THE TREATMENT OF HOURLY ELEVATIONS OF THE TIDE USING AN IBM 1620

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SUMMARY

At the Tidal Institute in recent months there has been assembled a family of mutually compatible computer programs for the treatment of tidal elevations at hourly intervals. These programs are written for the IBM 1620 computer and deal with the analysis and prediction of tidal elevations together with the extraction of residuals. In view of the comparatively small size and limited speed of this computer in relation to those more frequently favoured by workers in the tidal field, every attempt has been made to optimise the programs with the result that their ability to execute the tidal calculations can be compared with programs designed for more powerful computers. The aim has been to reduce the bulk of calculation at the expense of more elaborate programming, and without any reduction in precision.

Features of the IBM 1620

The IBM 1620 with the Card read-punch unit, IBM 1622, and with a core storage capacity of 40 K decimal digits is in many ways a convenient tool for the treatment of tidal data. In fact with little loss in efficiency the basic 20 K version of the computer can be equally applied to the task. Among its virtues can be listed those of which specific advantage was taken in the arrangement of the programs :

(1) The card reader-punch unit and also the line printer, IBM 1443, where this is available, are equipped with a buffer store so that in operation the input buffer for example contains the contents of the next card required by the program. When the program calls for this information, it is therefore immediately accessible without any delay caused by the initiation of card handling mechanics, whereupon the buffer is filled by the contents of the next card in sequence while computation proceeds virtually uninterrupted. Similar facilities are available in the output process.

Careful programming can therefore arrange to input and output data during useful computation time. This point is a significant one in tidal work since large quantities of data are of necessity handled in the process. The analysis of 12 months' tidal observations requires the input of over 8 500 units of observational data whereas the prediction program calls for the output of the same quantity of data.

(2) The 1620 computer is a variable word length machine in which arithmetic is performed on "fields" containing a variable number of binary-coded decimal digits. Calculations proceed serially digit by digit until a "flag" over the senior digit of a field indicates that the particular operation is complete. It is possible therefore to reduce the bulk of arithmetical calculations by a careful consideration of the number of significant digits required in each stage of a calculation. Hourly tidal elevations can be defined by 3 significant decimal digits so that much of the computation can be concerned with arithmetic applied to very small numerical fields. Moreover since it is known that no amplitude of a harmonic constituent will exceed 15 feet in magnitude and since it is customary to work to a nominal accuracy of one thousandth part of a foot, then it is seldom necessary to exceed six significant figures in any part of the computation. Unlike work on most other electronic calculators it is therefore possible on the IBM 1620 to tailor the program to the specific needs of the tidal calculation and so compensate for the limited speed of this small computer by eliminating spurious accuracy in the arithmetic.

(3) Although this computer is generally classed as small, nevertheless 40 thousand decimal digits represents a capacity sufficient to accommodate both a sizeable program and also an appreciable data storage. Moreover these 40 thousand digits are all accommodated in an immediate access store. This being so, it is possible to store within the computer a large matrix of tabular material for immediate use during the program. For example the analysis program at one stage stores a matrix of 1766 factors required in the calculation and again, in one of the prediction programs, rather than make use of trigonometrical subroutines or alternatively generate trigonometrical functions by other means, it is possible to store a complete cosine table for all four quadrants of arc containing values at intervals of one tenth of a degree. The latter makes it possible to predict hourly elevations of tide with a very swift table look-up procedure, without call for interpolation or quadrant adjustment.

(4) The availability of 4 Program Switches on the Console can with ease alter the flow of a program, so that a master program can be written designed to perform more than one task according to the switch arrangement. The obvious application of this feature within the programs discussed here is to control the form of output. Again intermediate stages of calculation are made available for scrutiny by this means. In the final stage of the analysis program however the arrangement of the program switches is significant in determining whether this program should in fact perform the analytical process or whether alternatively it should provide the requisite data concerning the nodal tide and the phase of the equilibrium tide for input into the prediction program.

(5) The possible configuration of ancillary equipment, particularly the availability of the 1311 Disk-Pack backing store and the 1443 Line Printer

can reduce time taken for the input of program instructions and the output of printed results to negligible quantities.

(6) Finally the computer is provided with an automatic parity check both in the input/output device and also in the course of calculation. When every digit is handled, it is examined in order to confirm that its representation by the magnetic fields of 6 individual cores forms a valid code. This has in our experience proved to be a powerful check. The Tidal Institute has now accumulated considerable experience with the computer which it has employed for numerous tasks and although every opportunity has been taken to check results, sometimes by hand calculations and sometimes by specially devised tests, we have always been satisfied with the arithmetical accuracy of the results.

In taking advantage of the above facilities, it has been possible to devise a program for the harmonic analysis of a one year's record of tidal elevations involving sixty constituents which can be completed in from 3 to 12 minutes, depending upon the particular configuration of ancillary equipment available. Again tidal hourly height predictions can be prepared using card or line printer output at a rate of 8.5 minutes per thirty constituent year upon the 1620 MK II. These figures compare favourably upon an economic basis with existing programs for more powerful computers.

Card coding

In the use of cards, economy can only be achieved by attempting to fill both input and output cards with useful information. Careful consideration of card coding is therefore advisable.

In the case of tidal elevations the foot scale has certain advantages. One-tenth part of a foot is a satisfactory order of accuracy both in observation and in prediction, and moreover since the maximum tidal range found in nature does not exceed 500 of these units, any tidal elevation can be expressed in 3 digits while still allowing considerable latitude in the choice of datum. In the metric system the equivalent convenient unit is the centimetre and although in this convention the 3 digit range —999 to +999 adequately covers the definition of tidal phenomena, it is necessary to place severe restrictions on the datum. This is seldom convenient in practice. Standardization has therefore been made on the one-tenth foot unit, although a trivial program has been written to convert data punched in the metric system into the standard convention and at the same time to apply a datum adjustment.

With a 3 digit convention it is possible to punch the data for a complete day upon a single card. The hourly elevations are punched in the first 72 card columns and where negative the junior digit of the three is overpunched with a negative sign. In this way 8 columns of the card remain and these are used to punch a code which identifies the data. A typical code would be : — $\bar{6}$4123456 which would indicate that the card contains data for year 1964 and for the 123rd day in 1964, i.e. May 2nd. The last three digits of this code define the port or location to which the data refers. Observed and predicted levels together with residuals are punched in this same convention but a colour coding is used to define the type of material. This system has the advantage that each card is unique and easily identifiable. Moreover the code punched in columns 73 to 80 makes it possible for all programs to have internal checking devices.

In this connexion, all programs are arranged, in so far as this is possible, to exercise control by typing appropriate instructions to the operator and moreover, by an examination of the card coding of the input, to accept only appropriate data and to inform the operator of the nature of any error if necessary. Again it has been arranged that whatever program output is obtained, whether it be on card, typewriter or line printer, each item of output is identified according to its geographical location, its reference date and its nature.

