DETERMINATION OF AVERAGE OCEAN DEPTHS FROM BATHYMETRIC DATA

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1. Introduction

The problem of determining average ocean depths for a region of the world-ocean, or for all oceans together, is encountered in theoretical oceanography (such as determining the ocean tides theoretically on the basis of Laplace's tidal equations), where numerical computations have to be carried out. In these cases it is usually required to represent the actual variable depths by average values taken over some regular grid. It is the purpose of this paper to describe a method of computing such average depths from bathymetric soundings made at irregularly spaced stations in the ocean. The method is applied in order to determine average depths for the world oceans in a grid of one degree.

2. Statement of the Problem

A rough inspection of available bathymetric charts will illustrate the problem and its difficulties. The charts contain depths derived from soundings made at isolated points, as well as contour lines deduced from the individual soundings. The coverage is by no means uniform. Along the populated coasts, and in the North Atlantic Ocean at one extreme, the coverage is reasonably dense. In the other extreme, there are vast regions of the Arctic and of the Southern Hemisphere Oceans where, except for points on single tracks, there are many one-degree squares of ocean area which are completely empty of soundings. It seems that this problem has not yet been given proper attention in oceanographic literature.

The approach to the solution of this problem suggested here, draws from an analogy in meteorology, where a similar situation exists. The meteorological analogy referred to is known as the process of " objective analysis ". By this, meteorologists understand the preparation and transformation of widely scattered elements of observations, such as pressure, temperature, winds, etc. into input data at regular grid-points for use in the computation of weather forecasts. For a typical analysis of the Northern Hemisphere, there are many more reporting stations over land areas than over the oceans, and the incoming information has to be transferred to a regular grid on which the equations, describing the behaviour of the atmosphere, are defined. Since usually not much time is available from the time of receipt of the data to the time of release of the forecast, the mathematical structure of the analysis has to be as simple as possible, aiming to achieve the greatest amount of accuracy with the least amount of computation.

3. The Numerical Procedure

In line with this analogy, the following numerical procedure for the computation of average ocean depths has been developed.

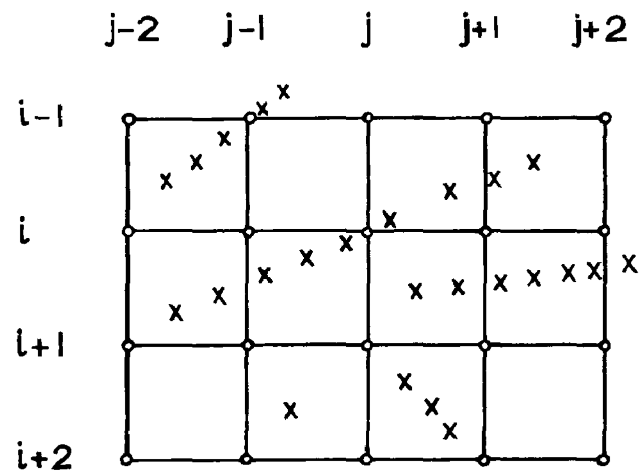

FIG. 1. — The Numerical Grid.
x Observation points (mostly along tracks)
o Grid points (where averages are required)
Initial Step : Assignment of Starting Values h_t at every Grid Point.

A rectangular grid is superimposed on the ocean area under consideration where depth-soundings are given at irregularly spaced observation points. An initial trial value h_t, let us say zero, is assigned to every grid point (fig. 1). Then the computation will go on in a 3-step repetitive cycle of scans through the area. At the first step, an interpolated depth h_{int} is calculated at every observation point from values of the 4 surrounding grid points (fig. 2) :

$$\begin{aligned} h_{int} = h_{i,j} &+ \Delta j(h_{i,j+1} - h_{i,j}) \\ &+ \Delta i(h_{i+1,j} - h_{i,j}) \\ &+ \Delta i \Delta j(h_{i+1,j+1} - h_{i+1,j} - h_{i,j+1} + h_{i,j}) \end{aligned} \quad (1)$$

Then the difference is formed between the interpolated and the observed depth at this point :

$$h_c = h_{obs} - h_{int} \quad (2)$$

Here h_{obs} and h_{int} denote the observed and interpolated depth at the observation point. At the second step, all grid-points are scanned successively in a systematic manner, and a correction $W \cdot h_c$ is computed. W is a

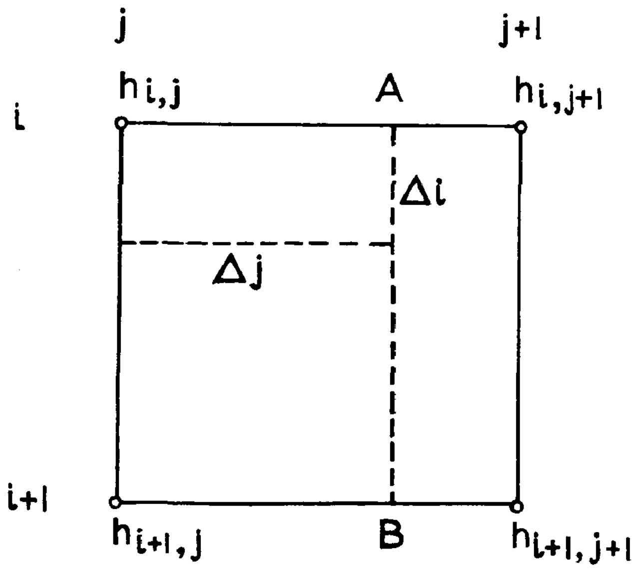

FIG. 2. — First Step of Iteration : Computation of Interpolated Depths at Observation Points.

