.

Method

Firstly, it is necessary to fix the tidal representation to be used. As mentioned in the previous section, it is recommended that the harmonic representation of tidal height should be used. In this method,

$$h = z_0 + \sum_{n=1}^{N} H_n f_n \cos (V_n + u_n - g_n + \sigma_n t) \quad \ldots (1)$$

where :

- h is the tidal height.
- z_0 is the value of mean sea level, in terms of some reference datum.
- H_n, g_n are respectively the amplitude and phase of the n-th constituent, and depend only upon the site at which the observations are made.
- f_n, u_n are slowly varying functions, having a period of about nineteen years. In principle, these also should be determined from the analysis of the observations, but usually the period of observation is too short for this to be done at all accurately. The values of f_n and u_n for the tide-generating potential can be determined from astronomical observations; it is usually assumed that for this long-period cycle, the tide varies in the same way as the potential.
- V_n is the phase of the n-th constituent of the tide-generating potential at Greenwich at $t = 0$.
- σ_n is the angular speed of the n-th constituent.

This representation can be used to provide accurate predictions of all except shallow water tides. In the case of shallow water tides, many terms are needed to get an accurate description of the tidal height, and even more terms would be needed to compute the times of tides accurately. For predominantly semidiurnal regular tides, shallow water corrections can be applied to the predictions of times and heights of high and low water (DOODSON, reference [5]); it seems quite feasible that some correction system could be devised to improve the prediction of hourly heights of tide in these cases. As far as this paper is concerned, however, the representation given in equation (1) will be used.

The purpose of the analysis is to determine the $(2N + 1)$ unknown constants (z_0 and the sets H and g). Normally, there will be far more than $(2N + 1)$ observations available, and it is very desirable that this should be so, as otherwise small errors in a few of the observations could lead to excessively large errors in the results of the analysis. However, the relatively large number of observations means that some form of optimisation procedure must be used, the aim being to obtain values for the (2N + 1) constants so that equation (1) fits the set of observed values as well as possible. The author hopes to discuss in a later paper this concept of a " best fit " between equation (1) and the set of observations; at this stage however, it will merely be pointed out that the least squares optimisation is a sensible method (in that it minimizes the variance, which is a measure of the difference between equation (1) and the set of observations), and that it is numerically a very convenient method. The least squares method will therefore be used in this paper to determine the set of (2N + 1) constants.

Equation (1) will be re-written as :

$$h_p = \sum_{n=0}^{2N} x_n \Phi_n(t) \tag{2}$$

where :

$$\left.\begin{aligned} x_0 &= z_0 \\ \Phi_0 &= 1 \\ x_{2n-1} &= H_n \cos g_n \\ \Phi_{2n-1} &= f_n \cos (V_n + u_n + \sigma_n t) \\ x_{2n} &= H_n \sin g_n \\ \Phi_{2n} &= f_n \sin (V_n + u_n + \sigma_n t) \end{aligned}\right\} \tag{3}$$

In equation (2), the symbol h_p is used to represent a prediction-formula height, as opposed to h_0, an observed height.

The least squares criterion is that :

$$\sum_{\text{(all observations)}} (h_0 - h_p)^2 = \text{minimum} \tag{4}$$

In future, the symbol Σ' will be used to imply a summation over all the observations. It follows from equation (4) that

$$\frac{\partial}{\partial x_m} \Sigma' [h_0 - \sum_{n=0}^{2N} x_n \Phi_n(t)]^2 = 0$$

for each x_m

$$\Sigma' [h_0 \Phi_m(t) - \sum_{n=0}^{2N} x_n \Phi_n(t) \Phi_m(t)] = 0 \tag{5}$$

$$\sum_{n=0}^{2N} x_n \Sigma' \Phi_n(t) \Phi_m(t) = \Sigma' h_0 \Phi_m(t) \tag{6}$$

Equation (6) is one of a set of (2N+1) similar equations, as the suffix m is allowed to take each of the (2N + 1) available values. It will be convenient to consider the matrix F, where :

$$F_{nm} = \Sigma' \Phi_n(t) \Phi_m(t) \tag{7}$$

and the vector Q, where :

$$Q_m = \Sigma' h_0 \Phi_m(t) \tag{8}$$

Equation (6) then becomes a matrix equation

$$Fx = Q \tag{9}$$

The order of the matrix F is equal to (2N + 1), which may be up to about 120-200. The problem, then, is to generate the matrix F and vector Q, and to solve the matrix equation (9) for x. Other methods of tackling the analysis problem have been suggested, such as that described by MOUNT and LUND (reference [6]), but these do not seem very applicable to the field of tidal analysis, where a choice of predictor formulae does not really arise.