Where the output of one program provides the input to another, it is arranged that the above identification is all that the second program requires in order to identify its own computations and its own output. In this way input data is reduced to a minimum and the possibility of confusion by the operator is eliminated. For example the analysis program requires as input only the observed elevations over a period of at least 358 days. The program checks that, given the first card, all later cards apply to the same port and are in sequence even if the data spans the year end and in this it is able to discriminate between leap years and common years. The program selects the data which it requires and the day code of the central time origin of the process is calculated to be carried forward automatically to the final stage of the analysis when it is used to compute the mean longitudes of the moon, sun, moon's perigee and moon's ascending node and thereafter the phase of each constituent in the equilibrium tide together with the appropriate corrections to the phase and amplitude of each constituent for the nodal tide.

THE TREATMENT OF OBSERVATIONAL DATA

Verification of data

The observed elevations of the tide are normally extracted from automatic tide gauge records which are available in a variety of forms. These are invariably graphical records in which the curves representing the tidal heights for 7 or 14 consecutive days are made to overlap on a single chart. The task of transferring this great mass of data to punched cards, at the same time applying time and height corrections indicated by the authority which maintains the gauge, is no easy one and it is inconceivable that this can be accomplished without error. A device for checking the prepared data is therefore highly desirable, other than a repetition of the work on a card verifier. On a large computer one might consider testing the validity of each tidal elevation against a multi-term polynomial fitted to neighbouring values and where an item of data appears to be suspect by this test, to replace it by a more reasonable value. On the 1620, a less elaborate technique is indicated, whereby each elevation is verified by a smoothness check, and the computer is made to indicate unsmooth values which can later be examined and corrected manually.

An examination of tabulated hourly tidal elevations shows that the array is most smooth in a 25 hourly sense. A simple way in which advantage can be taken of this feature is to apply the following numerical check : —

$$\zeta_t \fallingdotseq 1/6 \; (-\zeta_{t-50} + 4\zeta_{t-25} + 4\zeta_{t+25} - \zeta_{t+50})$$

where ζ_t is the tidal elevation at time t hours.

A program designed to test this condition against a tolerance is relatively trivial but in practice leaves much to be desired since it gives a large volume of output in which the majority of items are not errors in the normal sense but are correctly punched versions of the tidal elevation concerned. The reason for this failure is found in another feature of tidal records. All elevations of the tide are liable to perturbation by meteorological phenomena and since the passage of barometric systems responsible for these perturbations occupies upwards of 18 hours, it follows that the elevations for one day can be raised or lowered by an appreciable amount compared with those for the previous or following day. Consequently in such cases the above program tends to output a large number of consecutive hours as unsmooth.

If this were the only problem involved, it could be overcome by a similar trivial check on a shorter span of data, using the expression

$$\zeta_t \fallingdotseq 1/6 \; (-\zeta_{t-2} + 4\zeta_{t-1} + 4\zeta_{t+1} - \zeta_{t+2})$$

However this test also errs in a similar respect and results in an equivalent volume of pseudo-error output. Where the tidal range is large, and particularly where shallow water effects destroy the symmetry and simplicity of the conventional sinusoidal record, then the latter test is unable to cope with the conditions which exist at or near high water, low water and points of inflexion.

A combination of the two tests, however, has proved to be quite satisfactory and has been adopted as general practice. The principle used is that if an hourly elevation passes either one of the above tests it is acceptable but if it fails by both then it requires examination, with a view to correction. The case of Avonmouth tidal records was selected as a rigorous test of the smoothing process since Avonmouth has *a*) a 40 foot range of tide *b*) great susceptibility to meteorological perturbations *c*) a large shallow water contribution to the tide. In testing the procedures on a set of 80 days' records the application of either one of the tests produced an output of the order of 250 'errors', whereas the two tests combined produced only 17. This was considered to be quite satisfactory.

The appropriate flow diagram is shown in figure 1.

In operation, the computer is able to accomplish the test on 24 hourly values at approximately the same speed as it is capable of reading cards. If no error occurs in the punched data, the running time is therefore roughly equivalent to the time taken to read cards viz. 1 ½ minutes per 12 months' data. Actual running time depends upon the characteristics of the tide and its perturbation at the port, the tolerance parameter and also the quality of the punching but where output is arranged for cards or line-printer the running time does not exceed 1 ½ minutes by a significant margin.

Figure 2 gives a specimen output from this program for tidal observations made at the port of Jabal Dhanna in the Persian Gulf.

THE HARMONIC ANALYSIS OF HOURLY ELEVATIONS OF THE TIDE

A number of computer techniques have recently been devised for the task of determining the amplitudes and phases of the harmonic constituents of the tide. The basic problem of tidal analysis lies in the difficulty of separation of constituents which are very close in angular speed. Some techniques seek to take advantage of the fact that a span of data may be selected, usually of 355 or 369 days, in which most of the major constituents, as a first approximation, complete an integral number of oscillations. Suitable corrections can then be devised for the influence of one constituent upon another in order to allow for imperfection in the above approximations. This procedure is employed by DOODSON [1], MIYAZAKI and UENO [2] and also CARTWRIGHT and CATTON [3]. Other methods like those of MURRAY [4] and GROVES [5] give in the first instance less weight to the particular relationships of the harmonic constituents of the tide and seek, by matrix arithmetic and least squares principles, to solve a large number of simultaneous equations in terms of a specified number of independent constituent speeds. However, where the span of observational material is short it is necessary even here to recognize the problem of separation by the insertion at a later stage of probable constituent relationships where speeds are near. More recently still, power spectral techniques have also entered the analytical field and again in the first instance it is seen that equal weight is given to each constituent speed. In fact in power spectral techniques even the known constituent speeds may be disregarded and the analytical process may first be required to determine the full spectrum of harmonic response throughout the tidal range.

In a choice of analytical techniques one is therefore presented with a range of possibilities in which the significance which is placed upon the known relationships between the tidal constituents varies through an enormous range, from total predominance to complete disregard. On the one hand each constituent may be given specialized treatment according to its magnitude, and the manner in which it is likely to affect, and in turn be affected by, its neighbours. On the other hand in a more powerful technique all constituents, together perhaps with non-existent tidal constituents, may receive virtually identical treatment. From the programmer's point of view, this choice is significant. In general it can be said that the further the technique proceeds toward the latter extreme, the more the program lends itself to a more systematic treatment in which great advantage can be taken of program loops. The more individual the treatment required for each constituent, the more complex must the logic of the program become. In contrast however one can also state that as more advantage is taken of the available knowledge of tides, then the easier it becomes to reduce the bulk of calculation required by the computer. The choice therefore is between a small amount of arithmetic coupled with a complex program and a mass of calculation coupled with an elegant program.

