ASTROFIX BY ALTITUDES OF TWO PAIRS OF STARS ONE NEAR MERIDIAN AND ANOTHER NEAR ELONGATION

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ABSTRACT

With a view to improving upon the existing methods of simultaneous determination of latitude and longitude by equal or arbitrary altitudes and of separate determination of latitude by meridian and circum-meridian altitudes of stars, an attempt has been made in this paper to introduce an alternative method of astrofix from observations of only two pairs of stars — one for latitude near meridian transit and the other for longitude (time) near elongation. This method is claimed to be not only simpler and quicker but also more completely free from the usual errors of both vertical collimation and constant of atmospheric refraction.

Apart from its general use in geodetic surveys, the method is also of considerable importance in topographic and hydrographic surveys, and in navigation.

INTRODUCTION

By astrofix is meant the determination of coordinates of points on the earth's surface by astronomical methods. The precision of astrofix is however dependent on the quality of the angular work and the efficiency of the method of observation actually employed, and is accordingly classified into the following catagories :

(1) First-order astrofix, giving a standard error for the result of $0''.15$ in latitude and $0^s.015$ in longitude, obtained from observations on a single night. This is needed for Laplace points in geodetic surveys.

(2) Second-order astrofix, giving a standard error for the result of $1''.0$ in latitude and $0^s.15$ in longitude or less. This is mainly used for astro-geodetic deflections in geodetic surveys but has ample applications in topographic as well as hydrographic surveys.

(3) Third-order astrofix, giving an error for the result of 10″ in latitude and 1^s in longitude or less. This can be profitably utilised in navigation as well as in explorative surveys.

There are however various methods of determination of astronomical latitudes and longitudes (time) from observations of altitudes of stars. Among these, the more important and now in common use are :

(1) Latitude : The method most generally applied is Talcott, using either a transit or a theodolite provided with a Talcott level. The main drawback of this otherwise theoretically excellent method is that one has often to wait long and to choose stars whose declinations are of only moderate accuracy. As regards the method of circum-meridian altitudes, the computations involved are less simple than in the case of meridian transits. Moreover the results are likely to be affected by the uncertainty in the calculated value of the constant of atmospheric refraction.

(2) Longitude : The usual method is the observation of equal or arbitrary prime vertical altitudes with either a transit or a theodolite fitted with an impersonal micrometer or a Hunter shutter. The main difficulty with the method is that it often keeps the observer waiting indefinitely for completion of the observational programme, due to stars of observation not being always available at all latitudes.

(3) Longitude simultaneous with latitude : The method in common use is the Gauss method. It comprises observations with a prismatic astrolabe of stars of equal altitude. The method of arbitrary altitudes is also gaining ground because of its flexibility. But all the same, the methods have many practical difficulties. As remarked by ROELOFS, "A peculiarity of both methods, particularly the Gauss method, is that the observations pass off so easily and smoothly that many observers are inclined to make an exorbitantly large number of observations for safety's sake and realising that they cannot anyhow go to sleep. The computer is thus overwhelmed by this super-abundant material. The obvious remedy is to limit the number of observations to a minimum by a careful pre-selection of stars. It should be added that this does not mean a simple regular distribution of stars in all directions ". Among other disadvantages are long computations, a troublesome programme and loss of accuracy.

An attempt has therefore been made in the present paper to evolve an alternative method of simple and rapid astrofix which can be practically free from the various defects enumerated above, and thus prove of real value to the surveying profession in general.

LATITUDE

Selection of stars : A star-pair, of magnitudes varying generally from 2.0 to 6.0 depending on the order of accuracy needed, one situated to the north and the other to the south but both within about 15 minutes of their time of meridian transit, must be selected for observation from the fourth fundamental star catalogue FK4. In addition, the altitudes of the star-pair should preferably be above 30° and as nearly equal to each other as conveniently possible.

Observational programme : It is always necessary to have a carefully made programme, lasting for about an hour in the case of first-order determinations, about half an hour in the case of second-order determinations and about a quarter of an hour in the case of third-order determinations, showing for the selected star-pair names, aspects, altitudes (computed from the approximate relations : $h = \varphi \pm |90° - \delta|$ for north stars and $h = 90° - \varphi + \delta$ for south stars), and times (L.S.T.) of passages corresponding to the various azimuth positions of the star-pair (reduced from the relations $a'' = -15t^s \cdot \cos\delta \cdot \sec h$ for north stars and $180° - a'' = -15t^s \cdot \cos\delta \cdot \sec h$ for south stars).

Observational equipment : The astronomical theodolite Wild T4 with accessories, or any other theodolite of similar precision, equipped with a Talcott level, a good chronometer and a stop watch are all that are generally required for the first-order determinations. The Geodetic Tavistock and Wild T3 theodolites are usually used in the second-order determinations. In the case of third-order determinations, a Wild T2 or a Tavistock theodolite should be good enough.

Observational procedure : Before commencing actual observations, the line of horizontal collimation of the theodolite requires to be set in the meridian, correct to a quarter degree or so, knowing only the approximate value of the chronometer error and the chronometer time of meridian transit of a known circumpolar star. In the case where the chronometer error is abnormally large, it can be easily improved by including also a known high altitude meridian star. The routine of observation is that the theodolite is first set at the calculated altitude and azimuth readings of the stars of observation, as per programme, and then the required intersections of stars are made in quick succession with the help of the horizontal wire of the theodolite, noting the corresponding vertical circle readings side by side. In the case of Wild T4 the vertical circle of the theodolite requires to be clamped at the desired vertical settings and then, after carefully noting the respective vertical circle readings, the actual intersections of star-positions are made at each setting in quick succession with the help of the moving micrometer only. The vertical bubble readings are also to be carefully taken immediately after each intersection. The effect of any error in the vertical collimation being completely eliminated in the final results, the observations can be taken with advantage on one face of the theodolite only. For recording instants of intersections use can be made of only the stop watch and the chronometer provided. As regards the series of altitudes measured, these need not be symmetrical with respect to the meridian, but the number of intersections of star-positions for one member, either on the east or the west side of the meridian, should be about the same on the corresponding side of the meridian for the other member of the star-pair.

Computation : Let h be the altitude of a star of declination δ, corresponding to the hour angle t when the star is close to the meridian, and h_0 the altitude when the star is on the meridian.

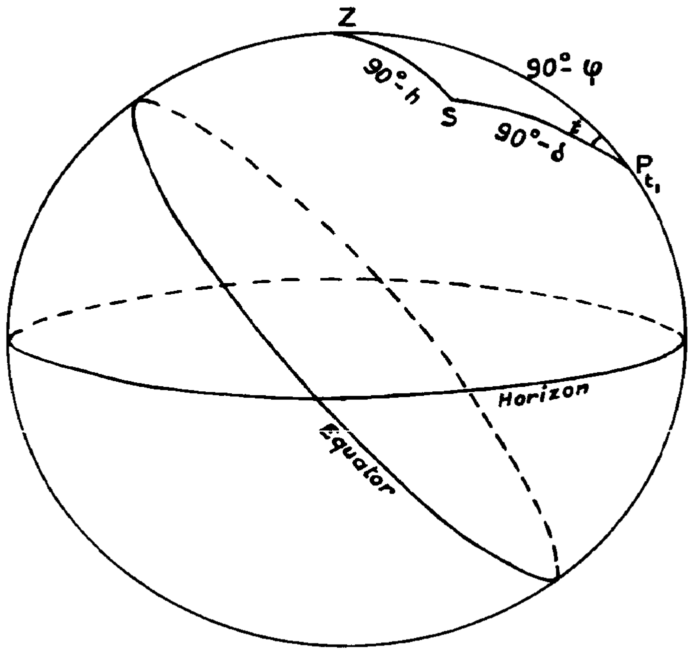

FIG. 1

Then on referring to figure 1 we have :

$\sin h = \sin \varphi \,.\, \sin \delta + \cos \varphi \,.\, \cos \delta \,.\, \cos t$ (1)

By making use of the Maclaurin formula and following DOOLITTLE's notations, the relation (1) can be reduced to the following forms :

$\varphi = h \mp (90° - \delta) + A \cdot m - B \cdot n + C \cdot o$ (2)

for upper (lower) transit of north stars

and

$\varphi = \delta + (90° - h) - A \cdot m + B \cdot n - C \cdot o$ (3)

for south stars

where

$$A = \cos \varphi \cdot \cos \delta \cdot \sec h_0$$

$$m = 2 \sin^2 \frac{t}{2} \cdot \operatorname{cosec} 1''$$

$$B = A^2 \cdot \tan h_0$$

$$n = 2 \sin^4 \frac{t}{2} \cdot \operatorname{cosec} 1''$$

$$C = A^3 \cdot \frac{2}{3} \cdot (1 + 3 \tan^2 h_0)$$

$$o = 2 \sin^6 \frac{t}{2} \cdot \operatorname{cosec} 1''$$

or after further simplification,

$\varphi = h \mp (90° - \delta) + A \cdot m - D + E$ (4)

for upper (lower) transit of north stars and

$$\varphi = \delta + (90° - h) - A \cdot m + D - E \tag{5}$$

for south stars

where

$A = \cos \varphi \cdot \cos \delta \cdot \sec h_0$

which is a constant for the particular star of observation;

$m = 2 \sin^2 \dfrac{t}{2} \cdot \operatorname{cosec} 1''$

which is a tabular quantity directly obtainable from table 1;

$D = 2.424 \cdot \left(\dfrac{A \cdot m}{1000}\right)^2 \cdot \tan h_0$

which can be directly read from chart 1, without requiring actual computations, much more easily and quickly than is normally possible in evaluating B and n separately, and

$$E = 0.0118 \left(\frac{A \cdot m}{1000}\right)^3 \cdot \tan^2 h_0 ,$$

which is mostly negligible, but which can be directly read from chart 2, without recourse to actual computations, much more easily and quickly than is normally possible in evaluating C and o separately.

