A RAPID MACHINE COMPUTATION METHOD OF CONVERSION BETWEEN LAMBERT GRID AND SPHERICAL CO-ORDINATES

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1. **Abstract.** — An attempt has been made to modify the existing formulae of conversion between Lambert grid and spherical co-ordinates and to design corresponding machine forms of computation, which are much simpler and less time-consuming than others, without requiring the use of elaborate trigonometrical tables (except for some listed values of grid constants and a special table of S^1 similar to N_{1K} of Survey of India or y' of U.S.C. & G.S.).

The suggested method of conversion, being significantly rapid and economical, is likely to prove of immense value to the surveying profession.

2. **Introduction.** — The Artillery requires a grid for convenient map references as well as for gunnery problems relating to position, range and direction finding. Grid maps are thus an essential requirement of the Army. But active service surveys are generally based on fixed points of existing surveys computed in spherical terms. These therefore have to be converted to grid terms. Similarly conversions are needed in the case of detached surveys based on astronomical co-ordinates. Conversion of co-ordinates from one grid to another is also sometimes required in areas of overlap between grids, and in that case the reverse process of conversion too becomes essential.

The method of conversion of co-ordinates is thus of special interest to the Army. Attempts have therefore been made at different times to improve upon the conventional method in various ways. The classical method of conversion is time-consuming as it requires use of 7- or 8-figure logarithmic tables even for ordinary accuracy, depending on the distance from the origin of grid. The method of conversion used in U.S.C. & G.S., though similar in principle to the classical one, has the advantage of being suitable for machine computations, and is considerably faster. But all the same, it does not possess the means to avoid the use of cumbersome 10-figure trigonometrical tables. The machine computation method of conversion, as used by Survey of India, on the other hand aims at completely eliminating the use of elaborate trigonometrical tables, by designing some special (not very long) tables. The method is quite fast, though a little less so than that used in U.S.C. & G.S., because of the great number of special tables needed in actual computations. However the main defect of the method is that it does not lend itself to conversions from Lambert grid to sphericals.

An attempt has therefore been made in the present paper to evolve a new machine computation method of conversion of co-ordinates which not only eliminates the use of cumbersome trigonometrical tables but also reduces the number of special tables to practically one, thus enabling computations to be carried out more rapidly and economically than by any other method.

3. **Conversion formulae.** — In the diagram below, let P be a point whose co-ordinates are to be converted between spherical : λ_P, L_P and Lambert grid : E_P, N_P where $E_P = \Delta E + E_0$ and $N_P = \Delta N + N_0$, the grid co-ordinates of the point P being ΔE, ΔN with respect to the true origin O of the grid whose spherical co-ordinates are λ_0, L_0 and grid co-ordinates with respect to the false origin O′ are E_0, N_0 expressed usually in round figures of the unit of grid.

From the diagram we have (*) the grid convergence :