Using a slow computer the obvious solution was to reduce calculation time at the expense of programming difficulties and no hesitation was experienced in selecting the proved technique of DOODSON [1] in that here was a method carefully devised in its origin for economic use by desk calculators. In this choice it was realized that certain restrictions are imposed. For example the span of data must remain constant and the data must be continuous throughout. The same constituents, 60 in all, must be determined for each case and the quality of correction applied to a constituent from its neigbours is restricted to a level which was considered adequate and practical in terms of hand calculation. In this connexion it is important to note that experience shows DOODSON's corrective processes to suffice for all cases except possibly those where large tidal ranges coincide with large shallow-water effects. In the latter circumstances the treatment of tidal phenomena by means of the conventional harmonic constituents breaks down in any case, so that it is possible to state that the technique does not prejudice the quality of results considered in terms of tidal prediction.

Doodson's yearly analysis

The procedure adopted by DOODSON is first to apply to a continuous set of data spanning 355 days a set of numerical filters each of which is designed to separate a particular species of tide; diurnal, semidiurnal, etc., from the series, one result being given for each day in each case. At this stage, one is presented with a set of X or Y functions in DOODSON's notation each with one suffix appropriate to the applied filter, e.g. X_2.

Each array is then combined several times month by month by means of a series of monthly multipliers in which a second numerical or literal suffix is acquired e.g. $X_{2.a}$. Each set of data at this stage comprises 29 separate values corresponding to the days of a composite month. The above sets of 29 values are then combined by a third set of multipliers in a daily sense in order to produce functions of the type X_{23a}, one in each case, appropriate to the central day of the observation set.

The 9 sets of hourly multipliers, the 7 sets of monthly multipliers and the 15 sets of daily multipliers represent a formidable number of possible linear combinations of this type, but by careful inspection of the hourly and daily increments in phase of the known tidal constituents some 213 in number are selected for calculation. These selected values have the virtue of receiving a dominant contribution from either the sine or cosine vector of either a single tidal constituent or of conjugate pairs, plus small contributions from the major tidal constituents.

The second stage of the analysis proceeds to extract from the linear combinations the appropriate sine or cosine vector by effecting suitable corrections for the influence of the major tidal constituents, which are themselves determined by similar linear combinations. The calculation of the values of the correction factors is a laborious task in view of the impurities involved in each item of data at this stage, but this task has been effectively accomplished by DOODSON in his original paper and the form of the requisite corrections is provided therein. In the case of some constituents their determination is in effect repeated by using up to four pairs of the above mentioned combinations so that a subsequent averaging process is required. For conjugate constituents which are spaced equally in speed on either side of a major constituent a simple subsequent separation process ensues.

At the close of the second stage of the analysis the process provides, in the form $R \cos \delta$ and $R \sin \delta$, both vectors of each of 60 tidal constituents and in this case δ is appropriate to a certain time on the central day of the analysis depending upon the species.

The third stage simply resolves the vectors to give an amplitude and phase of the constituent, calculates the phase of the appropriate constituent of the equilibrium tide, V, so as to establish a time scale, and computes and applies corrections for the nodal tide, f and u, to the amplitude and phase lag respectively of each constituent, and for the species phase which involves a quantity, Δ, again for each constituent.

The analysis program for twelve months' observations

The complete analytical computation which requires approximately 4 000 machine language instructions cannot be accommodated within the 1620 but in fact this proves to be of little disadvantage since it is often convenient to inspect the results of intermediate stages. In fact the program is written in 3 sections covering the three stages described above; the output of one section providing the input of that which follows.

The arrangement of the program in three sections is shown in the flow diagrams, figures 3 to 5.

Little comment is indicated by way of explanation of figure 3 since this is a simple computer application of the processes outlined in DOODSON's paper. It is interesting to note however that the application of the hourly multipliers for each day of the record can be accomplished in precisely the time required by the card reader to accept the requisite data and to transfer the information into the input buffer. The program is therefore arranged to perform these calculations during the card reading time. The basic observational data are contained on 358 cards which are read by the card reader in less than 1 ½ minutes by which time a large proportion of the cumbersome calculations of the whole program section are already complete. A further advantage of this arrangement is that storage need only be allocated for 96 hourly elevations and this storage area is continually overwritten as the data is read.

Figure 4 indicates the general facilities and order of calculation processes in stage 2 of the analysis. This program section is in fact built around a subroutine, consisting of only 19 machine language instructions, designed to apply all the corrections necessary to a single linear combination in order to determine either the sine or cosine vector of a single constituent or of a conjugate group. The procedure is as follows. For each species vector, the same linear combinations, up to 9 in number, are used as corrective terms. It is therefore necessary to specify these once only at the commencement of the calculations for each species vector. The corrections to each linear combination in the group can then be affected by a single " branch and transmit " instruction to the above subroutine. The factors to be used in the correction process are initially stored in a large array in the order required by the program and the address of the appropriate factor to be used is correctly incremented within the subroutine. The same is true of an array of divisors which are designed to evaluate the contribution of the particular constituent to the corrected linear combination. The address in which the result is to be stored is provided to the subroutine by the initiating " branch and transmit " instruction.

Stage 3 of the analysis program is more complex and is basically in two parts. The first part, on acceptance of the first data card, examines the code number of the central day of the analysis which is included as an identifying label on all cards output from stage 2. Upon this basis alone it proceeds to compute values of V (the phase of the appropriate constituent of the equilibrium tide) together with u and f (the nodal corrective terms) for the sixty constituents involved in the analysis. The second part makes use of a subroutine to resolve the vectors into an amplitude and phase appropriate to a central time origin. The quantities V, u, f and Δ are then applied and the results rounded off so as to obtain H and g of general tidal notation. In all cases other than the handling of the first data card the program proceeds directly to part 2 after checking the validity of the card contents.

This program section offers two further facilities as follows : —

a) The results are in all cases printed but in addition it is possible to provide output on sequentially numbered cards, one per constituent, which contain H, g and σ together with the identifying central day code, and port number.

b) The use of Program Switch 4 transforms this program section from an analysis computation into a means of computing values of V, u and f which are output on cards, one per constituent, together with σ and an identifying date code. The insertion of a data deck, each card containing a day code and the index number, one, will produce a set of cards containing values of the above quantities for each card input.

A specimen set of analytical results as output by the three stages of the program is given in figure 6. These data apply to the port of Swansea, Glamorgan, U.K.; central day 1st January, 1962.

The analysis program for 29 days' observations

The greatest restriction of the above program is the need to input a continuous set of tidal observations extending over a period of 358 days.