$$
\begin{aligned}
h_{int} &= h_A + \Delta i\,(h_B - h_A) \\
&= h_{i,j} + \Delta j\,(h_{i,j+1} - h_{i,j}) + \Delta i\,(h_{i+1,j} - h_{i,j}) \\
&\quad + \Delta i\,\Delta j\,(h_{i+1,j+1} - h_{i+1,j} - h_{i,j+1} + h_{i,j})
\end{aligned}
$$

Difference : $h_c = h_{obs} - h_{int}$

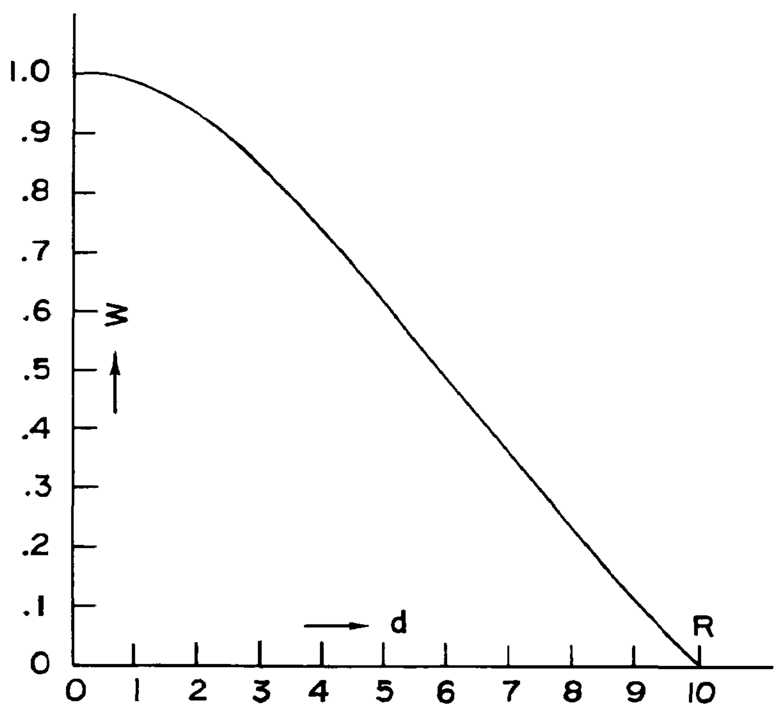

FIG. 3. — A Typical Weighting Function.

$$W = \frac{R^2 - d^2}{R^2 + d^2}$$

weighting factor which has the value 1 at the considered grid-point, and goes down monotonically to zero at a radius of R from the grid-point. All observation points inside the circle of radius R are considered, and for every observation point a weighted mean is determined, which is used to improve the initial trial value, h_t, — or the result of previous scan —, by adding the correction of $\frac{\Sigma (W \cdot h_c)}{\Sigma W}$. Such a weighting function is shown in fig. 3. Then, at the third step, all grid-points are scanned again, and a smoothing operator is applied with the purpose of filtering out small wave lengths of the order of about 2 grid-lengths, while leaving larger wave lengths (associated with the overall bottom-formation) substantially unchanged. (The development of such a smoothing procedure is given by SHUMAN [1]). The operator actually used has the form :

$$\bar{h}_{i,j} = \frac{1}{4} h_{i,j} + \frac{1}{8} [h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}] + + \frac{1}{16} [h_{i-1,j-1} + h_{i-1,j+1} + h_{i+1,j-1} + h_{i+1,j+1}] \quad (3)$$

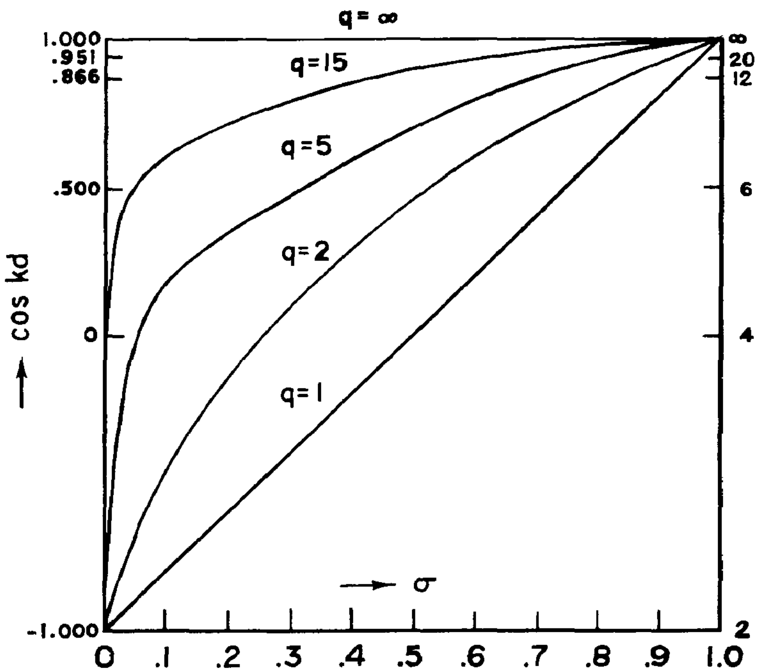

FIG. 4

$$\sigma^q = \left[1 - \frac{1}{2} [1 - \cos (kd)] \right]^q$$

$$kd = \frac{2\pi}{L} d$$

The increase of the selectivity of a smoothing operator σ (expressing the ratio of smoothed to unsmoothed amplitudes. k is the wave number, d the mesh length of the grid. L the wave length, q the number of successive applications of the operator).

The selectivity of the operator with regard to wave length can be increased by using it repetitiously, as shown in fig. 4, where the ratio of the amplitudes before and after smoothing is shown. This 31-step iteration cycle is repeated until the results of consecutive iterations do not differ at any grid-point by more than a predetermined arbitrary small constant.