Since in relations (4) and (5) A is a function of φ, which is an unknown quantity, and h_0, which is again a function of φ, the evaluation of A is not possible without assuming a value φ_m for φ, and in that case the computed value φ_c of φ, as derived from relation (4) and (5) on substituting φ_m for φ and $\varphi_m \pm (90° - \delta)$ for h_0 in the case of north stars and $\delta + (90° - \varphi_m)$ for h_0 in the case of south stars, will be in error by an amount equal to $\varphi - \varphi_c = \Delta\varphi_c$ due to :

(1) Error $\Delta\varphi_m$ in the assumed value φ_m of φ, where $\varphi - \varphi_m = \Delta\varphi_m$, a constant independent of the star observation.

(2) Error Δh in the observed altitudes of stars, on account of the assumed values of both the vertical collimation and the constant of atmospheric refraction. The error is thus practically constant during observations, unless the individual altitudes of the stars of observation differ considerably from each other due to faulty selection of stars.

(3) Error Δt in the assumed hour angle t, on account of the value of longitude λ of the place of observation being an unknown quantity.

But the above errors, when expressed in the form of differential formulae, take the following form :

$$\Delta\varphi_c = \sec a_N \cdot \Delta h - \tan a_N \cdot \cos \varphi_m \cdot \Delta t \tag{6}$$

for north stars

and

$$\Delta\varphi_c = - \sec a_S \cdot \Delta h + \tan a_S \cdot \cos \varphi_m \cdot \Delta t \tag{7}$$

for south stars

where a_N and a_S denote small azimuthal angles as approximately derived from the relations :

$a_N'' = -15 \cdot t^s \cdot \cos \delta \cdot \sec h_0$ and $180° - a_S'' = -15 \cdot t^s \cdot \cos \delta \cdot \sec h_0$

and suffixes N and S refer to north and south stars respectively.

TABLE I
for
values of $m : 2 \sin^2 \frac{t}{2} \cdot \operatorname{cosec} 1''$

Hour angle t	0^m	1^m	2^m	3^m	4^m
0^s	0″00	1″96	7″85	17″67	31″42
1	0.00	2.03	7.98	17.87	31.68
2	0.00	2.10	8.12	18.07	31.94
3	0.00	2.16	8.25	18.27	32.20
4	0.01	2.23	8.39	18.47	32.47
5	0.01	2.31	8.52	18.67	32.74
6	0.02	2.38	8.66	18.87	33.01
7	0.02	2.45	8.80	19.07	33.27
8	0.03	2.52	8.94	19.28	33.54
9	0.04	2.60	9.08	19.48	33.81
10	0.05	2.67	9.22	19.69	34.09
11	0.06	2.75	9.36	19.90	34.36
12	0.08	2.83	9.50	20.11	34.64
13	0.09	2.91	9.64	20.32	34.91
14	0.11	2.99	9.79	20.63	35.19
15	0.12	3.07	9.94	20.74	35.46
16	0.14	3.15	10.09	20.95	35.74
17	0.16	3.23	10.24	21.16	36.02
18	0.18	3.32	10.39	21.38	36.30
19	0.20	3.40	10.54	21.60	36.58
20	0.22	3.49	10.69	21.82	36.87
21	0.24	3.58	10.84	22.03	37.15
22	0.26	3.67	11.00	22.25	37.44
23	0.28	3.76	11.15	22.47	37.72
24	0.31	3.85	11.31	22.70	38.01
25	0.34	3.94	11.47	22.92	38.30
26	0.37	4.03	11.63	23.14	38.59
27	0.40	4.12	11.79	23.37	38.88
28	0.43	4.22	11.95	23.60	39.17
29	0.46	4.32	12.11	23.82	39.46
30	0.49	4.42	12.27	24.05	39.76
31	0.52	4.52	12.43	24.28	40.05
32	0.56	4.62	12.60	24.51	40.35
33	0.59	4.72	12.76	24.74	40.65
34	0.63	4.82	12.93	24.98	40.95
35	0.67	4.92	13.10	25.21	41.25
36	0.71	5.03	13.27	25.45	41.55
37	0.75	5.13	13.44	25.68	41.85
38	0.79	5.24	13.62	25.92	42.15
39	0.83	5.34	13.79	26.16	42.45
40	0.87	5.45	13.96	26.40	42.76
41	0.91	5.56	14.13	26.64	43.06
42	0.96	5.67	14.31	26.88	43.37
43	1.01	5.78	14.49	27.12	43.68
44	1.06	5.90	14.67	27.37	43.99

Hour angle t	0^m	1^m	2^m	3^m	4^m
45	1.10	6.01	14.85	27.61	44.30
46	1.15	6.13	15.03	27.86	44.61
47	1.20	6.24	15.21	28.10	44.92
48	1.26	6.36	15.39	28.35	45.24
49	1.31	6.48	15.57	28.60	45.55
50	1.36	6.60	15.76	28.85	45.87
51	1.42	6.72	15.95	29.10	46.18
52	1.48	6.84	16.14	29.36	46.50
53	1.53	6.96	16.32	29.61	46.82
54	1.59	7.09	16.51	29.86	47.14
55	1.65	7.21	16.70	30.12	47.46
56	1.71	7.34	16.89	30.38	47.79
57	1.77	7.46	17.08	30.64	48.11
58	1.83	7.60	17.28	30.90	48.43
59	1.89	7.72	17.47	31.16	48.76
60	1.96	7.85	17.67	31.42	49.09

Diff. per 1^s	$0^s.1$	$0^s.2$	$0^s.3$	$0^s.4$	$0^s.5$	$0^s.6$	$0^s.7$	$0^s.8$	$0^s.9$
0".02	0".00	0".00	0".01	0".01	0".01	0".01	0".01	0".02	0".02
0.04	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.04
0.06	0.01	0.01	0.02	0.02	0.03	0.04	0.04	0.05	0.05
0.08	0.01	0.02	0.02	0.03	0.04	0.05	0.06	0.06	0.07
0.10	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.12	0.01	0.02	0.04	0.05	0.06	0.07	0.08	0.10	0.11
0.14	0.01	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.13
0.16	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.13	0.14
0.18	0.02	0.04	0.05	0.07	0.09	0.11	0.13	0.14	0.16
0.20	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
0.22	0.02	0.04	0.07	0.09	0.11	0.13	0.15	0.18	0.20
0.24	0.02	0.05	0.07	0.10	0.12	0.14	0.17	0.19	0.22
0.26	0.03	0.05	0.08	0.10	0.13	0.16	0.18	0.21	0.23
0.28	0.03	0.06	0.08	0.11	0.14	0.17	0.20	0.22	0.25
0.30	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27
0.32	0.03	0.06	0.10	0.13	0.16	0.19	0.22	0.26	0.29

Hour angle t	5^m	6^m	7^m	8^m	9^m
0^s	49".09	70".68	96".20	125".65	159".02
1	49.41	71.07	96.66	126.17	159.61
2	49.74	71.47	97.12	126.70	160.20
3	50.07	71.86	97.58	127.22	160.80
4	50.40	72.26	98.04	127.75	161.39
5	50.73	72.66	98.50	128.28	161.98
6	51.07	73.06	98.97	128.81	162.58
7	51.40	73.46	99.43	129.34	163.17
8	51.74	73.86	99.90	129.87	163.77
9	52.07	74.26	100.37	130.40	164.37
10	52.41	74.66	100.84	130.94	164.97
11	52.75	75.06	101.31	131.47	165.57
12	53.09	75.47	101.78	132.01	166.17
13	53.43	75.88	102.25	132.55	166.77
14	53.77	76.29	102.72	133.09	167.37
15	54.11	76.69	103.20	133.63	167.97
16	54.46	77.10	103.67	134.17	168.58
17	54.80	77.51	104.15	134.71	169.19
18	55.15	77.93	104.63	135.25	169.80
19	55.50	78.34	105.10	135.80	170.41
20	55.84	78.75	105.58	136.34	171.02
21	56.19	79.16	106.06	136.88	171.63
22	56.55	79.58	106.55	137.43	172.24
23	56.90	80.00	107.03	137.98	172.85
24	57.25	80.42	107.51	138.53	173.47
25	57.60	80.84	107.99	139.08	174.08
26	57.96	81.26	108.48	139.63	174.70
27	58.32	81.68	108.97	140.18	175.32
28	58.68	82.10	109.46	140.74	175.94
29	59.03	82.52	109.95	141.29	176.56
30	59.40	82.95	110.44	141.85	177.18
31	59.75	83.38	110.93	142.40	177.80
32	60.11	83.81	111.43	142.96	178.43
33	60.47	84.23	111.92	143.52	179.05
34	60.84	84.66	112.41	144.08	179.68
35	61.20	85.09	112.90	144.64	180.30
36	61.57	85.52	113.40	145.20	180.93
37	61.94	85.95	113.90	145.76	181.56
38	62.31	86.39	114.40	146.33	182.19
39	62.68	86.82	114.90	146.89	182.82
40	63.05	87.26	115.40	147.46	183.46
41	63.42	87.70	115.90	148.03	184.09
42	63.79	88.14	116.40	148.60	184.72
43	64.16	88.57	116.90	149.17	185.35
44	64.54	89.01	117.41	149.74	185.99
45	64.91	89.45	117.92	150.31	186.63
46	65.29	89.89	118.43	150.88	187.27
47	65.67	90.33	118.94	151.45	187.91
48	66.05	90.78	119.45	152.03	188.55

