NAVIGATIONAL CHART OF THE NORTH ATLANTIC USING AN OBLIQUE CONFORMAL MAP PROJECTION

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The French Naval Hydrographic Office has recently published a navigational chart (No. 6504) of the northern portion of the Atlantic Ocean using an oblique conformal map projection. This chart, on a scale of about 1/6 000 000, is intended specifically for ships running between North America (from New York to Greenland) and the Eurafrican continental front (from Bergen to the Canary Islands), so that they may more easily make use of great circle tracks (figure 1).

In the present article it is proposed, after recalling the problem posed by the plotting of great circle tracks on a navigational chart, to give the principal characteristics of the Hydrographic Office's new chart, as well as the computation process used in its compilation.

I. — THE CARTOGRAPHIC REPRESENTATION OF ORTHODROMES

For a long while the Mercator projection was a necessity in maritime navigation because constant heading tracks are here represented by straight lines. Moreover this projection has two important advantages : on one hand the angles are retained in all areas of the chart, and on the other the rectangular grid representing the geographic meridians and parallels makes the scaling of a position's coordinates very easy, and likewise the plotting of a position by means of its coordinates.

In high and middle latitudes this system, due to the relatively rapid variation in local scale, which is nearly proportional to the secant of the latitude, has the disadvantage that great circles on the sphere are represented on the chart by arcs of rather great curvature.

Navigators have many simple methods at their disposal for computing or plotting orthodromic arcs corresponding to the shortest paths or to radio bearings. In fact, gnomonic projection charts of the oceans have been drawn up for this purpose. In the gnomonic system great circles of the spherical earth are shown on the chart as straight lines which are subsequently plotted point by point onto the Mercator projection navigational chart. In the case of orthodromic navigation these lines are easily plotted by joining the points of departure and of arrival, but this is not the case for radio bearings from a transmitting station in a given azimuth, for here the map projection used noticeably distorts angles, and it is necessary to show on the chart around each transmitting station a rose graduated in azimuth, thus supplying a second point for rectilinear plotting of bearings. The non-conformity of gnomonic projection also has the disadvantage that it prohibits practically any angular measurement on the chart, and in particular that of an orthodrome's azimuth at any point on its path. Similarly the significance of linear distortion prevents any distance measurement on the chart. Thus this cartographic solution is not as attractive as might be imagined at first sight. While it is strictly applicable to the problem of rectilinear representation of great circle arcs, it however lends itself very ill to the even approximate solution of any other problems concerning angles and distances.

Thus, for a long while, it was thought that a compromise solution should be sought, and that a plane representation system should be used which, though supplying only an approximate rectilinear representation of great circles, would keep the advantages of being conformal and permit distance measurement with a good degree of accuracy. These properties exist in the Mercator projection in the vicinity of the equator. Indeed this system is a true conformal one and the scale variation is only of the second order with respect to latitude, so long as this last remains low.

It suffices then to choose an " auxiliary " equator across the region being considered, and to take this line as a basis for a Mercator projection. In this new system, an oblique cylindrical conformal projection, the geographic meridians and parallels obviously no longer form a rectangular rectilinear grid as in the case of the Mercator projection, but nevertheless all the angular and metric properties of the Mercator projection are retained. In practice, for the scaling of geographic coordinates of a position or for the plotting of a position by its coordinates, it is necessary to graduate all the meridians and parallels of the grid as in the case of the gnomonic projection, whereas in the Mercator projection it is sufficient that the borders of the chart carry a scale. An oblique cylindrical conformal projection for a spherical earth has been used, by Mr. Louis KAHN in particular, to draw up small-scale charts of the principal routes for long-range aerial navigation.

II. — THE FRENCH HYDROGRAPHIC OFFICE OBLIQUE CONFORMAL CHART

In spite of the advantages of this system of representation it would appear not to have been yet advocated for maritime navigation. Therefore, as an experiment, we decided to make a first application of this system to a region sufficiently far from the equator in order that its value might become evident. From this point of view the northern part of the Atlantic Ocean between 40° and 50° North latitude where there are busy shipping routes was particularly suitable.

Choice of initial data

Much experimentation was necessary before fixing the limits of the chart, because in the first place we wished to keep to the 96 × 65 cm format used for French nautical charts, and then within this framework to give the chart the largest possible scale, taking in New York and the Bermudas to the west over to the inner part of the Bay of Biscay to the east, the Canary Islands to the south and finally the southern parts of Iceland and Greenland to the north. In fact a maximum scale corresponds to as small as possible an extent on each side of the auxiliary equator, that is to say, to " auxiliary " latitudes as low as possible for the auxiliary parallels forming the borders of the chart. As the scale on these extreme parallels is equal to the equatorial scale multiplied by the secant of their auxiliary latitude the scale variation in the area represented is thus reduced to the minimum (1).