4. Computations in a Trial Area

To test this method, a trial area of ocean has been chosen in the Southern Pacific, in a region where soundings are scarce and concentrated mostly along shipping tracks. The area is bounded in latitude by the equator and the parallel of 60° South, and in longitude by the meridians 90° West and 150° West (fig. 5). From all available data only those given on U.S. Navy Oceanographic Office charts Nos. 823 and 824 have been used as input data, making a total of 1131 points. The output grid, in Mercator coordinates, has about 4700 points.

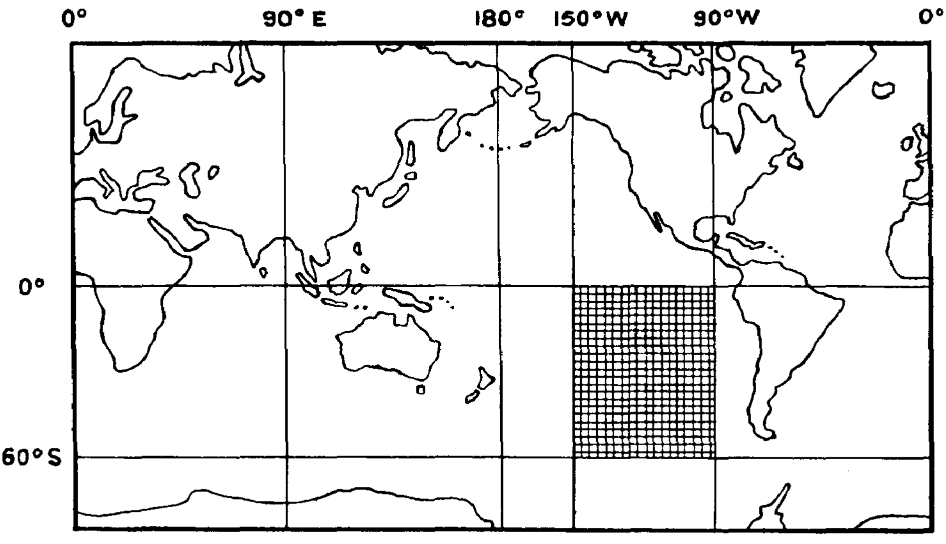

Fig. 5. — Trial Area for Computation of Average Depths.

As a quick and convenient visual means of display it is useful to draw contour lines from the averages obtained at the grid-points. This is shown for the region under consideration in fig. 6. A known mapped large-scale feature of the ocean bottom may then be used for comparison. In the trial area under consideration, we have the Pacific-Antarctic Ridge, defined on U.S. Navy Oceanographic Office Chart of the World No. 1262A by the 2 000 fathoms contours. The agreement between the mapped and plotted contours is quite good (see fig. 7), considering the incompleteness of the input data used. The overall smoothing effect is evident, and is even more impressive on a larger scale map of contour lines of 100 fathoms spacings.