Such a record is not always available and in order to minimize the restriction a further program has been written to analyse a 29 day period. It should be noted however that in order to fit the hourly multipliers to the data, observations from 32 days in all are required. The method is similar to that described by DOODSON [6] but not all the corrective processes described in this publication are applied. The calculations follow more closely those given by LENNON [7] although the number of tidal constituents extracted, nineteen in all, lies between the two techniques to which reference is made above. Those amendments to which DOODSON refers as additional refinements are normally quite small and have been ignored in the process. Had these processes been included, the 29 day analysis program would have been far more complex than that for twelve months' observations, and in view of the fact that the short period analysis in this context is considered as an interim procedure pending the accumulation of a more satisfactory record length, the labour was felt to be unjustified. As it exists at present the program does employ special separation processes based upon equilibrium relationships for those major constituents near together in speed which cannot otherwise be separated within a short period of data. For example K_1 is corrected for the P_1 contribution, S_2 for the T_2 and K_2 contributions, μ_2 for the $2N_2$ contribution and N_2 for ν_2. The above separation corrections also apply to constituents in higher species so that for example : —

MK_3 is corrected for MP_3
MS_4 for MK_4 and MT_4
$2MS_6$ for $2MK_6$ and $2MT_6$
MN_4 for $M\nu_4$
and $2MN_6$ for $2M\nu_6$

This procedure is well-tried as a practical means for the provision of valid constants in the absence of a long set of observations and has been in use at the Tidal Institute for many years.

A specimen program output which indicates the range of constituents extracted is given in figure 7. This set of results refers to Barrow Island, Australia; central day 15th July 1963. Before utilizing such results for prediction purposes it is normal practice to infer subsidiary constituents e.g. P_1, K_2, T_2, $2N_2$, ν_2, MK_4 and $2SM_6$, from equilibrium relationships or regional relationships where such information is available.

The complete program for the 29-day analysis involves some 2 500 instructions and is divided into two separate program sections : —

a) The first section incorporates all the processes previously described in association with sections 1 and 2 of the yearly analysis except for the application of the monthly multipliers which are not involved in a short period analysis.

b) The second section performs the processes previously assigned to the yearly analysis section 3 but for 19 constituents only, and in addition applies the individual corrective processes to K_1, S_2, N_2, μ_2, etc., mentioned earlier. The additional facilities described under yearly analysis section 3 relating to card output of H, *g*, V, *u*, *f* and σ are considered superfluous and are not available in the 29-day analysis program.

THE PREDICTION OF HOURLY ELEVATIONS OF THE TIDE

The prediction program

In comparison with the analytical problem, the programming of the prediction of hourly elevations of the tide is a less taxing matter and is concerned simply with the repetitive evaluation at hourly intervals of t of the expression : —

$$Z_0 + \sum_{c=1}^{c=C} f_c H_c \cdot \text{cosine } (\sigma_c t + V_c + u_c - g_c)$$

where Z_0 = mean level.

The interest lies in devising the most efficient performance upon a small computer of the great bulk of calculation involved and, in particular on the IBM 1620, with the use of the minimum number of significant digits in each stage of the work. The available possibilities are numerous. A library subroutine for the evaluation of each cosine can be used, the cosines can be incremented by a cosine (A + B) routine or again, a cosine table can be stored. The choice of process is critical involving a possible range of machine time from 3.5 minutes to less than 1.5 seconds per 30 constituent day.

The storage of a cosine table was selected as the most efficient process but further, it was found possible even within the capacity of a 20 K Core Store, to accommodate a table of sufficient size so as to avoid the necessity for interpolation or for quadrant adjustment. In fact a cosine table at intervals of 0.1° in arc and consisting of 4 digit items is input at the commencement of the program covering the angular range 0.0° to 89.9°. The program then extends this table to cover all four quadrants of arc. Here the value of 0.9999 is stored in preference to 1 000 for cosine 0° in order to restrict the items of this table to a uniform 4 digits' length. If the program is arranged to select the nearest tabular item appropriate to the angular argument it can be seen that the maximum error involved in a cosine is 0.00087 which, when used with a maximum constituent amplitude of say 10 feet, is tolerable.

In restricting the number of significant figures in the calculations to a minimum, an important decision is the prescription of the accuracy to which the hourly increment in constituent phase, σ, is given. The program accepts σ in degrees of arc rounded off to five decimal places so that the facility is given to prepare predictions correct to one decimal place in feet within a range of six months, plus or minus, from the time origin for which the values of V and u are defined. Since the values of u and f are slowly changing in time it is normal practice to replace these quantities by new values after a maximum period of 6 months' predictions. The restriction imposed here is therefore compatible with prediction requirements.

The obvious additional requirement of the program is to ensure that the basic calculation loop which determines the contribution of each constituent to any hourly elevation, increments the phase by σ, and also arranges the program addresses for the next calculation, is as economical in machine time as possible. This is in fact kept within the limit of 20 machine language instructions.

By outputting always upon cards but also with the opportunity to output on the line printer, output time is virtually zero. The format of card output is identical with that of analysis input.

Further details of the program flow can be ascertained by reference to figure 8.

An optimised program for hourly prediction

The program outlined above provides a satisfactory working process but does not give the most efficient prediction program for use with the IBM 1620. With a 40 K installation in mind and assuming the availability of certain special features, notably " move flag " and " indirect addressing ", a second program has therefore been written which introduces optimisation processes and in fact effects a saving in computer time in excess of 40 %. In order to achieve this end certain devices were employed, the most important of which being the choice of a new unit for angular measure.

It is interesting to note that an angular unit of $\pi/5\,000$ has two significant advantages in this connexion : —

a) The prediction process calls for successive incrementation of the constituent phase angle by the quantity σ. Using conventional angular measure it becomes necessary to check after each incrementation whether 360° has been exceeded and to effect adjustment if necessary. Using the unit $\pi/5\,000$, an angle which exceeds $2\,\pi$ involves a carry-over into a fifth integral digit. If, however, the appropriate numerical field within the computer is terminated by a flag over the most senior of four integral digits, then this carry-over is automatically lost whenever it occurs.

b) The precision required within the prediction calculations is compatible with the use of a cosine table with elements of four decimal digits spaced at intervals of $\pi/1\,250$. In this case the maximum error involved in a cosine is 0.00126. This is tolerable when considered alongside the fact that only the M2 constituent may approach in magnitude an amplitude of 10 ft while the amplitudes of all other constituents must be considerably less. The significant point here is that if the elements of the cosine table are stored sequentially as four digit items and if the constituent phases are evaluated in units of $\pi/5\,000$, then there is a simple relationship between the integral part of the constituent phase angle and the address of its cosine, provided that the former is divisible by four.

Having made this selection of angular measure, its introduction into the program requires further consideration. It is found most convenient to arrange for the program to compute initial constituent phases in a ten digit form as follows : —

$$2\bar{I}III0DDDD$$

Here $\bar{I}III$ represents the four digit integral part of the angle and is expressed in terms of $\pi/5\,000$. This integral part is a multiple of 4 and its senior digit is flagged to prohibit carry-over.

DDDD represents the four digit decimal part of the angle and is expressed in decimals of $\pi/1\,250$.

The integral and decimal parts of the angle are separated by a digit initially set at zero.