The three time-consuming parts of the analysis are the generation of the vector Q, the generation of the matrix F, and the solution of the matrix equation (9).

These three problems will be discussed in turn.

The elements of the vector Q have the form

$$\Sigma' f_n h_0 \cos (V_n + u_n + \sigma_n t)$$

or

$$\Sigma' f_n h_0 \sin (V_n + u_n + \sigma_n t) \qquad (10)$$

and one obvious way of computing the elements is simply to carry out these sums. It may be assumed, in order to reduce the volume of computation, that f_n and u_n remain constant for periods of up to a few months. In the analysis of a year's observations of hourly heights (to quote a typical analysis problem), about one million trigonometric functions will be required to compute Q; the use of a cosine table would generally be the most efficient way of obtaining these. An alternative approach to this problem was used by CARTWRIGHT (reference [2]) : he used the iteration process :

$$y_t = z_t + 2y_{t+1} \cos \theta - y_{t+2} \qquad (11)$$

When this is iterated $(M - 1)$ times with the initial values :

$$y_M = 0 \qquad y_{M-1} = z_{M-1} \qquad (12)$$

it yields finally y_1 and y_0, which satisfy :

$$y_0 - y_1 e^{-i\theta} = \sum_{r=0}^{M-1} z_r e^{ir\theta} \qquad (13)$$

This is a very efficient and useful process for computing the elements of Q. Because the f_n and u_n have to be assumed constant during this process, the evaluation of each of the elements of Q may have to be done in a number of stages. It has the disadvantage that under certain conditions y_0 or y_1 may become quite large in comparison with the elements of Q : in certain computers this might mean that either floating-point arithmetic, or double-length fixed-point arithmetic, would have to be used instead of single-length fixed-point arithmetic, and this could nullify its other advantages. The presence of a lot of gaps in the data will cause inconvenience, but the program could nevertheless be designed to accommodate these : the efficiency of the method would seem to be such as to make its use worthwhile.

The elements of the matrix F consist of series of the form :

$$\left.\begin{array}{l} \Sigma' f_n f_m \cos (V_n + u_n + \sigma_n t) \cos (V_m + u_m + \sigma_m t) \\ \Sigma' f_n f_m \cos (V_n + u_n + \sigma_n t) \sin (V_m + u_m + \sigma_m t) \\ \Sigma' f_n f_m \sin (V_n + u_n + \sigma_n t) \cos (V_m + u_m + \sigma_m t) \\ \Sigma' f_n f_m \sin (V_n + u_n + \sigma_n t) \sin (V_m + u_m + \sigma_m t) \end{array}\right\} \;\ldots\; (14)$$

When there are no gaps, these series can easily be summed analytically (over a period sufficiently short that f and u remain virtually constant), and even when the data is split by gaps into blocks, the series can still be analytically summed within each block.

There remains the question of the solving of the matrix equation (9). In many cases, the Gauss-Seidel iterative system will converge, and in such cases, this will probably be the best method to use. In this method, the matrix F is written as :

$$F = L + U \tag{15}$$

where L contains the diagonal and lower triangular elements of F (the upper triangle of L being zero), and U contains the upper triangle of F (the rest of U being zero). The iterative system is then that :

$$\left.\begin{aligned} Lx^{(k+1)} &= -Ux^{(k)} + Q \\ x^{(0)} &= 0 \end{aligned}\right\} \tag{16}$$

A sufficient (although not necessary) condition of convergence of this iterative system is that for each row of the matrix F, the absolute value of the diagonal element should be not less than the sum of the absolute values of all the other elements, i.e.