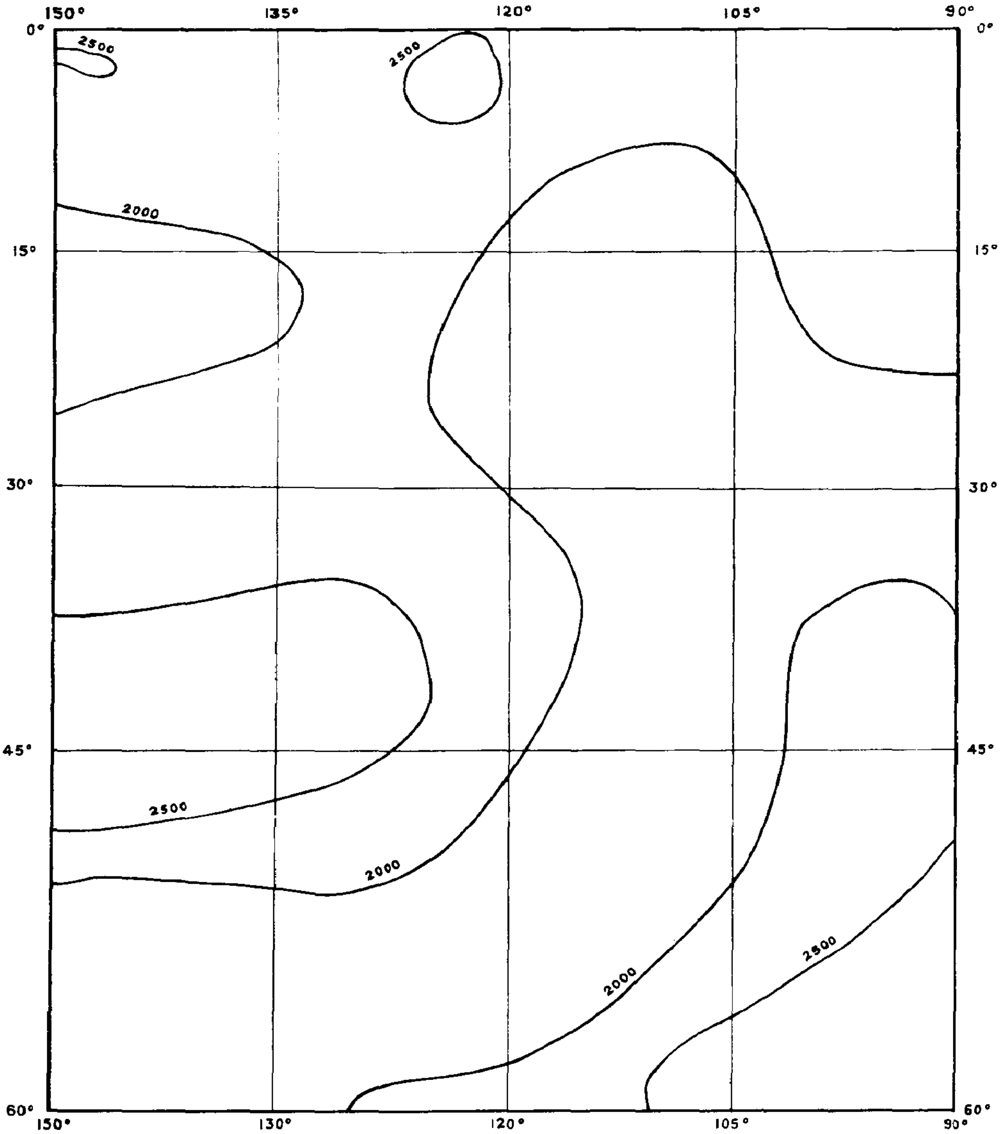

FIG. 6. — Contour lines of depth from numerical computation by the procedure described in this paper, using 1131 soundings from H.O. Charts 823 and 824. Depths in fathoms.

5. Average Depths of the World Oceans

By the method described above averages of depth have been determined on a grid, in Mercator coordinates τ, superimposed on the world oceans, where

$$\tau = \ln \tan \left(\frac{1}{2} \Phi + \frac{\pi}{4} \right). \quad (4)$$

Here Φ denotes geographical latitude in radians, and τ denotes Mercator latitude in radians.

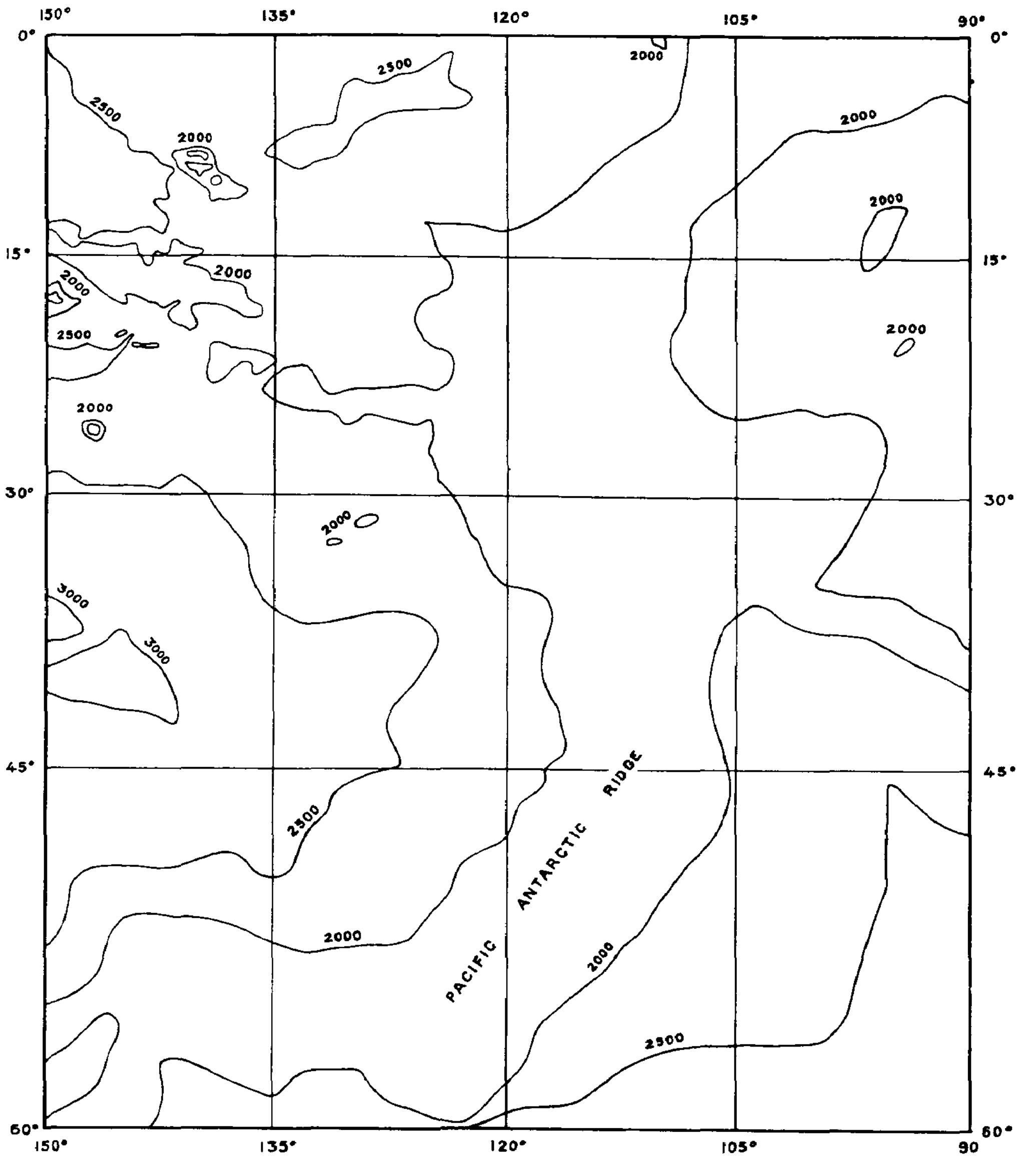

FIG. 7. — Contour lines of Depth from H.O. Chart 1262A, 10th edition, Jan. 1961. Depths in fathoms.