$$ORP = C = \Delta L \cdot \sin \lambda_0 ,$$

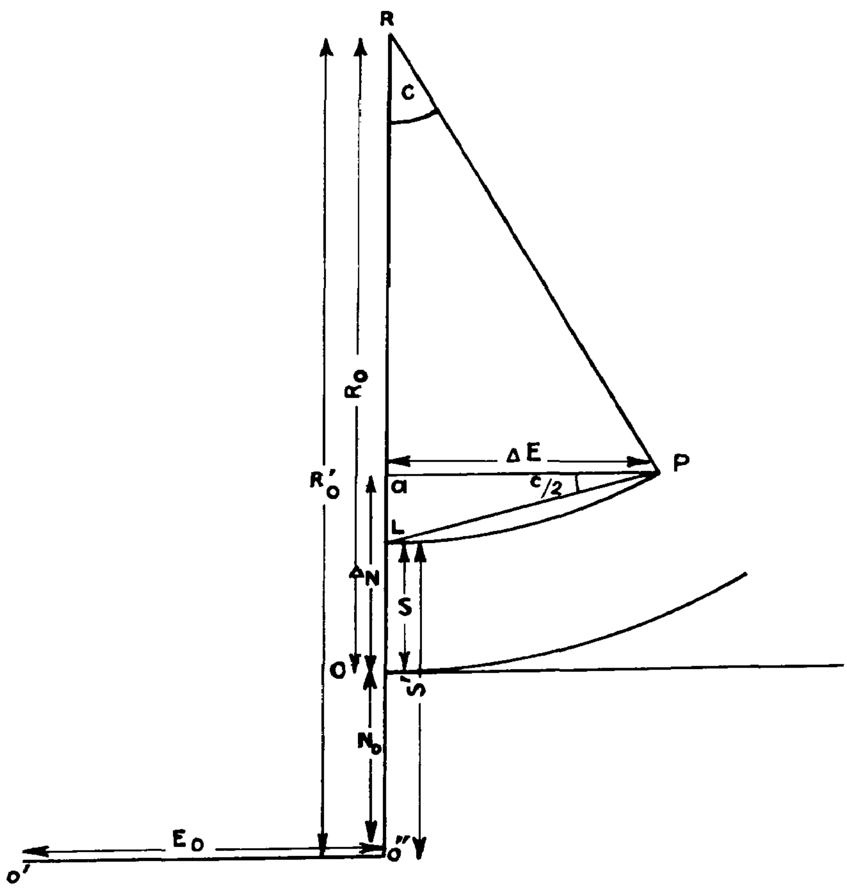

FIG. 1

(*) *Geodesy*, by Brig. G. BOMFORD, pp. 137-138.

where $\Delta L = L_P - L_0$, and $OL = S$ (*) where S is the spheroidal meridian distance so modified that the scale in meridian is everywhere equal to the slightly variable scale in parallel, thereby securing orthomorphism, the central scale factor being F_0 or $O''L = S + N_0 = S'$ which is equivalent to N_{1K} adopted in Survey of India or y' used in U.S.C. & G.S. S is, as usual, obtainable from the formula :

$$S = F_0 \left(m + \frac{m^3}{6\,\rho_0\,\nu_0} + m^4 \frac{\tan\lambda_0\,(1 - 4\,e^2 \cdot \cos^2\lambda_0)}{24\,\rho_0\,\nu_0^2} \right)$$
$$+ m^5 \left(\frac{5 + 3\,\tan^2\lambda_0}{120\,\rho_0^2\,\nu_0^2} \right) + m^6 \frac{\tan\lambda_0\,(7 + 4\,\tan^2\lambda_0)}{240\,\rho_0^2\,\nu_0^3}$$

where m is the true meridian distance from the latitude of the true origin of grid, ρ_0 and ν_0 the radii of curvature and e the eccentricity of the spheroid in use. S′ as deduced therefrom is thus easily expressible in convenient tabular forms correct to 0.1 of a yard/metre at regular intervals of one minute of latitude.

Also R_0, which is the radius of the developed parallel of the origin of grid, is equivalent to $F_0\,\nu_0 \cot\lambda_0$, or $R_0 + N_0 = F_0\,\nu_0 \cdot \cot\lambda_0 + N_0 = R_0'$, a known constant correct to only 0.1 of a yard/metre.

(*a*) CONVERSION FROM SPHERICAL TO LAMBERT GRID :

We have from the diagram,

$$\Delta E = (R_0 - S)\ \sin C = (R_0' - S')\ \sin C \tag{1}$$

and

$$\Delta N = S + \Delta E \cdot \tan\frac{C}{2} = R_0 - (R_0 - S) \cdot \cos C$$

or

$$R_0 - \Delta N = (R_0 - S)\ \cos C$$

or

$$R_0' - N_P = (R_0' - S') \cdot \cos C \tag{2}$$

Now expanding sin C in powers of C, we have from (1) :

$$\Delta E = (R_0' - S') \cdot \left(C - \frac{C^3}{6} + \frac{C^5}{120} - \frac{C^7}{5040} + \dots \right)$$
$$= (R_0' - S') \cdot C \cdot \left[1 - \frac{C^2}{12} \left(2 - \frac{C^2}{10} \right) \right] \text{, by approximation}$$

i. e. $\Delta E = (R_0' - S') \cdot C \cdot (1 - T)$ (3)

where

$$C = \Delta L'' \cdot 48481368 \cdot 10^{-13} \sin\lambda_0 = \Delta L'' \cdot K\,,$$