$$|F_{ii}| \geqslant \sum_{k \neq i} |F_{ik}| \quad \text{for all } i \tag{17}$$

That this condition will usually be satisfied can be seen by considering the matrix elements of F, in the case where an uninterrupted block of T observations is being analysed. The diagonal elements of F have the form :

$$\Sigma' f_n^2 \cos^2 (V_n + u_n + \sigma_n t)$$

or (18)

$$\Sigma' f_n^2 \sin^2 (V_n + u_n + \sigma_n t)$$

and both these are approximately equal to T/2 (or T, when $\sigma_n = 0$). For the off-diagonal elements, on the other hand,

$$|F_{ij}|_{i \neq j} \sim \left| \operatorname{cosec} \frac{T}{2} (\sigma_i - \sigma_j) \right| + \left| \operatorname{cosec} \frac{T}{2} (\sigma_i + \sigma_j) \right|$$

which will be rather small, unless the analysis is trying to separate constituents of very similar speeds, from a short period of observations.

In cases where condition (17) is not satisfied, provision must be made for solving the equation (9) by some other technique. The method of pivotal condensation (on the dominant elements) would perhaps seem the most suitable, since the matrix F would have to be seriously ill-conditioned before inaccurate results were produced by this method (the final processes of the analysis should check whether this has happened).

It is appropriate to discuss at this point the effect of inserting extra information (as discussed earlier in this paper). If the amplitude and phase are given at the outset for any constituent (say S_a and S_{sa}) then these can be allowed for when generating the elements of Q, by subtracting the contribution of these constituents from each member of the set h_0.

A rather more complicated case is where a relation is given between a pair of constituents (e.g. H (N_2) = H(M_2), $g\,(N_2) = g\,(M_2)$). In this example, either M_2 or N_2 will be left out of the vector x, and the matrix elements in the columns involving the other constituent will become rather more complicated.

Once the vector x has been found, the values z_0, H and g can quickly be deduced, and punched out on cards or tape. The final section of the program would then read these values back into the machine, and would predict the hourly heights for the period which had just been analysed. It would then print (in either tabular or graphical form) the frequency distribution of the differences between observed and predicted values.

Use

It is hoped to describe in this section a number of specific examples, and to discuss the sort of results which might be obtained. Firstly, however, it would seem useful to raise the question of data preparation and checking.

Normally the tidal observations will be available in either tabular or graphical form. The computer will require them in the form of either punched cards or punched paper tape. In preparing this data for the computer it is almost certain that mistakes will be made. To some extent these can be found by reading through a typescript copy of what has been punched, but although this form of checking can be used to reduce the number of errors, there is no guarantee that they will all be noticed. It is useful to include in the computer program a section which checks the data as it is read in, and which can draw the computer operator's attention to (or even eliminate) any gross errors. Given a sequence of, for example, five numbers, it is normally possible to deduce a good estimate of the central number in terms of the two numbers on either side, or of the last number in terms of the first four (provided always that the data is reasonably smooth). If the estimate differs substantially from the number's actual value, a gross error is indicated. Because time on large computers is so expensive, it may well be worthwhile to make the computer actually eliminate gross errors, rather than merely to point them out.

The most frequently encountered analysis problem is the analysis of about a year's observations of hourly heights in terms of about sixty constituents. All the constituents listed in Table 1, which is taken from DOODSON's paper (reference [7]), can be separated, and the Gauss-Seidel method can be used for solving the matrix equation (needing about five iterative cycles for convergence). If the port has shallow-water characteristics, it may be desired to include more constituents (perhaps eighth, tenth and twelfth diurnals) in the analysis, and this can be done. As mentioned earlier, values of H and g for the constituents S_a and S_{sa}, and also the value of z_0, can be obtained by analysing separately the monthly values of mean sea level, extending over a number of years. If more than one year's observations of hourly heights are available, then more accurate values for all the constituents should be obtainable. No extra constituent would be introduced which could not be considered in the analysis of one year's observations : the one gain of the longer period is greater accuracy. For estuary ports, the hydrography (and hence the tidal regime) may change quite rapidly for a district, and analyses for such a port would be carried out fairly frequently. The question arises as to how long a period of observations should be included in any analysis; an example would be a port where the data for 1953 and 1958 had already been analysed, and a further analysis using 1963 data is to be performed. To what extent the 1953 and 1958 data or results should be used will depend very much upon the particular port being considered. It is to be noticed that the earlier information may be used in either of two ways; an analysis could be carried out using all the data, or an analysis could be carried out using the 1963 data, and the earlier results (perhaps weighted according to age) could be added vectorially to the 1963 results. The second method will generally be the more economical, and both methods will produce very similar results unless the tidal constants have changed very rapidly with time, in which case the older data would not be used at all. Also, the second method enables a check to be kept on the annual variations of H and g, if any.