To obtain τ in degrees, as given in table 2 and in fig. 8, we must of course use the formula

$$\tau = \frac{180}{\pi} \; ln \tan \left(\frac{1}{2} \Phi + \frac{\pi}{4} \right) . \tag{5}$$

As input data, soundings at the intersections of one-degree geographical grid-lines have been chosen from the set of the GEBCO, of the IHB, Monaco, or from U.S. Navy Oceanographic Office charts, wherever a sounding at an intersection or near an intersection was available. For a considerable part of intersections no input data at all are at present available. A compilation of the input data, which were read manually from the charts, is given in table 1, arranged by meridians. The resulting averages of depths at grid-points of one Mercator degree are given in table 2.

(Four pages of Table 2 are reproduced hereunder)

TABLE 2

Average depths of world-oceans
Latitudes and longitudes in degrees, depths in metres
E = East, W = West, N = North, S = South
C = Point on continent, Parentheses ( ) = Location of island or peninsula
Phi = Geographical latitude, Tau = Mercator latitude

		LONGITUDE									
PHI	TAU	0	1 E	2 E	3 E	4 E	5 E	6 E	7 E	8 E	9 E
70.19	100 N	2943	3018	3073	3102	3103	3073	3011	2919	2796	2647
69.85	99 N	2986	3039	3069	3072	3047	2993	2910	2800	2666	2509
69.50	98 N	3018	3045	3046	3019	2964	2883	2778	2649	2501	2336
69.15	97 N	3039	3037	3006	2947	2861	2751	2620	2471	2309	2135
68.79	96 N	3047	3016	2952	2859	2740	2600	2443	2273	2095	1911
68.43	95 N	3042	2981	2885	2758	2607	2436	2253	2061	1867	1673
68.06	94 N	3022	2934	2807	2648	2465	2265	2056	1844	1634	1430
67.68	93 N	2986	2872	2717	2530	2318	2092	1859	1628	1403	1189
67.30	92 N	2930	2795	2618	2406	2170	1921	1668	1419	1182	959
66.91	91 N	2854	2704	2508	2278	2023	1756	1486	1224	976	748
66.51	90 N	2756	2596	2389	2146	1878	1598	1318	1047	793	561
66.11	89 N	2638	2473	2261	2012	1737	1451	1166	891	635	403
65.70	88 N	2499	2335	2124	1874	1600	1314	1029	756	503	277
65.29	87 N	2342	2183	1977	1735	1467	1187	909	644	400	183
64.87	86 N	2168	2018	1824	1592	1337	1070	805	553	323	120
64.44	85 N	1981	1843	1663	1448	1210	961	714	480	268	84
64.00	84 N	1782	1659	1496	1302	1085	859	635	423	233	70
63.56	83 N	1576	1468	1325	1153	962	761	563	377	211	71
63.11	82 N	1366	1273	1150	1003	839	666	496	337	197	C
62.66	81 N	1158	1079	976	853	716	572	431	300	186	C
62.20	80 N	955	890	806	706	596	480	368	C	C	C
61.73	79 N	765	711	644	566	481	393	C	C	C	C
61.25	78 N	591	547	496	438	376	312	C	C	C	C
60.76	77 N	439	403	365	325	283	242	C	C	C	C
60.27	76 N	311	282	255	230	206	184	C	C	C	C
59.77	75 N	210	186	168	155	146	141	140	C	C	C
59.26	74 N	134	114	104	101	104	112	124	C	C	C
58.75	73 N	82	66	61	66	78	96	118	C	C	221
58.23	72 N	51	37	37	48	66	89	117	149	186	227
57.70	71 N	34	23	26	41	63	90	120	153	188	224
57.16	70 N	26	18	24	41	65	93	123	153	183	(212)
56.61	69 N	21	17	26	45	70	96	123	147	170	(191)
56.06	68 N	14	16	29	50	73	97	118	136	150	(160)
55.49	67 N	2	12	29	52	74	94	109	119	123	C
54.92	66 N	0	4	27	51	72	87	96	97	93	C
54.34	65 N	0	0	18	39	57	77	79	73	60	42
53.76	64 N	0	0	10	30	45	54	55	50	29	3
53.16	63 N	0	0	6	24	36	40	36	23	2	C
52.56	62 N	0	0	7	25	34	33	24	C	C	C
51.95	61 N	(0)	0	15	33	42	32	C	C	C	C

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LONGITUDE

PHI	TAU	0	1 E	2 E	3 E	4 E	5 E	6 E	7 E	8 E	9 E
51.33	60 N	(0)	0	0	23	43	C	C	C	C	C
50.70	59 N	0	0	0	C	C	C	C	C	C	C
50.06	58 N	0	0	C	C	C	C	C	C	C	C
49.41	57 N	0	C	C	C	C	C	C	C	C	C
48.76	56 N	C	C	C	C	C	C	C	C	C	C
48.09	55 N	C	C	C	C	C	C	C	C	C	C
47.42	54 N	C	C	C	C	C	C	C	C	C	C
46.74	53 N	C	C	C	C	C	C	C	C	C	C
46.05	52 N	C	C	C	C	C	C	C	C	C	C
45.35	51 N	C	C	C	C	C	C	C	C	C	C
44.65	50 N	C	C	C	C	C	C	C	C	C	C
43.93	49 N	C	C	C	C	C	C	C	C	C	C
43.21	48 N	C	C	C	C	C	C	C	C	C	C
42.47	47 N	C	C	C	C	C	C	C	C	C	C
41.73	46 N	C	C	C	C	C	C	C	C	C	C
40.98	45 N	C	C	C	C	C	C	C	C	C	C
40.22	44 N	C	C	C	C	C	C	C	C	C	C
39.45	43 N	C	C	C	C	C	C	C	C	C	C
38.68	42 N	C	C	C	C	C	C	C	C	C	C
37.89	41 N	C	C	C	C	C	C	C	C	C	C
37.10	40 N	C	C	C	C	C	C	C	C	C	C
36.30	39 N	C	C	C	C	C	C	C	C	C	C
35.49	38 N	C	C	C	C	C	C	C	C	C	C
34.67	37 N	C	C	C	C	C	C	C	C	C	C
33.84	36 N	C	C	C	C	C	C	C	C	C	C
33.01	35 N	C	C	C	C	C	C	C	C	C	C
32.16	34 N	C	C	C	C	C	C	C	C	C	C
31.31	33 N	C	C	C	C	C	C	C	C	C	C
30.46	32 N	C	C	C	C	C	C	C	C	C	C
29.59	31 N	C	C	C	C	C	C	C	C	C	C
28.72	30 N	C	C	C	C	C	C	C	C	C	C
27.84	29 N	C	C	C	C	C	C	C	C	C	C
26.95	28 N	C	C	C	C	C	C	C	C	C	C
26.05	27 N	C	C	C	C	C	C	C	C	C	C
25.15	26 N	C	C	C	C	C	C	C	C	C	C
24.24	25 N	C	C	C	C	C	C	C	C	C	C
23.33	24 N	C	C	C	C	C	C	C	C	C	C
22.41	23 N	C	C	C	C	C	C	C	C	C	C
21.48	22 N	C	C	C	C	C	C	C	C	C	C
20.55	21 N	C	C	C	C	C	C	C	C	C	C