Hour angle t	5^m	6^m	7^m	8^m	9^m
49	66.43	91.23	119.96	152.61	189.19
50	66.81	91.68	120.47	153.19	189.83
51	67.19	92.12	120.98	153.77	190.47
52	67.58	92.57	121.49	154.35	191.12
53	67.96	93.02	122.01	154.93	191.76
54	68.35	93.47	122.53	155.51	192.41
55	68.73	93.92	123.05	156.09	193.06
56	69.12	94.38	123.57	156.67	193.71
57	69.51	94.83	124.09	157.25	194.36
58	69.90	95.29	124.61	157.84	195.01
59	70.29	95.74	125.13	158.43	195.66
60	70.68	96.20	125.65	159.02	196.32

Diff. per 1^s	$0^s.1$	$0^s.2$	$0^s.3$	$0^s.4$	$0^s.5$	$0^s.6$	$0^s.7$	$0^s.8$	$0^s.9$
0".32	0".03	0".06	0".10	0".13	0".16	0".19	0".22	0".26	0".29
0.34	0.03	0.07	0.10	0.14	0.17	0.20	0.24	0.27	0.31
0.36	0.04	0.07	0.11	0.14	0.18	0.22	0.25	0.29	0.32
0.38	0.04	0.08	0.11	0.15	0.19	0.23	0.27	0.30	0.34
0.40	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36
0.42	0.04	0.08	0.13	0.17	0.21	0.25	0.29	0.34	0.38
0.44	0.04	0.09	0.13	0.18	0.22	0.26	0.31	0.35	0.40
0.46	0.05	0.09	0.14	0.18	0.23	0.28	0.32	0.37	0.41
0.48	0.05	0.10	0.14	0.19	0.24	0.29	0.34	0.38	0.43
0.50	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
0.52	0.05	0.10	0.16	0.21	0.26	0.31	0.36	0.42	0.47
0.54	0.05	0.11	0.16	0.22	0.27	0.32	0.38	0.43	0.49
0.56	0.06	0.11	0.17	0.22	0.28	0.34	0.39	0.45	0.50
0.58	0.06	0.12	0.17	0.23	0.29	0.35	0.41	0.46	0.52
0.60	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54
0.62	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.50	0.56
0.64	0.06	0.13	0.19	0.26	0.32	0.38	0.45	0.51	0.58

Hour angle t	10ᵐ	11ᵐ	12ᵐ	13ᵐ	14ᵐ
0ˢ	196".32	237".54	282".68	331".74	384".74
1	196.97	238.26	283.47	332.59	385.65
2	197.63	238.98	284.26	333.44	386.56
3	198.28	239.70	285.04	334.29	387.48
4	198.94	240.42	285.83	335.15	388.40
5	199.60	241.14	286.62	336.00	389.32
6	200.26	241.87	287.41	336.86	390.24
7	200.92	242.60	288.20	337.72	391.16
8	201.59	243.33	289.00	338.58	392.09
9	202.25	244.06	289.79	339.44	393.01
10	202.92	244.79	290.58	340.30	393.94
11	203.58	245.52	291.38	341.16	394.86
12	204.25	246.25	292.18	342.02	395.79
13	204.92	246.98	292.98	342.88	396.72
14	205.59	247.72	293.78	343.75	397.65
15	206.26	248.45	294.58	344.62	398.58
16	206.93	249.19	295.38	345.49	399.52
17	207.60	249.93	296.18	346.36	400.45
18	208.27	250.67	296.99	347.23	401.38
19	208.94	251.41	297.79	348.10	402.32
20	209.62	252.15	298.60	348.97	403.26
21	210.30	252.89	299.40	349.84	404.20
22	210.98	253.63	300.21	350.71	405.14
23	211.66	254.37	301.02	351.58	406.08
24	212.34	255.12	301.83	352.46	407.02
25	213.02	255.87	302.64	353.34	407.96
26	213.70	256.62	303.46	354.22	408.90
27	214.38	257.37	304.27	355.10	409.84
28	215.07	258.12	305.09	355.98	410.79
29	215.75	258.87	305.90	356.86	411.73
30	216.44	259.62	306.72	357.74	412.68
31	217.12	260.37	307.54	358.62	413.63
32	217.81	261.12	308.36	359.51	414.59
33	218.50	261.88	309.18	360.39	415.54
34	219.19	262.64	310.00	361.28	416.49
35	219.88	263.39	310.82	362.17	417.44
36	220.58	264.15	311.65	363.07	418.40
37	221.27	264.91	312.47	363.96	419.35
38	221.97	265.68	313.30	364.85	420.31
39	222.66	266.44	314.12	365.75	421.27
40	223.36	267.20	314.95	366.64	422.23
41	224.06	267.96	315.78	367.53	423.19
42	224.76	268.73	316.61	368.42	424.15
43	225.46	269.49	317.44	369.31	425.11
44	226.16	270.26	318.27	370.21	426.07
45	226.86	271.02	319.10	371.11	427.04
46	227.57	271.79	319.94	372.01	428.01
47	228.27	272.56	320.78	372.91	428.97
48	228.98	273.34	321.62	373.82	429.93

Iour angle t	10^m	11^m	12^m	13^m	14^m
49	229.68	274.11	322.45	374.72	430.90
50	230.39	274.88	323.29	375.62	431.87
51	231.10	275.65	324.13	376.52	432.84
52	231.81	276.43	324.97	377.43	433.82
53	232.52	277.20	325.81	378.34	434.79
54	233.24	277.98	326.66	379.26	435.76
55	233.95	278.76	327.50	380.17	436.73
56	234.67	279.55	328.35	381.08	437.71
57	235.38	280.33	329.19	381.99	438.69
58	236.10	281.12	330.04	382.90	439.67
59	236.82	281.90	330.89	383.82	440.65
60	237.54	282.68	331.74	384.74	441.63

)iff. per 1^s	$0^s.1$	$0^s.2$	$0^s.3$	$0^s.4$	$0^s.5$	$0^s.6$	$0^s.7$	$0^s.8$	$0^s.9$
$0''.64$	$0''.06$	$0''.13$	$0''.19$	$0''.26$	$0''.32$	$0''.38$	$0''.45$	$0''.51$	$0''.58$
0.66	0.07	0.13	0.20	0.26	0.33	0.40	0.46	0.53	0.59
0.68	0.07	0.14	0.20	0.27	0.34	0.41	0.48	0.54	0.61
0.70	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63
0.72	0.07	0.14	0.22	0.29	0.36	0.43	0.50	0.58	0.65
0.74	0.07	0.15	0.22	0.30	0.37	0.44	0.52	0.59	0.67
0.76	0.08	0.15	0.23	0.30	0.38	0.46	0.53	0.61	0.68
0.78	0.08	0.16	0.23	0.31	0.39	0.47	0.55	0.62	0.70
0.80	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72
0.82	0.08	0.16	0.25	0.33	0.41	0.49	0.57	0.66	0.74
0.84	0.08	0.17	0.25	0.34	0.42	0.50	0.59	0.67	0.76
0.86	0.09	0.17	0.26	0.34	0.43	0.52	0.60	0.69	0.77
0.88	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.70	0.79
0.90	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81
0.92	0.09	0.18	0.28	0.37	0.46	0.55	0.64	0.74	0.83
0.94	0.09	0.19	0.28	0.38	0.47	0.56	0.66	0.75	0.85
0.96	0.10	0.19	0.29	0.38	0.48	0.58	0.67	0.77	0.86
0.98	0.10	0.20	0.29	0.39	0.49	0.59	0.69	0.78	0.88