During the course of computation the constituent phases are incremented each hour by σ, similarly expressed in the form $\bar{I}III0DDDD$, and at each stage the separating digit is monitored by a " branch on digit " instruction so that when a carry-over occurs from the decimal part into this location the quantity 39 is added to the field addressed by the separating digit; thereby this digit is reset to zero and the integral part of the angle is incremented by 4. This can be programmed with a minimum use of computer time.

If the cosine table is now arranged in the core store so that the address of cosine zero is 20 000 and the other elements are stored in sequentially higher locations and again if a digit, 2, is made to precede each constituent phase as shown above, then the five digit quantity $2\bar{I}III$ is always the address of the cosine of the particular constituent phase.

Other improvements are effected in the basic program by making free use of the ample storage area available within a 40 K core store. For example the basic constituent quantities representing amplitude, phase and speed are all stored in units of 10 storage locations even though they may consist of a smaller number of digits, while the three groups of data are spaced at intervals of 1 000 locations. In this way, if the address of the amplitude of one constituent is 31010 then the address of the amplitude of the next constituent is 31020, the address of the phase of the latter is 32020 and its speed can be found in 33020. This arrangement makes for efficiency in selecting the requisite operational data for each computational process. It will be seen that accommodation is provided for 100 constituents.

After the trial of many different computational schemes, that outlined above proved to be the most suitable for economic programming on the IBM 1620. In general terms, but with obvious exceptions in detail, the scheme illustrated in figure 8 applies to this optimised program and in fact the input data and form of output are identical in both programs. It is not possible to give an exact estimate of running time since this not only depends upon the number of constituents used but also upon the number of significant digits required to define their amplitudes. However, for an average case with, say, 30 harmonic constituents and a tidal range of the order of 6 to 7 feet, the running time on a MKII installation is approximately 8.5 minutes per thirty constituent year.

The preparation of data for prediction

Apart from a parameter card which defines the location, mean level, number of constituents and time limits for a prediction run, the input data of the prediction program consists simply of a card deck, one card per constituent, containing appropriate values of H, g, V, u, f and σ. An arbitrary maximum limit has been set at 60 constituents although there is room for twice this number within the basic 20 K computer. In this connexion it is significant that the analysis program can be used to assist in the generation of the input data, thereby removing the possibility of human error in data preparation.

A prediction run is prepared in the following way : —

1) Where a yearly harmonic analysis has been performed there is already provided a card deck containing H, g, and σ together with an identifying code for 60 constituents. The significant constituents only are selected. In the absence of an analysis only H, g and σ are punched manually.
2) Yearly Analysis Program, Section 3 is used with Sense Switch 4 to generate a card deck containing V, u, f and σ together with a date code for 60 constituents. It is normal to generate V, u, f for a date in the centre of the prediction period.
3) Both the above decks are input to a Prediction Data Combination Program of a trivial nature which prepares a further card deck combining H, g, V, u, f and σ for any selection of the 60 constituents and in addition can apply a time zone correction to the values of V. The latter deck is available for input into the prediction program. Internal checks ensure that the value of σ contained on the card from the first deck coincides with that from the second deck before the appropriate card of the third deck is punched.

The advantage of this system is that a minimum of manual data preparation is required. The analysis output cards remain constant for the particular port and can be filed for future use. The V, u and f cards are constant for a particular date and are on file whenever the appropriate period of predictions is required irrespective of the port location. At the same time the third pack can be retained on file as a complete record of the input data for a particular port run.

THE CALCULATION OF RESIDUALS

The residuals obtained by comparing predicted hourly elevations with the observed quantities are assuming considerable significance in current tidal investigations firstly as a measure of the accuracy of the harmonic method and secondly as a measure of the non-harmonic contributions to the tidal elevations; storm surges, transient oscillations and the like. With the observations and also the predictions available on file in this simple card format, the extraction of residuals is a small matter for the computer.

A further simple program is arranged to accomplish this, and at the same time to check that the identifying code of the observations agrees with that of the predictions and that the cards are in sequence.

This process can be performed quite rapidly. Apart from the time taken to input the predictions, 1.5 minutes per year, the remaining operations, namely the inputting of observations and the calculations, is controlled only by the time taken by the 1622 to output cards i.e. 2.92 minutes per year.

PROGRAM EFFICIENCY

The demands upon the computer in time and capacity by these programs are set out in figure 9 but it is important to note that entries in this table are to be considered as maximum values in that they apply to a first case for each program and to an installation with minimum facilities. For example many of the programs request data for a further case upon completion so that for subsequent runs the program input time can ignored. Again the replacement of the 1620 MK I by the MK II version reduces all computation time by a factor of four. This has its greatest effect in the prediction program which is concerned almost entirely with direct computation time. The MK II does not of itself significantly improve input/output time except in the case of the typewriter which in the later installation is of the "Golf-Ball" type and consequently some 50 % faster than the conventional writer. The MK II has its greatest advantage in offering the facilities of a Disk-Pack backing store which can be used for storing the programs and thereby can effectively remove all time entries under the Program Input heading. The line printer is another ancillary device available on-line with the computer which can reduce all entries under typewriter output time to a negligible quantity. It can therefore be appreciated that the MK II with Disk-Pack and line-printer will perform a harmonic analysis of one year's tidal observations in approximately 2.5 minutes.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation of the computer facilities made available by the Unilever Research Laboratory, Port Sunlight, which included direct access to the equipment so important in the proving of long programs together with timely solace on the many occasions of distress. The author is also indebted to Mrs. E. Murrell of the Tidal Institute for her considerable assistance in the preparation of program decks, computer operation and the verification of computer output by means of desk calculators.