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PHI	TAU	LONGITUDE 0	1 E	2 E	3 E	4 E	5 E	6 E	7 E	8 E	9 E
19.61	20 N	C	C	C	C	C	C	C	C	C	C
18.66	19 N	C	C	C	C	C	C	C	C	C	C
17.71	18 N	C	C	C	C	C	C	C	C	C	C
16.76	17 N	C	C	C	C	C	C	C	C	C	C
15.80	16 N	C	C	C	C	C	C	C	C	C	C
14.83	15 N	C	C	C	C	C	C	C	C	C	C
13.86	14 N	C	C	C	C	C	C	C	C	C	C
12.89	13 N	C	C	C	C	C	C	C	C	C	C
11.91	12 N	C	C	C	C	C	C	C	C	C	C
10.93	11 N	C	C	C	C	C	C	C	C	C	C
9.95	10 N	C	C	C	C	C	C	C	C	C	C
8.96	9 N	C	C	C	C	C	C	C	C	C	C
7.97	8 N	C	C	C	C	C	C	C	C	C	C
6.98	7 N	C	C	C	C	C	C	C	C	C	C
5.99	6 N	3428	3289	3130	2953	2760	2553	C	C	C	C
4.99	5 N	3497	3348	3173	2973	2752	2513	2260	C	1735	1477
4.00	4 N	3612	3456	3269	3051	2808	2543	2263	1975	1687	1411
3.00	3 N	3762	3603	3407	3177	2917	2632	2331	2023	1717	1427
2.00	2 N	3937	3778	3579	3340	3067	2767	2450	2126	1805	1504
1.00	1 N	4128	3974	3775	3532	3251	2939	2608	2270	1936	1622
.00	0	4327	4182	3987	3743	3456	3136	2794	2443	2095	1768
1.00	1 S	4524	4392	4204	3962	3673	3347	2995	2632	2271	1930
2.00	2 S	4712	4595	4416	4179	3890	3560	3201	2827	2454	2101
3.00	3 S	4881	4782	4616	4386	4100	3768	3404	3021	2639	2276
4.00	4 S	5028	4948	4796	4576	4295	3965	3598	3212	2823	2452
4.99	5 S	5149	5089	4854	4746	4474	4148	3783	3396	3004	2628
5.99	6 S	5244	5205	5087	4894	4633	4316	3957	3572	3180	2803
6.98	7 S	5315	5296	5197	5020	4774	4468	4118	3739	3350	2973
7.97	8 S	5367	5367	5287	5129	4899	4607	4269	3898	3514	3138
8.96	9 S	5409	5427	5365	5225	5011	4735	4409	4048	3670	3297
9.95	10 S	5446	5479	5435	5311	5115	4854	4540	4189	3818	3448
10.93	11 S	5480	5527	5497	5390	5209	4962	4661	4320	3956	3590
11.91	12 S	5407	5448	5423	5328	5167	4946	4675	4367	4039	3710
12.89	13 S	5339	5373	5348	5261	5114	4913	4667	4388	4091	3793
13.86	14 S	5305	5334	5308	5226	5088	4900	4670	4407	4128	3847
14.83	15 S	5280	5302	5273	5191	5058	4877	4655	4403	4134	3862
15.80	16 S	5247	5256	5218	5131	4997	4818	4602	4357	4095	3829
16.76	17 S	5197	5188	5135	5037	4897	4717	4504	4263	4007	3744
17.71	18 S	5135	5104	5033	4922	4772	4588	4373	4135	3880	3618
18.66	19 S	5070	5019	4931	4806	4647	4457	4238	3998	3740	3475