Hour angle t	15^m	16^m	17^m	18^m	19^m
0^s	441".63	502".46	567".19	635".85	708".42
1	442.62	503.50	568.30	637.02	709.66
2	443.60	504.55	569.42	638.20	710.90
3	444.58	505.60	570.53	639.38	712.15
4	445.56	506.65	571.65	640.56	713.39
5	446.55	507.70	572.76	641.74	714.64
6	447.54	508.76	573.88	642.93	715.89
7	448.53	509.81	575.00	644.11	717.14
8	449.51	510.86	576.12	645.30	718.39
9	450.50	511.92	577.24	646.48	719.64
10	451.50	512.98	578.36	647.67	720.89
11	452.49	514.03	579.48	648.86	722.15
12	453.48	515.09	580.61	650.05	723.40
13	454.48	516.15	581.73	651.24	724.66
14	455.47	517.21	582.86	652.43	725.91
15	456.47	518.27	583.99	653.62	727.17
16	457.47	519.34	585.12	654.80	728.43
17	458.47	520.40	586.24	656.01	729.69
18	459.47	521.47	587.38	657.21	730.95
19	460.47	522.53	588.51	658.40	732.21
20	461.47	523.60	589.64	659.60	733.48
21	462.48	524.67	590.77	660.80	734.74
22	463.48	525.74	591.91	662.00	736.01
23	464.48	526.81	593.05	663.20	737.27
24	465.49	527.89	594.18	664.40	738.54
25	466.50	528.96	595.32	665.61	739.81
26	467.51	530.03	596.46	666.81	741.08
27	468.52	531.11	597.60	668.02	742.35
28	469.53	532.18	598.74	669.22	743.62
29	470.54	533.26	599.88	670.43	744.89
30	471.55	534.33	601.03	671.64	746.17
31	472.57	535.41	602.17	672.85	747.44
32	473.58	536.50	603.32	674.06	748.72
33	474.60	537.58	604.47	675.28	750.00
34	475.62	538.67	605.61	676.49	751.28
35	476.64	539.75	606.76	677.70	752.56
36	477.65	540.83	607.91	678.92	753.84
37	478.67	541.91	609.06	680.13	755.12
38	479.70	543.00	610.22	681.35	756.40
39	480.72	544.09	611.37	682.57	757.68
40	481.74	545.18	612.53	683.79	758.97
41	482.77	546.27	613.68	685.01	760.26
42	483.79	547.36	614.84	686.23	761.54
43	484.82	548.45	616.00	687.46	762.83
44	485.85	549.55	617.15	688.68	764.12
45	486.88	550.64	618.31	689.91	765.41
46	487.91	551.73	619.48	691.13	766.7[illegible]
47	488.94	552.83	620.64	692.36	768.0[illegible]
48	489.97	553.93	621.80	693.59	769.2[illegible]

Hour angle t	15^m	16^m	17^m	18^m	19^m
49	491.01	555.03	622.97	694.82	770.58
50	492.05	556.13	624.13	696.05	771.88
51	493.08	557.24	625.30	697.28	773.18
52	494.12	558.34	626.47	698.51	774.48
53	495.15	559.44	627.63	699.75	775.78
54	496.19	560.55	628.80	700.98	777.08
55	497.23	561.65	629.98	702.22	778.38
56	498.28	562.76	631.15	703.46	779.68
57	499.32	563.87	632.32	704.69	780.98
58	500.37	564.98	633.49	705.93	782.29
59	501.41	566.08	634.67	707.17	783.59
60	502.46	567.19	635.85	708.42	784.90

Diff. per 1^s	$0^s.1$	$0^s.2$	$0^s.3$	$0^s.4$	$0^s.5$	$0^s.6$	$0^s.7$	$0^s.8$	$0^s.9$
$0''.98$	$0''.10$	$0''.20$	$0''.29$	$0''.39$	$0''.49$	$0''.59$	$0''.69$	$0''.78$	$0''.88$
1.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
1.02	0.10	0.20	0.31	0.41	0.51	0.61	0.71	0.82	0.92
1.04	0.10	0.21	0.31	0.42	0.52	0.62	0.73	0.83	0.94
1.06	0.11	0.21	0.32	0.42	0.53	0.64	0.74	0.85	0.95
1.08	0.11	0.22	0.32	0.43	0.54	0.65	0.76	0.86	0.97
1.10	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99
1.12	0.11	0.22	0.34	0.45	0.56	0.67	0.78	0.90	1.01
1.14	0.11	0.23	0.34	0.46	0.57	0.68	0.80	0.91	1.03
1.16	0.12	0.23	0.35	0.46	0.58	0.70	0.81	0.93	1.04
1.18	0.12	0.24	0.35	0.47	0.59	0.71	0.83	0.94	1.06
1.20	0.12	0.24	0.36	0.48	0.60	0.72	0.84	0.96	1.08
1.22	0.12	0.24	0.37	0.49	0.61	0.73	0.85	0.98	1.10
1.24	0.12	0.25	0.37	0.50	0.62	0.74	0.87	0.99	1.12
1.26	0.13	0.25	0.38	0.50	0.63	0.76	0.88	1.01	1.13
1.28	0.13	0.26	0.38	0.51	0.64	0.77	0.90	1.02	1.15
1.30	0.13	0.26	0.39	0.52	0.65	0.78	0.91	1.04	1.17
1.32	0.13	0.26	0.40	0.53	0.66	0.79	0.92	1.06	1.19
1.34	0.13	0.27	0.40	0.54	0.67	0.80	0.94	1.07	1.21

But since $\varphi - \varphi_c = (\varphi - \varphi_m) + (\varphi_m - \varphi_c)$, we have :

$$\Delta\varphi_c = \Delta\varphi_m + \Delta\varphi \tag{8}$$

where $\Delta\varphi_m$ stands for the constant error in the assumed value φ_m and $\Delta\varphi$ for the difference (assumed *minus* computed) between the assumed value φ_m and the computed value φ_c.

Accordingly the relations (6) and (7) can be re-written as :

$$\Delta\varphi_m = -\Delta\varphi_N + \sec a_N \cdot \Delta h - \tan a_N \cdot \cos \varphi_m \cdot \Delta t \tag{9}$$

for north stars

and

$$\Delta\varphi_m = -\Delta\varphi_S - \sec a_S \cdot \Delta h + \tan a_S \cdot \cos \varphi_m \cdot \Delta t \tag{10}$$

for south stars

Now eliminating Δh between relations (9) and (10) and dividing both sides by $\cos a_N + \cos a_S$, we have, after simplification,

$$\Delta\varphi_m = -\frac{\Delta\varphi_N \cdot \cos a_N + \Delta\varphi_S \cdot \cos a_S}{\cos a_N + \cos a_S} - 15 \cdot \tan \frac{a_N - a_S}{2} \cdot \cos \varphi_m \cdot \Delta t \tag{11}$$

where Δt is in time.

Since a_N and a_S are small and do not differ considerably from each other, the first term of the R.H.S. of relation (11) which is the weighted mean of $-\Delta\varphi_N$ and $-\Delta\varphi_S$ (the respective weights being $\cos a_N$ and $\cos a_S$) can be easily reduced to the form :

$$-\frac{\Delta\varphi_N + \Delta\varphi_S}{2}$$

which is obviously the arithmetic mean of $-\Delta\varphi_N$ and $-\Delta\varphi_S$, without introducing any appreciable error unless $\Delta\varphi_N$ and/or $\Delta\varphi_S$ are abnormally large quantities. Thus the relation (11) ultimately takes the following form :

$$\Delta\varphi_m = -\frac{\Delta\varphi_N + \Delta\varphi_S}{2} - 15 \cdot \tan \frac{a_N - a_S}{2} \cdot \cos \varphi_m \cdot \Delta t \tag{12}$$

Now a_N and a_S being again small and of the same sign, $\tan \dfrac{a_N - a_S}{2}$ is necessarily of smaller order of magnitude and therefore can be easily and quickly computed by making use of only 3-figure trigonometrical tables, without introducing any appreciable error unless Δt is abnormally large, in which case 4-figure tables may be necessary.

Accuracy : From the differential formulae (9), (10) and (12), it becomes obvious that the accuracy of the determination depends on :

(1) Precision of observed altitudes of stars, which is again dependent not only on the precision of the vertical circle readings of the theodolite in use, but also on that of the vertical collimation as well as the constant of atmospheric refraction. In the case of precision theodolites, viz. Wild T4, Wild T3 and the Geodetic Tavistock, the vertical angles are always read up to 0.1″, though the accuracy actually obtainable with these instruments is generally of the order of ± 1.0″ which is quite large. However in the case of Wild T4 the order of accuracy can be brought much lower down, say to about 0.2″ or 0.1″, by taking repeated readings of each vertical

setting, the actual intersections in each case being made by means of the moving micrometer only; but in the case of other theodolites it is unlikely to be improved by more than 50 per cent in any circumstances. As regards errors Δh due only to the vertical collimation and the constant of atmospheric refraction, as already stated these are bound to remain practically constant during observations, and as such, cannot affect the final results appreciably. Thus if, apart from the possible errors arising from the vertical collimation and the constant of atmospheric refraction, there is an error of $\pm 1.0''$ in the two vertical circle readings for a star-pair, the corresponding error in the mean value of the latitude obtained will be of the order of $0.7''$, which is highly satisfactory.

(2) Precision of observed hour angles of stars, which is again dependent on the precision of the chronometer time — the error (Δt or ΔT) in the latter being due to that in the assumed value λ_m of longitude. But as already indicated while describing the procedure of observation, the error in the chronometer time can, with proper care, be reduced to a very small order of magnitude, so that ultimately its effect :

$$-15 \cdot \tan \frac{a_N - a_S}{2} \cdot \cos \varphi_m \cdot \Delta t$$

on the final value of latitude becomes either negligible on making $a_N - a_S$ sufficiently small, or easily computable to the desired accuracy by making use of the provisional value of the chronometer error $\Delta T\,(= \Delta t)$, as determined from longitude (time) observations by neglecting the effect of $\Delta\varphi$.