K being a known constant, and

$$T = C^2 \cdot \left(2 - \frac{C^2}{10} \right) \cdot \frac{1}{12}$$

Again expanding cos C in powers of C, we have from (2) :

$$R_0' - N_P = (R_0' - S') \cdot \left(1 - \frac{C^2}{2} + \frac{C^4}{24} - \frac{C^6}{120} + \dots \right)$$
$$= (R_0' - S') - (R_0' - S') \cdot \frac{C^2}{2} \left(1 - \frac{C^2}{12} + \frac{C^4}{360} - \dots \right)$$

or

$$N_P - S' = (R'_0 - S') \cdot \frac{C^2}{2}\left[1 - \frac{C^2}{24}\left(2 - \frac{C^2}{10}\right)\right] \quad \text{, by approximation}$$

$$= (R'_0 - S') \cdot \frac{C^2}{2} \cdot \left(1 - \frac{T}{2}\right)$$

i. e.

$$N_P - S' = (R'_0 - S') \cdot C^2 \cdot (2 - T) \cdot \frac{1}{4} \tag{4}$$

Speed and accuracy : Since K is a known constant for a particular grid, values of C are easily obtainable one after another from the products of $\Delta L''$ and K, using calculating machines. Moreover the expression for T is given in the form most convenient for machine computation; for once the values of C^2 become known, the evaluation of T takes hardly any time. The formulae (3) and (4) are thus extremely rapid, involving no trigonometrical tables, and computable with the help of only one special table giving values of S′ to the nearest tenth of a yard/metre.

Both the conversion formulae are highly precise; the probable error of the computed values being of the order of only $\pm$ 0.1 yard/metre for any reasonably large longitudinal extent from the grid origins, which is very satisfactory.

(*b*) Conversion from Lambert grid to spherical :

Referring to the same diagram and following similar notations, we have,

$$R_0 - S = \sqrt{(R_0 - \Delta N)^2 + \Delta E^2}$$

or

$$R'_0 - S' = \sqrt{(R'_0 - N_P)^2 + \Delta E^2}$$

or by the method of successive approximations :

$$R'_0 - S' = (R'_0 - N_P) \cdot \left[1 + \frac{1}{4} \cdot \frac{\Delta E}{R'_0 - N_P}\left\{1 + \frac{1}{1 + \dfrac{\Delta E^2}{2\,(R'_0 - N_P)^2}}\right\}\right]$$

or

$$R'_0 - S' = (R'_0 - N_P) \cdot \left[1 + \frac{C_1}{4} \cdot \left(1 + \frac{1}{1 + \dfrac{C_1^2}{2}}\right)\right]$$

where

$$C_1 = \frac{\Delta E}{R'_0 - N_P}$$

or putting in a more convenient form,

$$N_P - S' = \left\{\frac{0.5}{1 + \dfrac{C_1^2}{2}} + 0.5\right\} \cdot \frac{C_1^2}{2}\,(R'_0 - N_P) \tag{5}$$

Again from (1) and (2) , we have :

$$\frac{\Delta E}{R'_0 - S'} = \sin C = C - \frac{C^3}{6} + \frac{C^5}{120} - \frac{C^7}{5040} + \ldots \tag{6}$$

and

$$\frac{\Delta E}{R_0' - N_P} = \tan\ C = C + \frac{C^3}{3} + \frac{2\,C^5}{15} + \frac{17\,C^7}{315} + \ldots \qquad (7)$$

or combining (6) and (7) :

$$\frac{2\Delta E}{R_0' - S'} + \frac{\Delta E}{R_0' - N_P} = 3C + \frac{3C^5}{20} + \frac{C^7}{18} \quad \text{approximately}$$