Finally, the following basic data were adopted as the characteristics of the system :

Central point : Latitude 45°02′ N, Longitude 37°43′ W
Major axis (auxiliary equator) : Azimuth at the central point 78°52′.5
: Total range 53°36′

Conformal representation of the ellipsoid on the sphere

As the use of an auxiliary equator is only convenient on a sphere it was first of all necessary to make a conformal representation of the terrestrial ellipsoid on an auxiliary sphere. By reason of the smallness of the chart's scale, and also of the fact that the direction of the chosen major axis is separated by hardly more than 10° from that of the central point's parallel, we adopted the simplest system of projection which consists of representing the ellipsoid on a sphere transversally osculatory at the central point, that is to say tangent to the ellipsoid along the parallel of this point, and having as radius the great normal to the ellipsoid at this latitude. In this system the spherical longitudes are equal to the ellipsoidal longitudes; the latitude of the central point is also retained, and for all other points, the difference in meridional parts between a given point and the central point has the same value on the sphere as on the ellipsoid. Among conformal representations of an ellipsoid on a sphere it is not the best from

(1) In fact when designing the chart it was possible to use a slightly larger format — 97.5 × 66.3 cm. Nevertheless the initial data used for the 96 × 65 cm format have been retained, in particular the scale deduced from this format. Thus the chart covers an expanse a little larger than that initially planned. All the computations given in the present article refer to the original format 96 × 65 cm.
However, in order to meet a demand expressed after the chart had already been compiled, a window was inserted to the left side of the frame so as to extend the chart as far as the port of Norfolk on the east coast of the United States of America.

the point of view of linear distortions, but these are much smaller in this system than those encountered later on in the conformal representation of the sphere on a plane, and there are no practical objections to their acceptance. In the most unfavourable case of chart coverage, the length distortion is in fact still below 1/2 000; and this is the distortion of the Mercator projection 1°46′ from the equator.

Chart scale

In fact the basic data adopted for the chart do not define a geodetic line on the ellipsoid, which would be useless, since this line is not represented by a great circle on the transversally osculatory sphere. These basic data, however, are representative of the auxiliary equator on the sphere. This consideration allows us a simplification which only involves a slight modification of the limits of the chart coverage and it in no way interferes with the angular and metric properties of the plane representation.

Given that an arc of 53°36′ on the auxiliary equator of the sphere whose radius is N_0 is represented on the chart by a length of 96 cm, it is easy to deduce the scale of the plane representation of the equator, i.e. 1/6 226 000, and this in practice can be taken as the scale on the major axis because, as we have seen, it is permissible to omit the change in scale introduced by the conformal representation of the ellipsoid on the sphere.

In these conditions half the length of the shorter border of the chart, i.e. 32.5 cm, represents the auxiliary meridional parts of the chart's borders; their auxiliary latitude, somewhat less than 18°, is easily deduced therefrom. In a zone extending a little less than 18° on each side of the equator the chart has, then, the properties of the Mercator projection. The ratio of the scale on the auxiliary parallels bounding the chart to the equatorial scale is 1.05057, so that the variation in scale within the chart's borders does not exceed 5 %, or only plus or minus 2.5 % in relation to the two auxiliary parallels of 12°40′ latitude where the scale is exactly intermediate between the minimum scale on the chart's major axis and the maximum scale on the longitudinal borders.

Computation of the geographical grid

To pass from geographic coordinates on the sphere to auxiliary coordinates serving as a basis for the plane representation, i.e. auxiliary latitudes and longitudes counted respectively from the auxiliary equator and from the auxiliary meridian of the central point, one must resolve the spherical triangle formed by the sphere's geographic pole, the auxiliary equator's pole and the point under consideration. The chart is afterwards compiled with the help of these auxiliary coordinates as in the case of a normal Mercator projection.

However, since it is the actual geographical grid, and not the auxiliary grid, that is to be shown on the chart it is necessary both for the working out and for the use of this grid to calculate the plane rectangular coordinates of the intersections of the geographic meridians and parallels after having determined their auxiliary coordinates.

For the conversion of coordinates on the sphere, the longitudes on the ellipsoid are naturally used as these are retained in the representation of this surface on the sphere. In the case of the latitudes, however, the representation of the ellipsoid on the sphere is taken into account by using the spherical latitudes deduced from the ellipsoidal latitudes under consideration.

The computation was made for geographic meridians and parallels at 2°30′ intervals, the distance between the corresponding intersections on the chart being 45-47 millimetres in latitude and considerably smaller in longitude.

Discrepancies between charted orthodromes and their chords

It is now appropriate to find out if the chart answers its purpose well, that is to say how much the orthodromes on the chart deviate from straight lines, not only from the point of view of angles, but also from the point of view of the difference in length between the arc of great circle and the corresponding straight line on the chart. To simplify the calculations these may be made from the sphere and not the terrestrial ellipsoid, thus involving a negligible discrepancy.