TABLE I

	degrees/ hour	degrees/ day						
S_0	0.0000000	0.000000	OQ_2	27.3416964	-63.799285	MN_4	57.4238337	-61.827990
S_a	0.0410686	0.985647	MNS_2	27.4238337	-61.827990	M_4	57.9682084	-48.762998
S_{sa}	0.0821373	1.971295	$2N_2$	27.8953548	-50.511484	SN_4	58.4397295	-37.446491
M_m	0.5443747	13.064993	μ_2	27.9682084	-48.762998	MS_4	58.9841042	-24.381499
MS_f	1.0158958	24.381499	N_2	28.4397295	-37.446491	MK_4	59.0662415	-22.410204
M_f	1.0980331	26.352793	ν_2	28.5125831	-35.698005	S_4	60.0000000	0.000000
	-	-	OP_2	28.9019669	-26.352793	SK_4	60.0821373	1.971295
$2Q_1$	12.8542862	-51.497131	M_2	28.9841042	-24.381499	-	-	-
σ_1	12.9271398	-49.748645	MKS_2	29.0662415	-22.401204	$2MN_6$	86.4079380	-86.209489
Q_1	13.3986609	-38.432139	λ_2	29.4556253	-13.064993	M_6	86.9523127	-73.144496
ρ_1	13.4715145	-36.683652	L_2	29.5284789	-11.316506	MSN_6	87.4238337	-61.827990
O_1	13.9430356	-25.367146	T_2	29.9589333	- 0.985600	$2MS_6$	87.9682084	-48.762998
MP_1	14.0251729	-23.395851	S_2	30.0000000	0.000000	$2MK_6$	88.0503457	-46.791703
M_1	14.4920521	-12.190749	R_2	30.0410667	0.985600	$2SM_6$	88.9841042	-24.381499
χ_1	14.5695476	-10.330859	K_2	30.0821373	1.971295	MSK_6	89.0662415	-22.401204
π_1	14.9178647	- 1.971248	MSN_2	30.5443747	13.064993	-	-	-
P_1	14.9589314	- 0.985647	KJ_2	30.6265120	15.036287	-	-	-
S_1	15.0000000	0.000000	$2SM_2$	31.0158958	24.381499	-	-	-
K_1	15.0410686	0.985647	-	-	-	-	-	-
ψ_1	15.0821353	1.971248	MO_3	42.9271398	-49.748645	-	-	-
Φ_1	15.1232059	2.956942	M_3	43.4761563	-36.572248	-	-	-
θ_1	15.5125897	12.302153	SO_3	43.9430356	-25.367146	-	-	-
J_1	15.5854433	14.050640	MK_3	44.0251729	-23.395851	-	-	-
SO_1	16.0569644	25.367146	SK_3	45.0410686	0.985647	-	-	-
OO_1	16.1391017	27.338441	-	-	-	-	-	-

Turning to shorter periods of observations, the question of analysing about six months' observations of hourly heights can be considered next. The difficulties here are typified by the constituents z_0, S_a and S_{sa}. The members of the pairs (z_0, S_a) and (S_a, S_{sa}) only change their phase relationships by 180° in six months, and this is the minimum change which can be used to get reliable separation of a pair of constituents. In such a case, the Gauss-Seidel method of solving the matrix equation would probably not converge, and the alternative method of solving the matrix equation would have to be used instead. Using this alternative method, it should be possible to separate fairly accurately the sixty constituents using six months' data, but this is the shortest period for which this should be attempted.