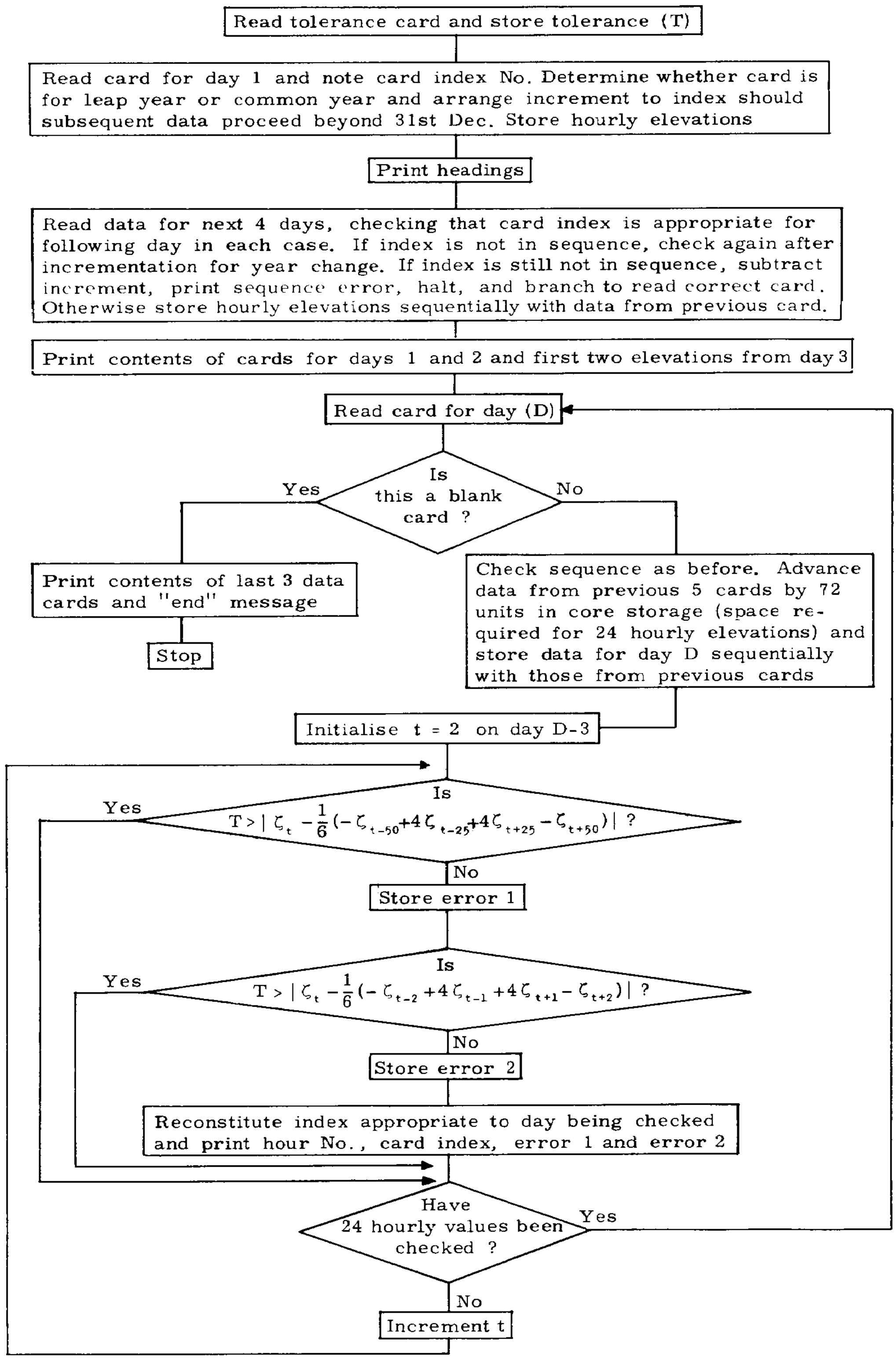

FIG. 1. — DATA VERIFICATION PROGRAM BLOCK DIAGRAM

```
Enter tolerance card

Enter data cards plus blank card

Tidal observations smoothness check
commencing with code 62233354
Tolerance = 0. 5 feet

0540500460400360330320300300320340390430480500500500520520500490490500506 2233354
0500500480440390360340330330340380420460520570580570560540520510490470466 2234354
044044
Hr Code No.   I      II
00 62236354 000. 6 000. 8
22 62281354 000. 5 000. 5
01 63005354 000. 9 001. 1
08 63061354 004. 8 005. 0
09 63061354 000. 5 003. 3
10 63061354 000. 5 000. 8
07 63092354 000. 6 001. 1
22 63109354 000. 9 000. 9
Data card out of sequence correct and press start
06 63121354 000. 9 000. 9
20 63122354 000. 9 001. 2
22 63122354 000. 5 000. 6
00 63126354 001. 1 000. 6
02 63127354 000. 8 000. 7
21 63201354 000. 8 000. 9
043037031026023020018017017017017018022028034040047054060063064063058062
045039034030025022021019019019019020021026032037043050056061064064061056
049043038035030026024022021022023023023026031037042047053058061063062058
End of smoothing process.
```

Explanation of Jabal Dhanna smoothness run

Since the test requires knowledge of ζ_{t-50} and ζ_{t+50} it is not possible to test the first and last 50 items of data. The first 50 hours are therefore printed out beneath the headings for subsequent check against the original data. Similarly the hourly values on the last 3 cards are output at the conclusion of the run.

The first "error" output questions the validity of the elevation appropriate to 0000 hrs on 24th August 1962 for this port :

Check (1) indicates an error of 0. 6 ft. Check (2) indicates an error of 0. 8 ft.