LONGITUDE

Selection of stars : A star-pair, of magnitudes varying generally from 2.0 to 6.0 depending on the order of accuracy needed, one situated to the east and the other to the west but both within about 15 minutes of their time of elongation, must be selected for observation from the precise

Table 2

Giving ranges r of declinations of stars at elongation at different latitudes for $t_e > 20°$ (numerically), $a_e > 50°$ (numerically) and $h_e > 30°$.

φ	r				φ	r			
0°		0°			45°	Between	47°	and	57°
5	Between	5.5°	and	10°	50	"	52	"	60.5
10	"	10.5	"	20.5	55	"	56.5	"	64
15	"	16	"	31	60	"	61.5	"	67.5
20	"	21	"	43	65	"	66.5	"	71
25	"	26.5	"	46	70	"	71	"	75
30	"	31.5	"	48.5	75	"	76	"	78.5
35	"	36.5	"	51	80	"	80.5	"	82.5
40	"	42	"	54	85	"	85.5	"	86

fourth fundamental star catalogue, FK4. In addition, the altitudes of the star-pair should preferably be above 30° and as nearly equal to each other as conveniently possible. While selecting a star-pair as above, use can however be made of table 2 which gives the various ranges of declinations and hour angles of stars at elongation for different latitudes, subject to the following conditions :

(*a*) Azimuths numerically $> 50°$;
(*b*) Hour angles numerically $< 20°$;
(*c*) Altitudes $> 30°$.

But since, for actual location in the fundamental star catalogue, the positions of stars must be known in terms of declinations and right ascensions, the range of hour angles required in table 2 cannot be usefully employed without reducing it to that of right ascensions by subtracting the values of hour angles from those of local sidereal times of observations derived directly from the corresponding standard mean times of observations by referring to table 3.

Table 3

Giving values of R
Change per day : 4 minutes
Day : from 0 to 24 hours U.T.

Date	R h	R m	Date	R h	R m
Jan. 6	7	00	July 8	19	02
22	8	03	23	20	01
Feb. 6	9	03	Aug. 7	21	00
21	10	02	23	22	03
Mar. 8	11	01	Sept. 7	23	02
23	12	00	22	0	02
April 8	13	03	Oct. 7	1	01
23	14	02	22	2	00
May 8	15	01	Nov. 7	3	03
23	16	01	22	4	02
June 8	17	04	Dec. 7	5	01
23	18	03	22	6	00

Observational programme : It is always essential to have a carefully-made programme lasting for about an hour in the case of first-order determinations, about half an hour in the case of second-order determinations and about a quarter of an hour in the case of third-order determinations, showing for the selected star-pair names, aspects, altitudes (computed from the approximate relation : $h = h_e + |t - t_e| \cos \delta$, where $\sin h_e = \sin \varphi \cdot \operatorname{cosec} \delta$ and $\cos t_e = \tan \varphi \cdot \cot \delta$), and times (L.S.T.) of passages corresponding to the various azimuth positions of the star-pair (reduced from the relation :

$$a = a_e - \tan a_e \cdot 2 \sin^2 \frac{t_e - t}{2} \operatorname{cosec} 1''$$

where $\sin a_e = -\cos \delta \cdot \sec \varphi$, or simply $\sin a \fallingdotseq -\cos \delta \cdot \sec \varphi$).

Observational equipment : The astronomical theodolite Wild T4 equipped with an impersonal micrometer, or any other theodolite of similar precision provided with a Talcott level, a good chronometer with chronograph equipment and a wireless set are all that are generally required for the first-order determinations. The Geodetic Tavistock with a Hunter Shutter and Wild T3 theodolites are usually used in the second-order determinations, though the results in respect of the former are slightly better because of the impersonal shutter eye-piece employed. In the case of the third-order determinations, a Wild T2 or a Tavistock theodolite should be good enough.

Observational procedure : Before commencing actual observations, the line of horizontal collimation of the theodolite requires to be set in the meridian, correct to a quarter degree or so, knowing only the approximate value of the chronometer error and the chronometer time of meridian transit of a known circumpolar star. When necessary an approximate value of the latitude can also be deduced from the same observational results by making use of the relations : $\varphi = h_0 \mp (90° - \delta)$ or $\varphi = \delta + 90° - h_0$. The routine of observation is the same as in the case of latitude except that the system of time-recording in this case is more rigorous than in the case of latitude. While observing with the astronomical theodolite Wild T4 for first-order determinations, the procedure followed is similar to that of time-determination by meridian transits of stars observed only on one face of the theodolite at different vertical settings, their vertical circle readings being noted every time with adequate care. As usual the eye-piece micrometer of the theodolite in this case has to be turned through 90°, and the vertical bubble readings for each intersection taken from the Talcott level instead of the striding level. The case with the Geodetic Tavistock theodolite is similar. For its fuller description, reference may however be made to one of my previous papers : "A Method of Determining Astronomical Latitude and Longitude by Observing only Time and Horizontal Angles between Pairs of Stars", published in the *Proceedings of the National Institute of Sciences of India*, Vol. 26, A, No. 2, 1960. In the case of Wild T3, Wild T2 or Tavistock theodolites, the intersections of stars are carried out with the help of the horizontal wire and the instants of intersections noted with a tappet or simply a stop-watch, following the usual ear and eye method. As regards the series of altitudes measured, these need not be exactly symmetrical with respect to the meridian, but the number of intersections for the west star should in any case be the same as for the east star.

Computation : Let h be the altitude of a star of declination δ, corresponding to the hour angle t when the star is near elongation, and h_e be the altitude, corresponding to the azimuth a_e and the hour angle t_e, when the star is at elongation.

Then on referring to fig. 2, we have :

$$\sin h = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos t \tag{1}$$

$$\sin h_e = \sin \varphi \cdot \operatorname{cosec} \delta \tag{13}$$

$$\cos t_e = \tan \varphi \cdot \cot \delta \tag{14}$$

and

$$\sin a_e = - \cos \delta \cdot \sec \varphi \tag{15}$$

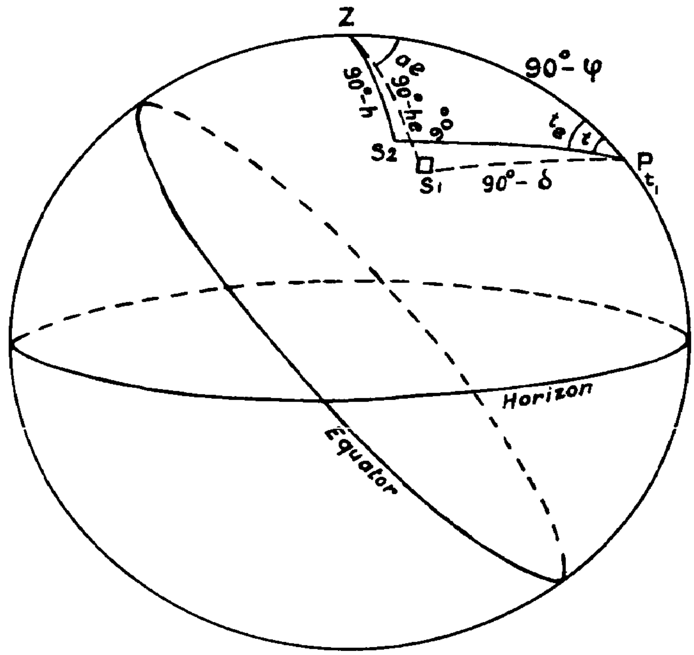

FIG. 2

Denoting $h - h_e$ by x and $t - t_e$ by y, we have from relations (1), (13) and (14) :

$$\frac{\cos(y+t_e) - \cos t_e}{\cos t_e} = \frac{\sin(x+h_e) - \sin h_e}{\cos\varphi \cdot \cos\delta}$$

or

$$\cos y - \sin y \cdot \tan t_e - 1 = \frac{\sin x \cdot \cos h_e + \cos x \cdot \sin h_e - \sin h_e}{\cos\varphi \cdot \cos\delta \cdot \tan\varphi \cdot \cot\delta}$$

or

$$1 - \cos y + \sin y \cdot \tan t_e = (1 - \cos x - \sin x \cdot \cot h_e) \cdot \sec^2\delta$$

or

$$\sin^2\frac{y}{2} + \sin\frac{y}{2} \cdot \cos\frac{y}{2} \cdot \tan t_e = \sin^2\frac{x}{2} \cdot \sec^2\delta - \sin\frac{x}{2} \cdot \cos\frac{x}{2} \cdot \cot h_e \cdot \sec^2\delta$$

or

$$\left(\frac{y}{2} - \frac{y^3}{48} + \frac{y^5}{3840} - \ldots\right)^2 + \left(\frac{y}{2} - \frac{y^3}{48} + \frac{y^5}{3840} - \ldots\right)\left(1 - \frac{y^2}{8} + \frac{y^4}{384} - \ldots\right)$$
$$= \left(\frac{x}{2} - \frac{x^3}{48} + \frac{x^5}{3840} - \ldots\right)^2 \cdot \sec^2\delta - \left(\frac{x}{2} - \frac{x^3}{48} + \frac{x^5}{3840} - \ldots\right)\left(1 - \frac{x^2}{8} + \frac{x^4}{384} - \ldots\right) \cdot \cot h_e \cdot \sec^2\delta$$

or retaining terms up to only the 5th powers of x and y :

$$y + \frac{y^2}{2} \cdot \cot t_e - \frac{y^3}{6} - \frac{y^4}{24} \cdot \cot t_e + \frac{y^5}{120} = -x \cdot \sec\delta + \frac{x^2}{2} \cdot \cot t_e \cdot \sec^2\delta +$$
$$+ \frac{x^3}{6} \cdot \sec\delta - \frac{x^4}{24} \cdot \cot t_e \cdot \sec^2\delta - \frac{x^5}{120} \cdot \sec\delta \qquad (16)$$