or

$$3C = \frac{2\Delta E}{R_0' - S'} + \frac{\Delta E}{R_0' - N_P} - \frac{3C^5}{20} - \frac{C^7}{18}$$

or

$$C_3 = C_2 - \alpha \qquad (8)$$

where

$$C_2 = \frac{2\Delta E}{R_0' - S'} + \frac{\Delta E}{R_0' - N_P}$$

$$\alpha = 0.000622 \cdot C_2^5$$

and

$$C_3 = 3\ C$$

But

$$\Delta L'' = C \times 206264.81 \times \operatorname{cosec} \lambda_0$$
$$= C_3 \times 68754.937 \times \operatorname{cosec} \lambda_0$$

i. e.

$$\Delta L'' = C_3 \cdot K',\ K' \text{ being a known constant} \qquad (9)$$

Speed and accuracy : Since K' is a known constant for a particular grid, the values of $\Delta L''$ are easily obtainable one after another from the products of C_3 and K', using calculating machines. Moreover the expression for $R_0' - S'$ though comparatively long, is given in a form most suitable for machine computation; for once the values of $\frac{C_1^2}{2}$ become known, the evaluation of $R_0' - S'$ takes hardly any time. The conversion formulae are thus very rapid, involving no trigonometrical tables, and are as before computable with the help of only one special table.

The formulae are also highly precise, the probable error of computed values being of the order of 0.01 second for any reasonably large longitudinal extent from the grid origins, which is very satisfactory.

4. **Computations.** — Sample computations worked out in relevant forms have been appended in order to make the method clearer. The necessary instructions, and the tables of S' and grid constants as given, are likely to prove useful for carrying out computations with extreme rapidity and economy.

Instructions for using Form 1

1. Complete headings, entering grid, spheroid in use and striking out metres or yards whichever is not applicable. The spheroid is Everest in the case of India.
2. Complete lines (1) — (5) as instructed on the form.

3. Set the values of K (from *Table of Grid Constants*) on the Setting Lever and multiply by $\Delta L''$ i.e. line (5). Record the product from the Product Register in line (6) with the same sign as line (5), noting the decimal point carefully. Clear S.L. and the Multiplier Register.
4. Transfer the product from P.R. to S.L., clear P.R. and M.R. and multiply by itself i.e. by C. Record the product from P.R. in line (7). Clear S.L. and M.R.
5. Transfer the product from P.R. to S.L. and clear P.R. First turn the handle backwards twice and then after setting the gear lever to the mark " ÷ ", continue turning the handle forward till the figures on M.R. equal $2 + \frac{C^2}{10}$. Record $\frac{1}{12}$th of the product from P.R. in line (8). Clear S.L., M.R. and P.R.
6. Put the gear lever back to the mark 'X'. Set the upper tabular value of S' (from *Table of S'*) on S.L. and transfer the same from S.L. to P.R. Clear S.L. and M.R. Set the corresponding tabular value of diff. for 1'' on S.L. in correct position and multiply by the seconds part in λ_P so that the sum obtained in P.R. may equal S' for λ_P. Record the same sum from P.R. in line (9).
7. Set line (10) on S.L. and multiply by C i.e. line (6) Clear S.L. and M.R. Set the product on S.L., turn the handle backwards once and then after setting the gear lever to the mark " ÷ ", continue turning the handle forward till the figures on M.R. equal $1 + T$. Record the product on P.R. in line (11) with the same sign as line (5). Set the gear lever back to the mark 'X'.
8. Set R'_0 (from *Table of Grid Constants*) on S.L. and transfer the same from S.L. to P.R. Clear S.L. Set S' i.e. line (9) on S.L. in correct position and turn the handle forward once. Record the difference obtained from P.R. in line (10) and turn the handle backwards once so that R'_0 may reappear on P.R. to be used for another value of S'.
9. Complete line (12).
10. Set line (10) on S.L. and multiply by C i.e. line (6). Set the product on S.L. and clear P.R. and M.R. Turn the handle backwards twice and then after setting the gear lever to the mark " ÷ " continue turning the handle forward till the figures on M.R. equal $2 + T$. Record 1/4th of the product from P.R. in line (13). Clear S.L., M.R. and P.R.
11. Complete line (14) as instructed on the form.