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LONGITUDE

PHI	TAU	0	1 E	2 E	3 E	4 E	5 E	6 E	7 E	8 E	9 E
19.61	20 S	5007	4942	4842	4708	4543	4347	4124	3879	3615	3341
20.55	21 S	4946	4875	4771	4635	4467	4270	4044	3794	3523	3237
21.48	22 S	4888	4817	4717	4585	4422	4227	4001	3747	3470	3173
22.41	23 S	4835	4771	4680	4558	4403	4215	3993	3739	3456	3148
23.33	24 S	4791	4740	4663	4557	4415	4237	4021	3767	3477	3156
24.24	25 S	4707	4674	4619	4535	4416	4259	4061	3822	3544	3232
25.15	26 S	4630	4616	4584	4524	4430	4295	4117	3895	3631	3330
26.05	27 S	4562	4568	4560	4525	4455	4344	4186	3983	3735	3447
26.95	28 S	4505	4531	4545	4535	4490	4402	4267	4082	3851	3577
27.84	29 S	4457	4502	4538	4552	4531	4466	4351	4186	3972	3714
28.72	30 S	4419	4480	4535	4570	4570	4527	4432	4286	4090	3849
29.59	31 S	4387	4461	4530	4582	4601	4576	4500	4372	4194	3970
30.46	32 S	4356	4435	4511	4571	4604	4598	4546	4447	4302	4116
31.31	33 S	4333	4415	4494	4562	4605	4614	4582	4506	4387	4229
32.16	34 S	4320	4405	4490	4564	4617	4640	4623	4565	4465	4326
33.01	35 S	4322	4414	4506	4591	4657	4694	4693	4649	4563	4437
33.84	36 S	4339	4444	4551	4653	4740	4798	4815	4788	4715	4597
34.67	37 S	4365	4491	4622	4750	4863	4950	4997	4995	4939	4828
35.49	38 S	4373	4503	4637	4769	4887	4980	5036	5043	4996	4896
36.30	39 S	4376	4506	4640	4773	4894	4992	5055	5072	5037	4951
37.10	40 S	4374	4500	4632	4762	4883	4984	5053	5080	5059	4989
37.89	41 S	4367	4488	4614	4739	4856	4956	5029	5065	5058	5006
38.68	42 S	4358	4472	4589	4704	4813	4909	4982	5026	5032	4998
39.45	43 S	4351	4454	4558	4661	4757	4844	4913	4959	4976	4961
40.22	44 S	4345	4437	4525	4611	4691	4763	4823	4868	4892	4893
40.98	45 S	4343	4420	4491	4557	4617	4671	4719	4757	4785	4798
41.73	46 S	4339	4402	4455	4501	4541	4576	4608	4637	4662	4684
42.47	47 S	4328	4377	4415	4443	4465	4482	4499	4517	4538	4563
43.21	48 S	4304	4343	4368	4383	4391	4394	4398	4405	4420	4444
43.93	49 S	4264	4295	4312	4318	4317	4311	4306	4305	4313	4334
44.65	50 S	4205	4230	4243	4245	4239	4229	4219	4213	4217	4234
45.35	51 S	4126	4148	4159	4160	4153	4143	4131	4124	4125	4139
46.05	52 S	4027	4048	4058	4060	4055	4046	4037	4031	4032	4044
46.74	53 S	3910	3930	3941	3945	3944	3939	3933	3930	3934	3945
47.42	54 S	3776	3796	3809	3817	3820	3821	3821	3823	3830	3842
48.09	55 S	3628	3649	3667	3680	3690	3697	3704	3712	3722	3735
48.76	56 S	3470	3495	3518	3538	3557	3573	3588	3603	3617	3633
49.41	57 S	3309	3339	3369	3399	3427	3454	3479	3502	3523	3543
50.06	58 S	3149	3186	3226	3267	3307	3346	3383	3416	3445	3472
50.70	59 S	2999	3045	3095	3149	3203	3256	3306	3352	3393	3428

Using the present computations, a bathymetric map of the oceans in Mercator coordinates has been constructed (fig. 8). The irregular coastlines have been replaced by straight sections which are multiples of the unit grid length of one degree. With a view to applications to numerical analysis the coastline was chosen so that every grid-point in the ocean has at least one neighbor in each direction.

In drawing the coastline, the following computational policy has been adopted.

The large continental masses, including Australia and Greenland, have been regarded as real barriers. The remaining islands and some peninsulas have been included, whenever their extent is comparable with the basic one degree square of the Mercator grid, but have been disregarded in carrying out the computations leading to the average depths at their positions. This was done for the reason that when a larger grid-step than 1° will be used, some or all of the islands will not be recognizable in the coarser grid. Accordingly, these islands and peninsulas are shown as land-areas on the map (fig. 8), but the computed depths at corresponding grid-points have also been retained, and are given in table 2 (indicated by parentheses).

At some points of the larger islands or near the continental coasts, the computation yielded negative depth-values resulting from the rising trend of the ocean bottom towards the coast. Wherever this happened a value of zero has been substituted in table 2.

The radius R of the weighting function W, described in section 3, has been chosen as 15 Mercator grid-units everywhere, except in the Atlantic Ocean, where, due to the relative abundance of input data, a value of 12 units was deemed sufficient.

For the actual choice of a particular weighting function W there exist many possibilities. One would like to account for known special features of the ocean bottom in the vicinity of the point considered. This means that besides weighting with respect to distance only, angular weighting needs to be considered, and the function becomes two-dimensional. The weighting function itself, and particularly its maximum radius R of applicability, may vary from point to point. These refinements will undoubtedly be valuable and will be used in more advanced stages of the computational program. At present we do not yet possess enough detailed structural information of the ocean bottom topography on a world-wide basis to warrant the construction of a more sophisticated weighting function than the one given in fig. 3.