Now by the method of successive approximations, we obtain the required solution of the equation (16) in the following simplified form :

$$Y \text{ in time} = -\frac{X}{15} - \frac{X^3}{90}\cdot \sin^2\delta \cdot \sin^2 1'' - \frac{X^4}{120}\cdot \sin^2\delta \cdot \cot t_e \,.\, \sin^3 1''$$
$$-\frac{X^5}{1800}\sin^2\delta\,(9-\cos^2\delta)\cdot \sin^4 1''$$

neglecting terms containing higher powers of X, where :

$$X = x \cdot \sec\delta \qquad (17)$$

Since the term containing the 5th power of X in relation (17) is smaller than 0.01s for observations of stars within about 20 minutes of their time of elongation we have finally :

$$t \text{ in time} = t_e \text{ in time} - \frac{X}{15} - P - Q \qquad (18)$$

where

$$P = 0.2612 \cdot \left(\frac{X}{1000}\right)^3 \cdot \sin^2\delta$$

which can be directly read from chart 3 without recourse to actual computations.

$$Q = 0.364\left(\frac{P}{10}\right)\left(\frac{X}{1000}\right)\cot t_e$$

which can also be directly read from chart 4 without requiring actual computations. And

$$t_e > 20^\circ \text{ numerically}$$

Since the hour angle t of a star is equivalent to the chronometer time T (L.S.T.) of observation *minus* the right ascension α of the star, relation (18) ultimately reduces to :

$$T = t_e + \alpha - \frac{X}{15} - P - Q$$

and (19)

$$\Delta t = \Delta\,(T - \alpha) = \Delta T$$

Obviously the computed value T_c of T, as derived from relation (19) by making use of the assumed values of φ and T, will be in error by an amount equal to $T - T_c = \Delta T_c = \Delta t_c$ due to :

(1) Error ΔT_m in the assumed value T_m of T, where $T - T_m = \Delta T_m$, a constant independent of the star of observation on account of the assumed value λ_m of longitude being in error by the same amount ΔT_m.

(2) Error Δh in the observed altitudes of stars on account of the assumed values of both the vertical collimation and the constant of atmospheric refraction. The error is thus practically constant during observations unless the individual altitudes of the stars of observation differ considerably from each other due to faulty selection of stars.

(3) Error $\Delta\varphi$ in the assumed value of the unknown quantity φ.

But the above errors, when expressed in the form of differential formulae, take the following form :

$$\Delta T_c = \operatorname{cosec} a_E \cdot \sec\varphi \cdot \Delta h - \cot a_E \cdot \sec\varphi \cdot \Delta\varphi \qquad (20)$$

for east stars

and

$$\Delta T_c = -\text{cosec}\ a_W \cdot \sec\varphi \cdot \Delta h + \cot a_W \cdot \sec\varphi \cdot \Delta\varphi \tag{21}$$

for west stars

where a_E and a_W denote azimuthal angles as approximately determined from the relations :

$$\sin a_E = -\cos\delta \cdot \sec\varphi \quad \text{and} \quad \sin a_W = -\cos\delta \cdot \sec\varphi$$

and suffixes $_E$ and $_W$ refer to east and west stars respectively.

But since $T - T_c = (T - T_m) + (T_m - T_c)$ we have :

$$\Delta T_c = \Delta T_m + \Delta T$$

where ΔT_m is the constant error in the assumed value T_m of the chronometer time T, and ΔT is the difference (assumed *minus* computed) between the assumed value T_m and the computed value T_c.

Accordingly the relations (20) and (21) can be re-written as :

$$\Delta T_m = -\Delta T_E + \text{cosec}\ a_E \cdot \sec\varphi \cdot \Delta h - \cot a_E \cdot \sec\varphi \cdot \Delta\varphi \tag{22}$$

for east stars

and

$$\Delta T_m = -\Delta T_W - \text{cosec}\ a_W \cdot \sec\varphi \cdot \Delta h + \cot a_W \cdot \sec\varphi \cdot \Delta\varphi \tag{23}$$

for west stars

Now eliminating Δh between relations (22) and (23) and dividing both sides by $\sin a_E + a_W$, we have after simplification :

$$\Delta T_m = -\frac{\Delta T_E \cdot \sin a_E + \Delta T_W \cdot \sin a_W}{\sin a_E + \sin a_W} + \frac{1}{15} \cdot \tan\frac{a_E - a_W}{2} \cdot \sec\varphi_m \cdot \Delta\varphi \tag{24}$$

Obviously the first term of the R.H.S. of relation (24) is the weighted mean of $-\Delta T_E$ and $-\Delta T_W$, the respective weights being $\sin a_E$ and $\sin a_W$, where a_E and a_W lie between 50° and 90° and are easily computable by making use of 3-figure trigonometrical tables only, unless ΔT_E and/or ΔT_W are abnormally large quantities. Again $a_E - a_W$ being small, $\tan\frac{a_E - a_W}{2}$ is of a smaller order of magnitude, and as such is easily computable by making use of 3-figure trigonometrical tables only, without introducing any appreciable error unless $\Delta\varphi$ is abnormally large in which case 4-figure tables may be necessary.

For longitude difference, however, it is necessary to determine at two stations the difference of correct local time of some event which can be perceived at both. The event is usually provided by some wireless rhythmic time-signals giving the seconds beats of the emitting local clock. These seconds beats are compared at the receiving station against those of the local clock whose error and rate have been known from astronomical observations, to determine the required longitude difference in time, after having corrected for propagation of signal and for a small error in the reputed time of emission of signal which is regularly published in the Admiralty Notices to Mariners and in the *Bulletin Horaire*. A detailed description of the procedure of determining accurately this longitude difference in time in the case of the Geodetic Tavistock fitted with shutter eyepiece, has been given in one of my previous papers referred to on page 111.

Accuracy : From the differential formulae (22), (23) and (24), it becomes obvious that the accuracy of the determination depends on :

(1) Precision of observed altitudes of stars, which is again dependent not only on the precision of the vertical circle readings of the theodolite in use, but also on that of the vertical collimation as well as the constant of atmospheric refraction. In the case of precision theodolites, viz. Wild T4, Wild T3 and the Geodetic Tavistock, the vertical angles are always read up to 0.1″, though the accuracy actually obtainable with these instruments is generally of the order of ± 1.0″, which is quite large. However in the case of Wild T4, the order of accuracy can be brought much lower down, say to 0.2″ or 0.1″, by taking repeated readings of each vertical settings, the actual intersections in each case being automatically made by the movement of the moving micrometer only. In the case of other theodolites, it is unlikely to be improved in any circumstances by more than 50 per cent, except in the case of the Geodetic Tavistock with which the results obtained are slightly better due to automatic time-recording carried out with a Hunter Shutter. As regards errors Δh, due only to the vertical collimation and the constant of atmospheric refraction, as already stated these are bound to remain practically constant during observations, and as such, cannot affect the final results appreciably. Thus if, apart from the possible errors arising from the vertical collimation and the constant of atmospheric refraction, there is an error of ± 1.0″ in the two vertical circle readings for a star-pair, the corresponding error in the mean value of the chronometer time or longitude in time at a place of moderately high latitude is of the order of 0.05s, which is highly satisfactory.

(2) Precision of the assumed value of φ. As already indicated while describing the procedure of observation the error $\Delta\varphi$ when very large can, with proper care, be reduced to a very small quantity, so that ultimately its effect : $\frac{1}{15} \cdot \tan \frac{a_E - a_W}{2} \cdot \sec \varphi \cdot \Delta\varphi$ on the final value of chronometer time or longitude in time becomes either negligible on making $a_E - a_W$ sufficiently small, or easily computable to the desired accuracy by making use of the provisional value of latitude error $\Delta\varphi$ as determined from latitude observations by neglecting the effect of Δt.

CONCLUSION

Apart from its being extremely rapid and simple in its application, the method has many other practical advantages. Among these, the more important are :

(1) It is free from the error of vertical collimation, thereby obviating the necessity of observing on both faces of the theodolite as in other methods. This saves time and simplifies computations without sacrificing accuracy.

(2) It needs a much lesser number of stars, e.g. only two pairs of stars for the entire observations — one for latitude and the other for time or

longitude — and thus provides ample scope for the proper selection of stars whose declinations and right ascensions may be of dependable accuracy, which is not always possible in other methods.