Instructions for using Form 2

1. Complete headings, entering grid, spheroid in use and striking out metres or yards whichever is not applicable. The spheroid is Everest in the case of India.
2. Complete lines (1) — (4) as instructed on the form.
3. Set R'_0 (from *Table of Grid Constants*) on S.L. and transfer the same from S.L. to P.R. Clear S.L. Set line (3) on S.L. in correct position

and turn the handle forward once. Record the difference obtained from P.R. in line (5) and turn the handle backwards once so that R'_0 may reappear on P.R. to be used again for another value of N_P. Clear S.L., M.R. and P.R.

4. Divide ΔE i.e. line (4) as usual by line (5). Record the dividend from M.R. in line (6) with the same sign as line (4). Clear S.L., M.R. and P.R.
5. Set line (6) on S.L. and multiply by itself i.e. by C_1. Record half the product from P.R. in line (7). Clear S.L., M.R. and P.R.
6. Divide 0.5 as usual by $1 + \frac{C_1^2}{2}$ i.e. 1 *plus* line (7). Clear S.L. and P.R. Set the dividend (from M.R.) plus 0.5 on S.L., clear M.R. and then multiply by line (7). Clear S.L. and P.R. Set the dividend (from M.R.) plus 1 on S.L., clear M.R. and then multiply by line (5). Record the product from P.R. in line (8). Clear S.L., M.R. and P.R.
7. Complete line (9) as instructed on the form.
8. Divide ΔE i.e. line (4) as usual by line (9). Clear S.L. and P.R. Set the dividend (from M.R.) on S.L. and multiply by 2. Clear M.R. and S.L. Set C_1 i.e. line (6) on S.L. in correct position and turn the handle backwards once. Record the sum obtained from P.R. in line (10) with the same sign as line (4). Clear S.L., M.R. and P.R.
9. Set C_2 i.e. line (10) on S.L. and multiply by itself i.e. by C_2. Clear S.L. and M.R. Set the product from P.R. on S.L., clear P.R. and multiply by itself i.e. by C_2^2. Clear S.L. and M.R. Set the product from P.R. on S.L., clear P.R. and multiply by C_2 i.e. line (10). Clear S.L. and M.R. Set the product from P.R. on S.L., clear P.R. and multiply by 0.000622. Record the product from P.R. in line (11) with the same sign as line (10). Clear S.L., M.R. and P.R.
   N.B. Never use more than 4 figures in the above operation.
10. Complete line (12) as instructed on the form.
11. Set K' (from *Table of Grid Constants*) on S.L. and multiply by C'_3 i.e. line (12). Record the product from P.R. in line (13) with the same sign as line (12). Clear S.L., M.R. and P.R.
12. Complete lines (14) — (15).
13. Set R'_0 (from *Table of Grid Constants*) on S.L. and transfer the same from S.L. to P.R. Clear S.L. Set line (9) on S.L. in correct position and turn the handle forward once. Record the difference obtained in line (16) and turn the handle backwards once so that R'_0 may reappear on P.R. to be used again for another value of line (9). Clear S.L., M.R. and P.R.
14. Set the lower tabular value of S' (from *Table of S'*) on S.L. in correct position and turn the handle forward once. Divide the product in P.R. as usual by the corresponding tabular value of diff. for 1″. Obtain the seconds part in λ_P from the dividend (from M.R.) and then record the complete value of λ_P in line (17). Clear S.L., M.R. and P.R.

Table of grid constants
(..... spheroid)

		Grid X		Grid Y	
		°	′	°	′
Grid origin :	L_0	...	...	...	...
	λ_0	...	...	...	...
$K = 48\,481\,368 \cdot \sin \lambda_0 \cdot 10^{-13}$		. . . . .		. . . .	
$K' = 68\,754.937 \text{ cosec } \lambda_0$		. . . . .		. . . .	
$R'_0 = R_0 + N_0$ (yards/metres)		. . . . .		. . . .	
$E_0 =$ Grid easting of origin (yards/metres)		....		....	
$N_0 =$ Grid northing of origin (yards/metres)		....		....	