FIG. 2

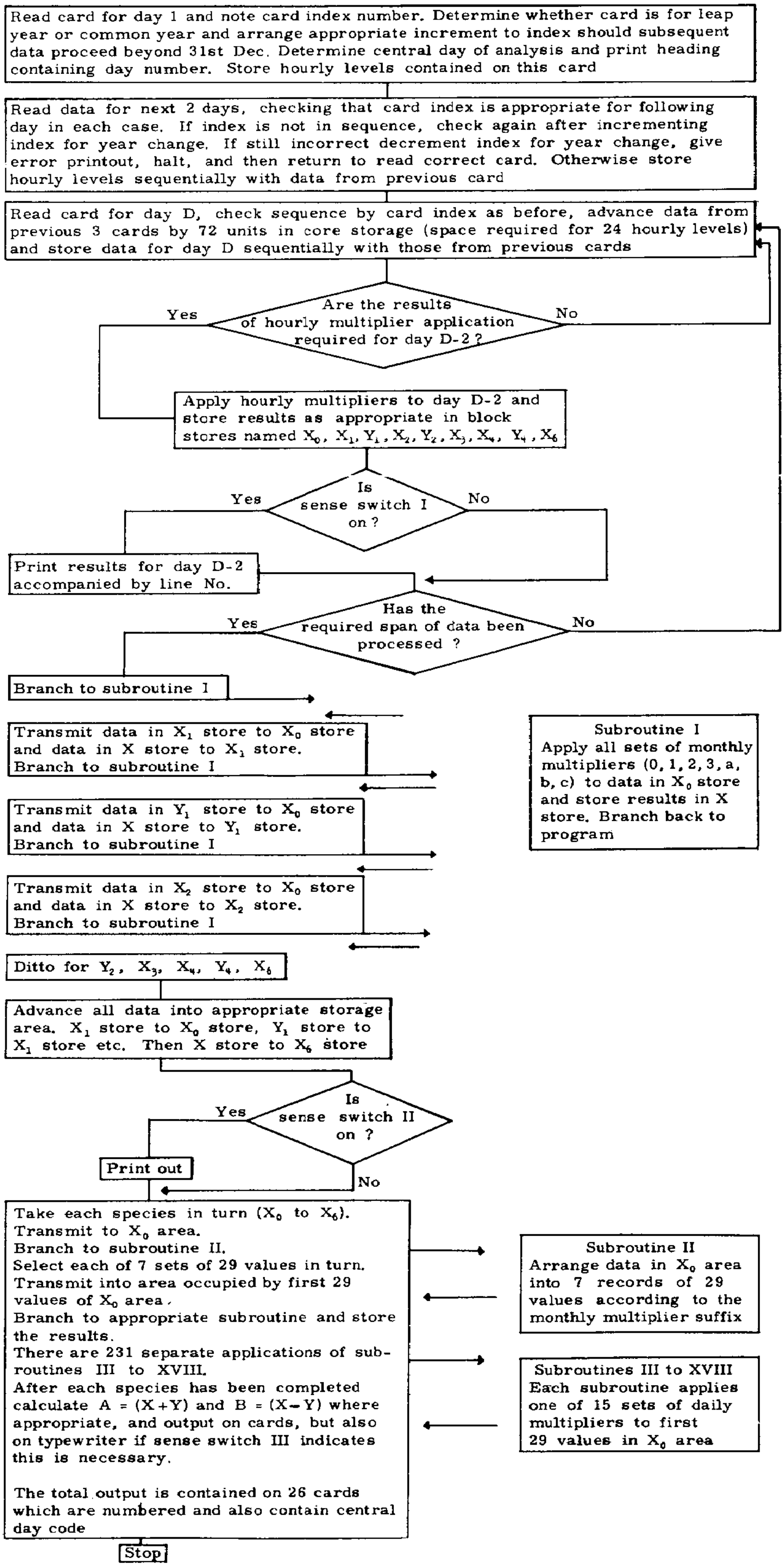

FIG. 3. — ANALYSIS STAGE I — BLOCK DIAGRAM

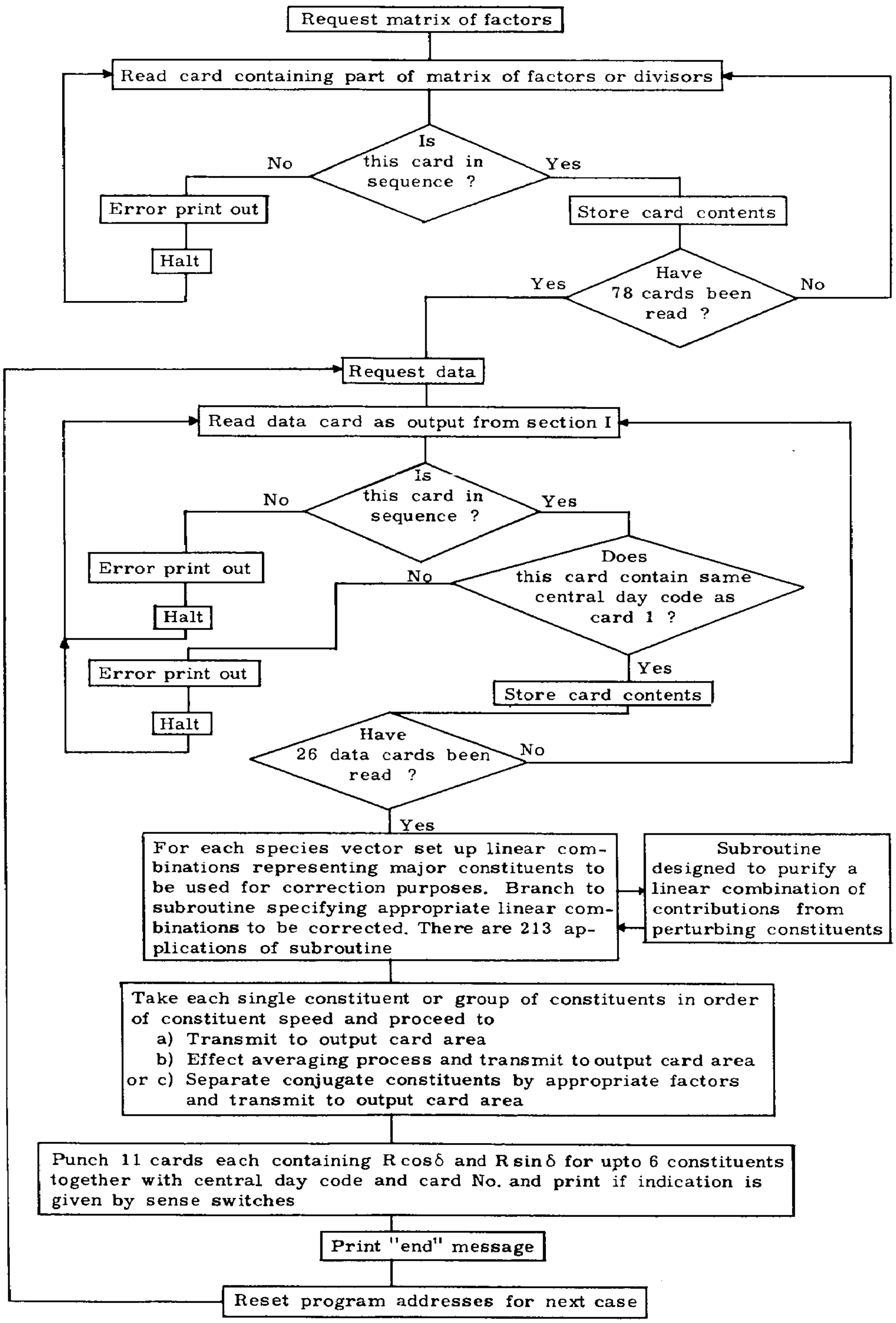

FIG. 4. — ANALYSIS STAGE II. — BLOCK DIAGRAM

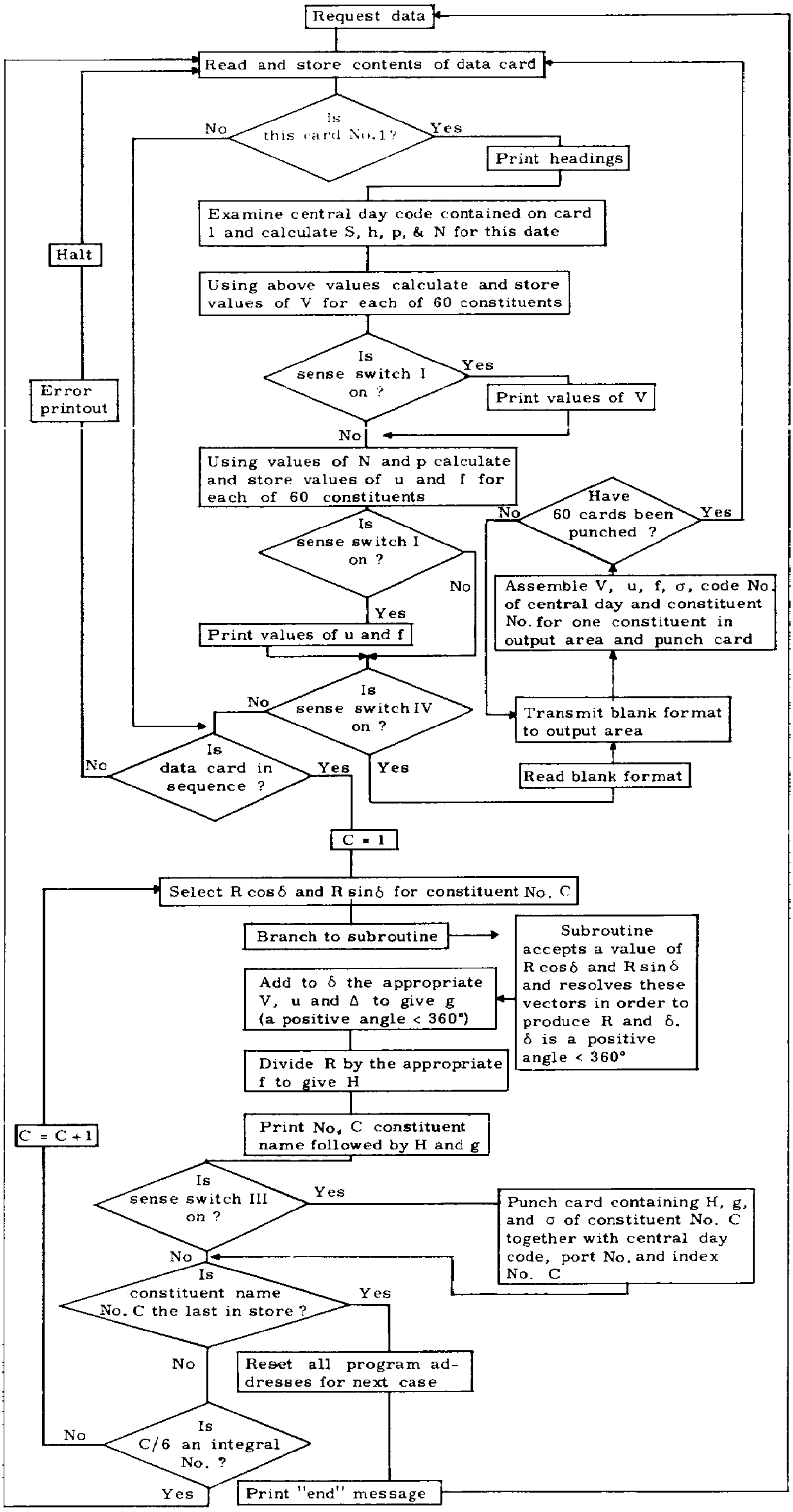

FIG. 5. — ANALYSIS STAGE III. — BLOCK DIAGRAM

Enter observation pack
Yearly analysis by Doodson
Central day code $\bar{6}$1366058
End of process

Enter factors
Enter data cards (26) from section 1

Doodsons analysis section 2
Central day code $\bar{6}$1366058
End

Load data 11 cards from section 2
Yearly analysis by Doodson
Central day code $\bar{6}$1366058
Units - H in feet* 10 to −3
- G in degrees* 10 to −2

	H	G			
SO	26149	00000			
Load blank format (4, 8 punch)					
SA	00387	22449	MKS2	00046	27295
SSA	00139	09309	LDA2	00205	15979
MM	00109	10527	L2	00609	17708
MSF	00144	04681	T2	00158	22957
MF	00119	10346	S2	03709	21995
2Q1	00039	27542	R2	00039	13952
SIG1	00037	16307	K2	01089	22008
Q1	00078	31821	MSN2	00127	03390
RHO1	00027	26386	KJ2	00079	11602
O1	00222	35678	2SM2	00133	05507
MP1	00022	30331	MO3	00021	20840
M1	00013	12031	M3	00091	14098
CHI1	00010	32927	SO3	00016	03308
PI1	00012	01843	MK3	00014	23592
P1	00080	11798	SK3	00076	22408
S1	00042	07968	MN4	00065	33933
K1	00200	12271	M4	00192	02915
PSI1	00021	05669	SN4	00014	27339
PHI1	00012	29994	MS4	00127	10989
THE1	00003	21571	MK4	00031	10935
J1	00026	28051	S4	00025	29771
SO1	00022	03935	SK4	00004	28297
OO1	00025	10575	2MN6	00060	33189
OQ2	00083	08100	M6	00123	01099
MNS2	00128	18918	MSN6	00039	05963
2N2	00344	15629	2MS6	00156	06421
MU2	00447	22070	2MK6	00040	04752
N2	01958	15461	2SM6	00021	16121
NU2	00396	14056	MSK6	00022	10380
OP2	00058	15021			
M2	10331	17318	End		

FIG. 6

Load data 4 cards from section 1