(3) Altitudes of each star-pair being above 30° and lying close to each other, it is practically free from the error of constant of atmospheric refraction — a factor of vital importance in astro-determinations by altitudes of stars.

(4) Computations involved are so simple that these can be easily worked out with the help of a few simple tables and charts specially designed for the purpose, and trigonometrical tables not exceeding 5 figures, except in the case of first-order determinations, when 6-figure tables are needed in computing the values of X, t_e and h_e.

The present method of astrofix is thus likely to prove more useful than any other method, especially in the field when both latitude and longitude are wanted in various survey operations including Laplace stations and astro-geodetic deflections at a large number of stations required in geodetic triangulations.

Specimen examples have been added at the end of this article in order to make the method clearer.

Acknowledgement

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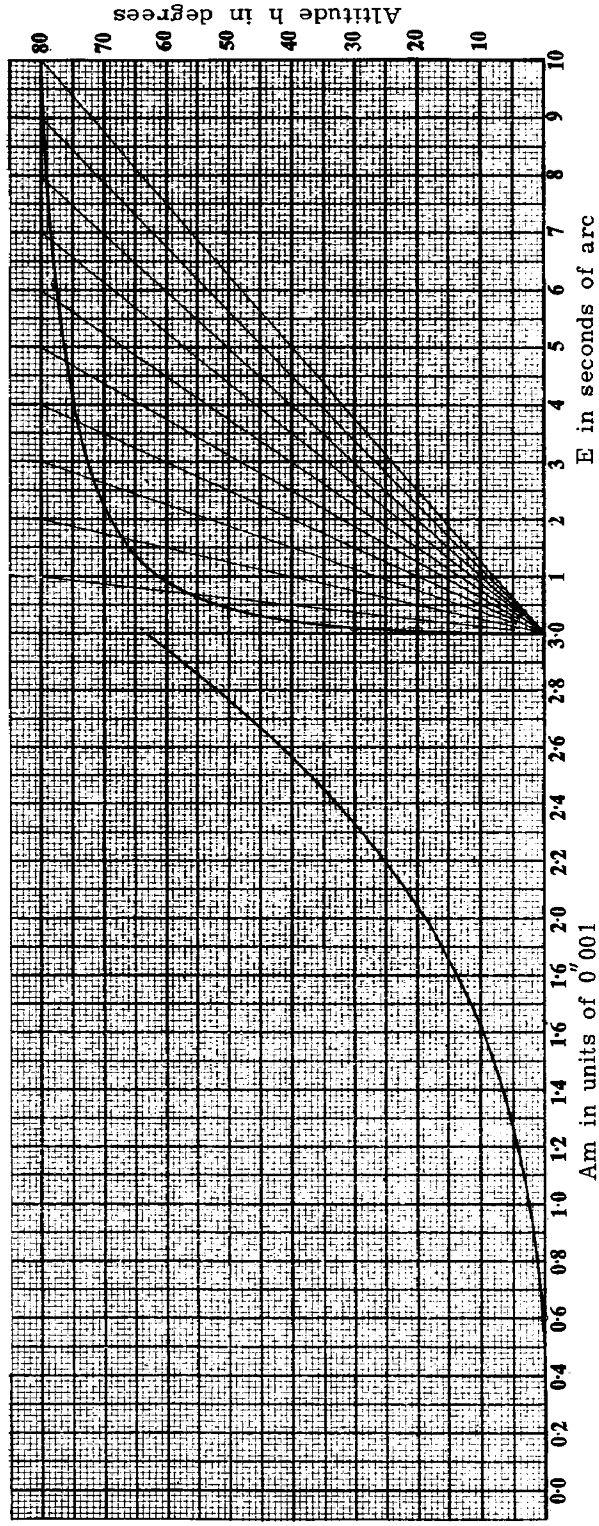
LATITUDE CHART 2
for
E in seconds of arc
Argument : Am & h



EXAMPLES

1. Computation of latitude observations.

(*a*) ***First-order determination :***

FORMULAE :

(1) $h_0 = \varphi_m \pm (90° - \delta)$ for upper (lower) transit of north stars.
$h_0 = 90° - \varphi_m + \delta$ for south stars.

(2) $A = \cos \varphi_m \cdot \cos \delta \cdot \sec h_0$

(3) $\varphi_c = h \mp (90° - \delta) + A \cdot m - D + E$ for north stars near upper (lower) transit.
$\varphi_c = \delta + (90° - h) - A \cdot m + D - E$ for south stars.

(4) $a''_N = -15 \cdot t^s \cdot \cos \delta \cdot \sec h_0$ for north stars.
$180° - a''_S = -15 \cdot t^s \cdot \cos \delta \cdot \sec h_0$ for south stars.

(5) $\Delta\varphi_m = -\dfrac{\Delta\varphi_N + \Delta\varphi_S}{2} - 15 \cdot \tan \dfrac{a_N - a_S}{2} \cdot \cos \varphi_m \cdot \Delta t$

1. Star : N/S	A Draconis : N	12 Ophiuchi : S
2. Altitude : h (1)	51° 21′ 41″2	57° 13′ 41″6
3. L.S.T. of observation : T	16^h 08^m 27^s36	16^h 18^m 59^s70
4. R.A. : α from FK4	16 28 05.46	16 34 29.50
5. H.A. : t	— 0 19 38.10	— 0 15 29.80
6. Decl. : δ from FK4	68° 50′ 57″3	— 2° 14′ 56″6
7. cos δ	0.3608	0.9992
8. φ_m	30 19 10	30 19 10
9. cos φ_m	0.8632	0.8632
10. h_0 from formula (1)	51 28 13	57 25 53
11. sec h_0	1.6053	1.8577
12. A from formula (2)	0.5000	1.6023
13. m from table 1	756″53	471″45
14. A · m	378.3	755.4
15. A · m in min. and sec.	6′ 18″3	12′ 35″4
16. D from chart (2)	0.4	2.2
17. E from chart (3)	0.0	0.0
18. φ_c from formula (3)	30° 18′ 56″4	30° 18′ 48″6
19. $\varphi_m - \varphi_c$: $\Delta\varphi$	+ 13.6	+ 21.4
20. Mean of $-\Delta\varphi_N$ and $-\Delta\varphi_S$	— 17.5	
21. a in min. from formula (4)	170′	431′
22. a in degr. and min.	2 50	7 11
23. $(a_N - a_S)/2$	— 2 10	
24. tan $(a_N - a_S)/2$	— 0.038	
25. Δt (4)	— 1^s00	
26. Correction for Δt from formula (5)	— 0″5	
27. $\Delta\varphi_m$ from formula (5)	— 18″0	
28. $\varphi_m + \Delta\varphi_m$: φ	30° 18′ 52″0	

(1) Corrected for vertical inclination, collimation and refraction.

(2) Enter with A · m reading, ascend vertically to the curve on the left-hand side and then move horizontally towards the right. Enter again with h_0 reading, move horizontally to the curve on the right-hand side, ascend vertically up to the bounding horizontal line and move downwards following the slanting line (or the interpolated slanting line and move from the common position thus attained, descend vertically downwards to obtain the figure for D from the horizontal scale on the right-hand side.

(3) Proceed as in (2) above after substituting E for D.

(4) Only provisional value as obtained from time observations without considering the effect of $\Delta\varphi$.

(*b*) *Second-order determination :*

Proceed as in (*a*) above but the permissible error in the last decimal place for each figure in the present case is about ± 5.

(*c*) *Third-order determination :*

FORMULAE :

(1) $h_0 = \varphi_m \pm (90° - \delta)$ for upper (lower) transit of north stars.
$h_0 = 90° - \varphi_m + \delta$ for north stars.

(2) $A = \cos\varphi_m \cdot \cos\delta \cdot \sec h_0$.

(3) $\varphi = h \mp (90° - \delta) + A \cdot m$ for north stars near upper (lower) transit.
$\varphi = \delta + (90° - h) - A \cdot m$ for south stars.

1. Star : N/S	A Draconis : N	12 Ophiuchi : S
2. Observed altitude : h	51° 21′ 40″	57° 13′ 40″
3. L.S.T. of observation : T	$16^h\ 08^m\ 30^s$	$16^h\ 19^m\ 03^s$
4. R.A. : α from any star almanac	16 28 05	16 34 29
5. H.A. : t	— 0 19 35	— 0 15 26
6. Decl. : δ from any star almanac	68° 50′ 57″	— 2° 14′ 57″
7. cos δ	0.361	0.999
8. φ_m	30° 18′ 00″	30° 18′ 00″
9. cos φ_m	0.863	0.863
10. h_0 from formula (1)	51° 27′	57° 27′
11. sec h_0	1.605	1.859
12. A from formula (2)	0.500	1.602
13. m from table 1	753″	468″
14. A · m	377	750
15. A · m in min. and sec.	6′ 17″	12′ 30″
16. φ from formula (3)	30° 18′ 54″	30° 18′ 53″
17. Mean φ	30 18 54	

2. Computation of clock correction observations.

(*a*) *First-order determination :*

FORMULAE :

(1) $\cos t_e = \cot\delta \cdot \tan\varphi_m$, t_e $+ve$ for west stars and $-ve$ for east stars.