Table of S′
(...... spheroid)

Grid X							
Latitude		S′	Diff. for 1″	Latitude		S′	Diff. for 1″
°	′	Yard/Metre	Yard/Metre	°	′	Yard/Metre	Yard/Metre
...	00	....	....	...	00	....	....
...	01	....	....	...	01	....	....
...	...	....	....	...	...	....	....
...	...	....	....	...	...	....	....

FORM 1

Machine deduction of grid co-ordinates from spherical

Formulae :

(1) $C = K \cdot \Delta L''$

(2) $T = C^2 \left(2 - \frac{C^2}{10}\right) \frac{1}{12}$

(3) $\Delta E = (R'_0 - S') \cdot C\,(1 - T)$

(4) $N_P - S' = (R'_0 - S')\, C^2\,(2 - T) \frac{1}{4}$

Grid : Spheroid : Yards/Metres

1. Point P	1
2. Latitude of P = ...	14° 01′ 40″.56
3. Longitude of P = L_P	65 42 28.40
4. $\Delta L = L_P - L_0$ (*)	— 14 17 31.60
5. $\Delta L''$	— 51 451.60
6. C (8 places) [Formula (1)]	— 0.081 211 15
7. C^2 (8 places)	0.006 595 25
8. T (8 places) [Formula (2)]	0.001 098 85
9. S' (**) for λ_P	398 291.7
10. R' (*) — S'	20 839 116.4
11. ΔE (1 place) [Formula (3)]	— 1 690 509.0
12. Grid easting of P = E_P : line (11) + E_0	1 309 491.0
13. $N_P - S'$ (1 place) [Formula (4)]	68 681.8
14. Grid northing of P = N_P : line (9) + line (13) ..	466 973.5

(*) Values to be taken from *Table of Grid Constants* for the grid in use.
(**) Values to be taken from *Table of S′* for the grid in use.

FORM 2

Machine deduction of spherical co-ordinates from grid

Formulae :

(1) $C_1 = \dfrac{\Delta E}{R'_0 - N_P}$

(2) $N_P - S' = \left(\dfrac{0.5}{1 + \dfrac{C_1^2}{2}} + 0.5\right) \dfrac{C_1^2}{2} (R'_0 - N_P)$

(3) $C_2 = \dfrac{\Delta E}{R'_0 - S'} \cdot 2 + C_1$

(4) $\alpha = C_2^5 \cdot 0.000622$

(5) $C_3 = C_2 - \alpha$

(6) $\Delta L'' = K' \cdot C_3$

Grid : Spheroid : Yards/Metres :

1. Point P	1
2. Easting of $P = E_P$	1 309 491·0
3. Northing of $P = N_P$	466 973.5
4. $\Delta E = E_P - E_0$ (*)	— 1 690 509.0
5. $R'_0 - N_P$: R'_0 (*) — line (3)	20 770 434·6
6. C_1 (8 places) [Formula (1)]	— 0.081 390 16
7. $\dfrac{C_1^2}{2}$ (8 places)	0.003 312 18
8. $N_P - S'$ (1 place) [Formula (2)]	68 681.9
9. $R'_0 - S'$: line (5) + line (8)	20 839 116.5
10. C_2 (8 places) [Formula (3)]	— 0.243 633 99
11. α (8 places) [Formula (4)]	— 0.000 000 53
12. C_3 : line (10) — line (11)	— 0.243 633 46
13. $\Delta L''$ (2 places) [Formula (6)]	— 51 451.60
14. ΔL	— 14° 17′ 31″60
15. Longitude of $P = L_P$: line (14) + L_0 (*)	65 42 28·40
16. S' : R'_0 (*) — line (9)	398 291.6
17. Latitude of $P = \lambda_P$ (**) for S' (2 places)	14° 01′ 40″56

(*) Values to be taken from *Table of Grid Constants* for the grid in use.
(**) Values to be taken from *Table of S′* for the grid in use.