To carry out the actual calculations of the averages, the ocean area of fig. 8 has been divided in 35 computational square regions, i.e. each region is bounded by two meridians and two parallels of latitudes. About 50 iterations were required to obtain an average at every grid-point, accurate to two fractional decimal units, as defined in section 3. The smoothing operator (3) has been applied q (= 5) times in each iteration. As expected, slight discontinuities developed in areas where the computational regions join, although some overlapping has been provided in choosing the regions. To compensate for this effect, the sets of depth-values at the overlapping joints were first averaged, and then a further smoothing operation was

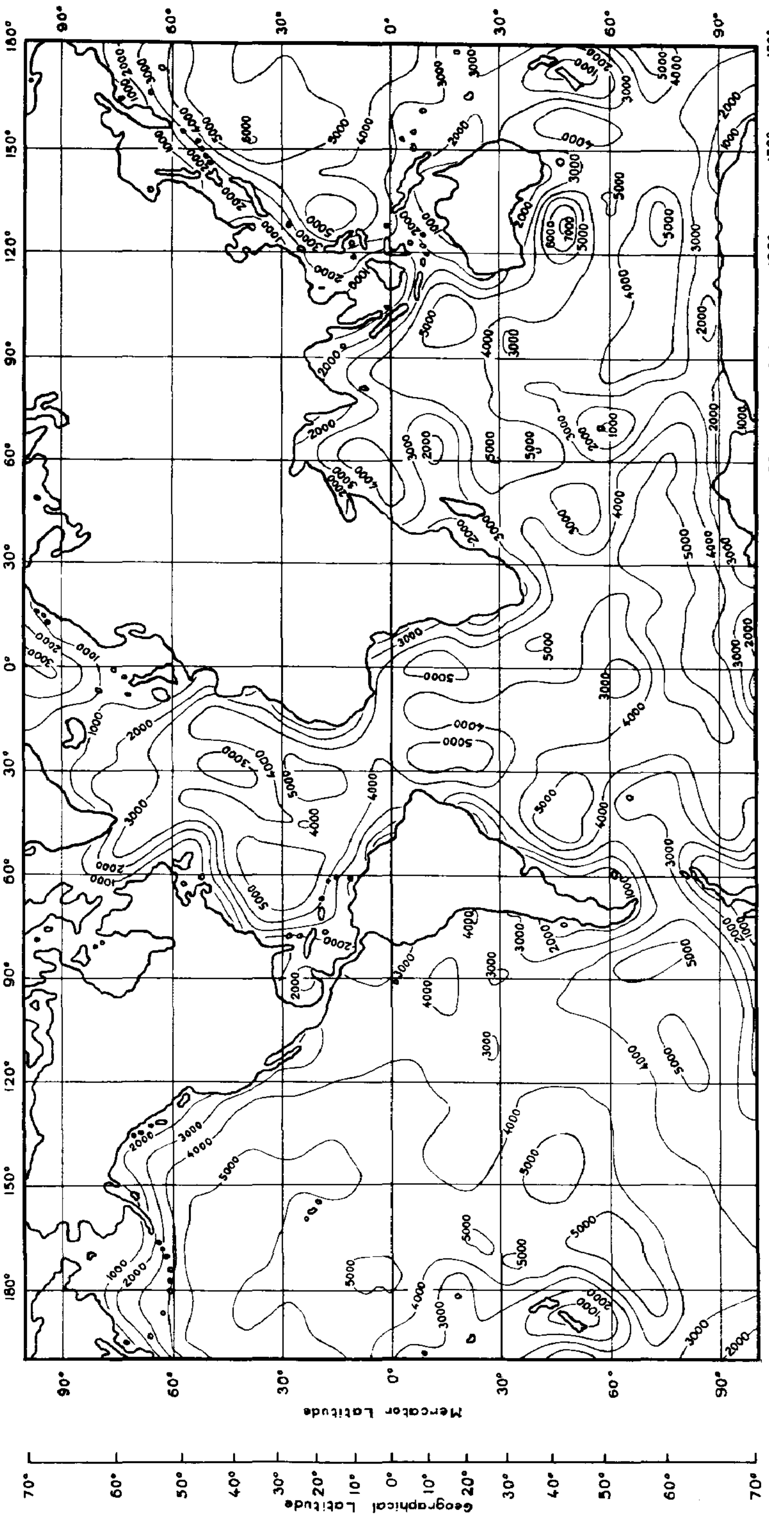

FIG. 8. — Bathymetric map of the world — oceans with contour lines of average depths from numerical computation by the procedure described in this paper. Depths in metres.

carried out. The final results of these computations are given in table 2 up to 100° Mercator latitude τ North and 99° Mercator latitude τ South, corresponding to about 70 geographical degrees North and South. Most of the Arctic and Antarctic regions are not covered. Fig. 8 contains contour lines of depth based on the results given in table 2.

All computations have been carried out on the CDC 1604-A computer of the Weizmann Institute of Science, Rehovoth, Israel.

6. Remarks on Smoothing

A final point to be mentioned concerns smoothing. It may be argued that smoothing in this case is more of a convenience than a necessity. The actual bottom configuration of the oceans, even on a large scale, is quite rugged, with plenty of mountain ranges, ridges, valleys, basins and trenches. Though in averaging itself the smoothing effect is inherent, the pattern of the averages may be quite irregular. Now, most of the mathematical applications for whose sake the averages have been computed, and especially those which instigated this research, require a smooth field, particularly if derivatives of the depth-field are needed. These have little meaning when sharp local variations are preserved. It has also been found that the computation of the average depth is more stable, and convergence is reached more quickly, if smoothing is present. For both these reasons a smoothing cycle has been included in the computation scheme.

The particular smoothing operator used preserves the overall average for the world oceans as a whole and for bounded regions as well.

7. Conclusion

In conclusion, it should be pointed out that the bathymetric averages presented in this paper are based on sparse, irregularly spaced, and possibly not too reliable input data. Nevertheless, the results obtained may serve as a tentative model-ocean for numerical oceanographic computations. They form the first presentation of its kind.

Acknowledgements

The author expresses his gratitude to Professor C. L. PEKERIS for advice and help.

Mr. Amir PNUELI aided considerably in the later stages of the work, by speeding up the programmes and by carrying out the final compilations of the entire programme.

This research was supported by the National Science Foundation under NSF Grant — G 15310 and by the Office of Naval Research under Contract Nonr-1823(00).

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