(2) $\sin h_e = \operatorname{cosec}\delta \cdot \sin\varphi_m$

(3) $T_c = t_e + \alpha - \dfrac{X}{15} - P - Q$, last three terms beings of opposite sign in case of east stars.

(4) $\sin a = -\cos\delta \cdot \sec\varphi_m$

(5) $$T_m = -\frac{\Delta T_E \cdot \sin a_E + \Delta T_W \cdot \sin a_W}{\sin a_E + \sin a_W} + \frac{1}{15} \cdot \tan\frac{a_E - a_W}{2} \cdot \sec\varphi_m \cdot \Delta\varphi$$

1. Star : W/E	23 Canum. Venat : W	Lyræ : E
2. Altitude : h (1)	48° 16′ 20″.3	58° 49′ 15″.0
3. L.S.T. of observation : T_m	16^h 40^m $15^s.97$	16^h 48^m $01^s.94$
4. R.A. : α from FK4	13 18 42.85	19 15 09.13
5. H.A. : t	3 21 33.12	—2 27 07.19
6. Decl. : δ from FK4	40° 20′ 31″.8	38° 04′ 09″.9
7. cot δ	1.177402	1.276750
8. φ_m	30° 19′ 10″	30° 19′ 10″
9. tan φ_m	0.584808	0.584808
10. cos t_e from formula (1)	0.688554	0.746654
11. t_e in arc	46° 29′ 03″.3	—41° 41′ 55″.0
12. t_c in time	3^h 05^m $56^s.22$	— 2^h 46^m $47^s.67$
13. sin φ_m	0.504821	0.504821
14. cosec δ	1.544757	1.621755
15. sin h_e from formula (2)	0.779826	0.818696
16. h_e	51° 14′ 40″.6	54° 57′ 16″.0
17. $h - h_e$: x	—2 58 20.3	3 51 59.0
18. x in seconds	—10700″.3	13919″.0
19. sec δ	1.312005	1.270222
20. x sec δ	—14038″.8	17680″.2
21. X in time (opp. sign for east stars)	— 0^h 15^m $35^s.92$	— 0^h 19^m $38^s.68$
22. P from chart (2) (sign same as line 21)	—0.30	—0.55
23. Q from chart (3) (sign same as line 21)	0.00	—0.04
24. $T_e + \alpha$	16 24 39.07	16 28 21.46
25. T_c from formula (3)	16 40 15.29	16 48 00.73
26. $T_m - T_c$: ΔT	0.68	1.21
27. cos δ	0.762	0.787
28. sec φ_m	1.158	1.158
29. sin a from formula (4)	0.882	0.911
30. Weighted mean of $-\Delta T_W$ and $-\Delta T_E$ from formula (5)	— $0^s.95$	
31. a	62° 00′	65° 47′
32. $(a_E - a_W) / 2$	—1 54	
33. tan $(a_E - a_W) / 2$	— 0.0332	
34. $\Delta\varphi$ (4)	—18″.0	
35. Correction for $\Delta\varphi$ from formula (5)	— $0^s.05$	
36. Clock correction : ΔT_m from formula (5)	— $1^s.00$	

(1) Corrected for vertical inclination, collimation and refraction.

(2) Proceed as in (2) of example 1 after substituting X for $A \cdot m$, δ for h_0 and P for D.

(3) Enter with X reading, ascend vertically to the slanting line (or the interpolated slanting line) representing P on the left-hand side and move horizontally towards the right. Enter again with t_e reading, move horizontally to the curve on the right-hand side, ascend vertically up to the bounding horizontal line and then move downwards following the slanting line (or the interpolated slanting line), and finally from the common position thus attained, descend vertically downwards to obtain the figure for Q from the horizontal scale on the right-hand side.

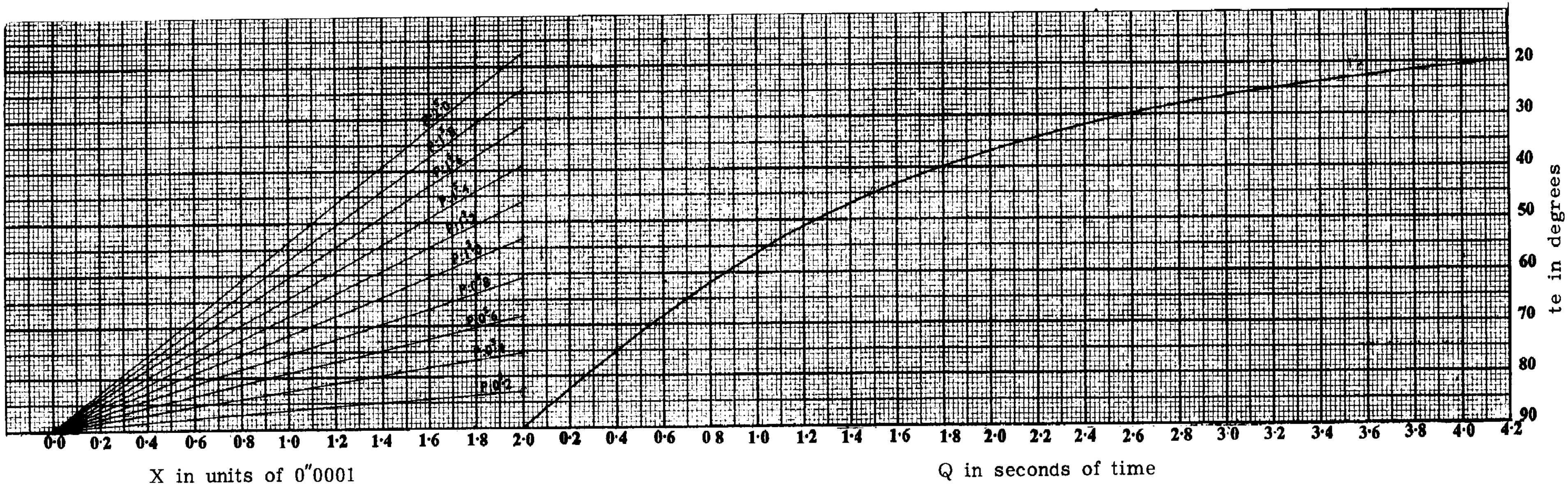
(4) Only provisional value as obtained from latitude observations without considering the effect of Δt.

LONGITUDE CHART 2

for

Q in seconds of time. Q is always + ve in case of east stars and - ve in case of west stars.

Argument : P, X & te



(*b*) ***Second-order determination :***

Proceed as in (*a*) above but the permissible error in the last decimal place for each figure in the present case is about ± 5.

(*c*) ***Third-order determination :***

FORMULAE :

(1) $\cos t_e = \cot \delta \cdot \tan \varphi_m$, t_e $+ve$ for west stars and $-ve$ for east stars.
(2) $\sin h_e = \operatorname{cosec} \delta \sin \varphi_m$
(3) $T_c = t_e + \alpha - \dfrac{X}{15}$, sign of $\dfrac{X}{15}$ being opposite in case of east stars.
(4) $\Delta T_m = -\dfrac{\Delta T_E + \Delta T_W}{2}$

1. Star : W/E	23 Canum. Venat : W	Lyræ : E
2. Observed altitude : h	48° 16′ 20″	58° 49′ 15″
3. L.S.T. of observation : T_m	16h 40m 19s.0	16h 48m 04s.9
4. R.A. : α from any star almanac ..	13 18 42.9	19 15 09.1
5. H.A. : t	3 21 36.1	— 2 27 04.2
6. Decl. : δ from any star almanac ..	40° 20′ 32″	38° 04′ 10″
7. cot δ	1.17740	1.27675
8. φ_m	30° 18′ 00″	30° 18′ 00″
9. tan φ_m	0.58435	0.58435
10. cos t_e from formula (1)	0.68801	0.74607
11. t_e in arc	46° 31′ 38″	—41° 44′ 56″
12. t_e in time	3h 06m 06s.5	— 2h 46m 59s.7
13. sin φ_m	0.50453	0.50453
14. cosec δ	1.54476	1.62175
15. sin h_e from formula (2)	0.77938	0.81822
16. h_e	51° 12′ 14″	54° 54′ 25″
17. $h - h_e$: x	— 2 55 54	3 54 50
18. x in seconds	— 10554″	14090″
19. sec δ	1.31201	1.27022
20. x sec δ : X	— 13847″	17897″
21. X in time (opp. sign for east stars)	— 0h 15m 23s.1	— 0h 19m 53s.1
22. T_c from formula (3)	16 40 12.5	16 48 02.4
23. $T_m - T_c = \Delta T$	6.5	2.4
24. Mean clock correction : ΔT_m from formula (4)	— 4.4	

Conversion of time

L.S.T. = standard time *plus* (local long. *minus* standard long.) in time *plus* R (interpolated for the date and the fraction of the day).

N.B. — The table is correct to 2 minutes for all years. For better accuracy, however, the tabular values may be taken to be true for :

(1) 1962, 1966, 1970, 1974 ... i.e. two years after the leap years.

(2) All leap years after increasing the values by 2 minutes except for the months of January and February in which case the tabular values are to be decreased by 2 minutes.

(3) Years following the leap years after increasing by 1 minute.

and (4) Remaining years after decreasing by 1 minute.