If only 1-3 months' observations of hourly heights are available, a more limited analysis must be accepted. Considering the list of sixty constituents again, the members of the groups :

z_0, S_a, S_{sa}	MS_f, M_f
$2Q_1$, σ_1	Q_1, ρ_1
O_1, MP_1	M_1, χ_1
π_1, P_1, S_1, K_1, Ψ_1, Φ_1	θ_1, J_1
SO_1, OO_1	OQ_2, MNS_2
$2N_2$, μ_2	N_2, ν_2
OP_2, M_2, MKS_2	λ_2, L_2
T_2, S_2, R_2, K_2	MSN_2, KJ_2
SO_3, MK_3	MS_4, MK_4
S_4, SK_4	$2MS_6$, $2MK_6$
$2SM_6$, MSK_6	

cannot be separated at all accurately. Given a suitable method of solving the equations, answers could be obtained for an analysis in terms of the whole sixty constituents, and the histogram at the end of the analysis might well suggest that the results were satisfactory; but if predictions and observations were compared for a period separated by about six months from the analysis period, it would probably be found that the agreement was unsatisfactory. There would seem to be three approaches to the solution of this problem of analysing 1-3 months' observations. The first approach relies upon the existence of some nearby port for which a complete analysis has already been performed. If it can be said of the nearby port that $H(2N_2) = \alpha H(\mu_2)$ and that $g(2N_2) = g(\mu_2) + \beta$, then the use of the same constants α and β should give sensible results for the station under examination, except perhaps for the shallow-water constituents. Because the values of H and g for shallow-water constituents are very dependent upon local conditions, this approach of linking one port with another is at its weakest for these constituents (the following constituents depend substantially upon shallow-water effects : MS_f, MP_1, SO_1, MNS_2, OQ_2, μ_2, OP_2, L_2, MKS_2, MSN_2, KJ_2, $2SM_2$ and all third-, fourth-, or sixth-diurnal constituents). The same method could be used for separating the members of all the groups except the first; in this case z_0 would normally be deduced from the analysis of the 1-3 months' data, the constants for S_a and S_{sa} being taken to be the same as those at the nearby port. Generally, the position is greatly simplified because the information for the nearby port will show that the amplitudes of many constituents are so small that these can be ignored altogether.

The second approach to the problem is very similar in procedure, and is used when there is no suitable fully-analysed nearby port. In this case the equilibrium tide is used in place of the nearby port as a source of amplitude ratios and phase differences (for those constituents which are present in the equilibrium tide). Again the answers obtained should be sensible, and much better than those obtained by attempting a full analysis of such a limited amount of data : however, the first approach would generally be preferred when there is a suitable port nearby.

The third approach seems interesting in principle, but has been very little used. In this approach, a full analysis in terms of the sixty constituents is carried out on a number of short periods of observations (e.g. four periods, each of a month's duration, separated by gaps of three months). The example quoted would probably be successful, and the results obtained yield good predictions, but this approach would have to be used with the greatest care, and each case would have to be considered individually.

If only a week's or fortnight's observations of hourly heights are available, then the analysis results become less reliable. The same approaches can be used as those described above for the rather longer periods of observation, but it must be accepted that the shorter the period of observations being analysed, the greater the reliance that must be placed upon the nearby port or upon the equilibrium tide. Another point that becomes exceedingly important for these short-period analyses is the effect of meteorological disturbances on the constants. Winds or abnormal barometric pressures are going to affect any analysis, but the short-period analyses are particularly vulnerable; it is desirable before starting such an analysis to check that conditions were meteorologically normal throughout the period of observations. Given suitable weather conditions, useful predictions can be obtained with the results of an analysis of a fortnight's observations for ports where the range of tide is small, and which do not exhibit marked shallow-water effects.

Conclusions

Now that the Liverpool Tidal Institute has access to a large and fast computer (English Electric KDF9), it is intended that a program along the lines described in this paper should be written, as it is felt that this should provide a powerful, economical and generally applicable analysis tool.

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