Harmonic analysis - Doodsons 29 day
Central day code $\bar{6}$3196283
Units - H in feet* 10 to −3
- G in degrees* 10 to −2

	H	G
S0	06436	00000
Q1	00074	25537
O1	00483	27065
K1	00641	29147
J1	00057	20013
MU2	00057	31229
N2	00414	27511
M2	02630	30198
L2	00092	33660
S2	01463	01112
2SM2	00075	19088
MO3	00026	35169
M3	00029	08009
MK3	00000	35688
MN4	00048	23690
M4	00132	24575
MS4	00153	30011
2MN6	00039	08104
M6	00026	12215
2MS6	00098	17963

Corrected
MU2 00058 26398

FIG. 7

Program	Storage capacity required	Time on basis of IBM 1620 Mk I with minimum facilities (Card read-punch and typewriter only)				
					Output	
		Program input	Data input	Computation	Card	Typewriter
Metric to feet conversion program	2 870	5 secs	negligible	negligible	2 min 55 secs per year	-
Data Verification	6 570	19 secs	1 min 30 secs per year	negligible	negligible	
Yearly Analysis I " " II " " III	31 403 19 905 21 676	1 min 35 secs 1 min 13 secs 1 min 42 secs	1 min 35 secs 6 secs negligible	1 min 5 secs 24 secs Approx 1 min	negligible negligible negligible	- - Approx 3 mins
Generation of V, u & f	21 676	1 min 42 secs	negligible	Approx 8 secs	29 secs	-
29 Day Analysis I " " " II	21 864 19 954	1 min 25 secs 50 secs	8 s[illegible] negligible	Approx 28 secs Approx 30 secs	negligible -	- Approx 30 secs
Hourly Height Predictions	18 652	27 secs	15 secs max	one hour per 30 constituent year	negligible	-
Optimised Hourly Height Predictions	34 000	20 secs	15 secs max	34 mins per 30 constituent year	negligible	-
Prediction Data Combination	13 770	11 secs	15 secs max	zero	29 secs	-
Extraction of Residuals	10 164	9 secs	1 min 30 secs per year	zero	2 min 55 secs per year	-

Fig. 9

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