Obviously the most unfavourable case concerns the great circle joining two corners of the chart both situated on the same side of its great axis. Its plane representation falls outside the chart and at its extremities makes an angle of 8°48′ with the chart's border, its dip in relation to the border being very nearly 2°, i.e. 119 nautical miles. However the error in distance is very small; the border of the chart measures 3 061 miles, whereas the orthodrome is 3 050 miles long (figure 2).

If we consider a slightly less extreme case, one nearer that encountered in actual practice, the charted orthodromic arc comes appreciably nearer its chord. For example, taking two points whose difference in auxiliary longitude is 36°, i.e. about 2/3rds of the longitude range of the chart, and whose auxiliary latitude, 12°, is equally about 2/3rds that of the chart's borders, we find that the orthodrome only makes an angle of 3°53′ with its chord at its extremities, that it has a dip of 36 nautical miles and that the length of the arc is 2 111 miles as against 2 113 measured on its chord.

These figures show that, in the normal use of the chart, whose most useful part is situated in the vicinity of its major axis, the auxiliary equator, there will in general be no disadvantage in using the plotting of straight lines instead of the charted great circles.

This substitution allows a considerable benefit to be gained in comparison with similar substitutions made on the usual Mercator chart, i.e. in comparison to the use of loxodromes (figure 3). The following table sums up the comparison for the two extreme cases, the upper and the lower borders of the oblique chart.

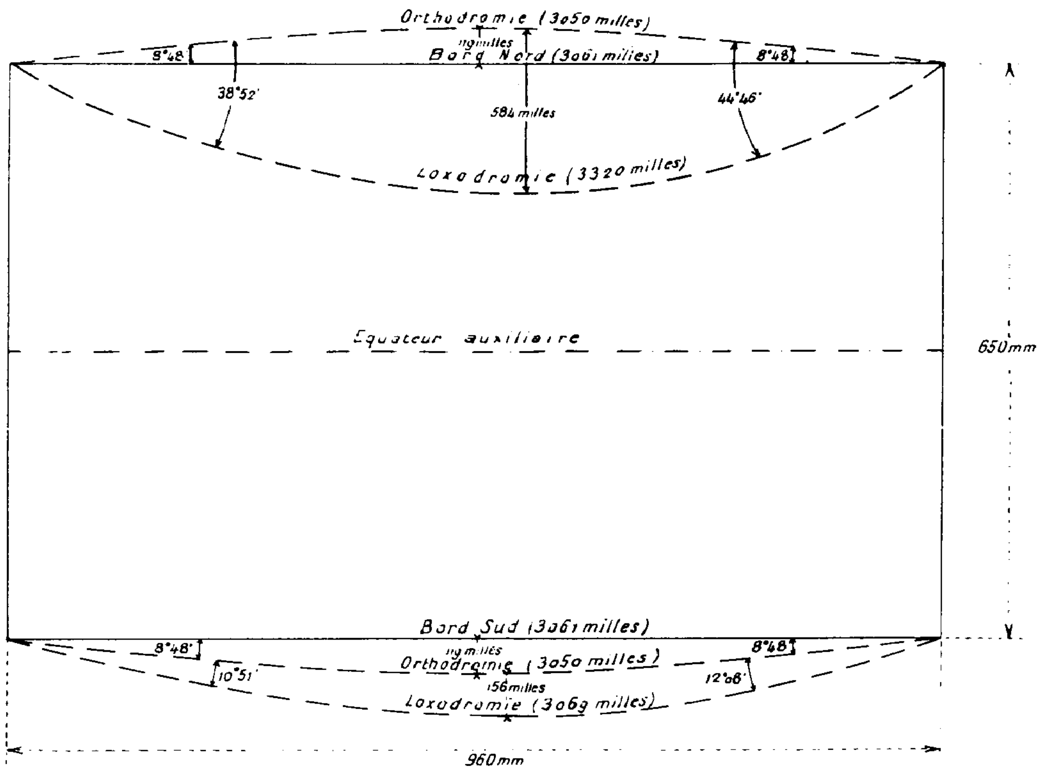

FIG. 2. — Layout of the oblique conformal chart.

		Azimuths		Length in nautical miles	Max. distance to the orthodrome in nautical miles
		at the left hand border	at the right hand border		
Northern border	orthodrome	38° 57′	122° 35′	3 050	
	straight line plotted on the oblique chart	47° 45′	113° 47′	3 061	119
	loxodrome on the Mercator chart	77° 49′	77° 49′	3 320	584
Southern border	orthodrome	70° 58′	93° 56′	3 050	
	straight line plotted on the oblique chart	62° 10′	102° 45′	3 061	119
	loxodrome on the Mercator chart	81° 49′	81° 49′	3 069	156

For the lower border of the chart, involving geographic latitudes between 19° and 27°, the loxodrome with reference to the orthodrome presents deviations which only slightly exceed those existing between the

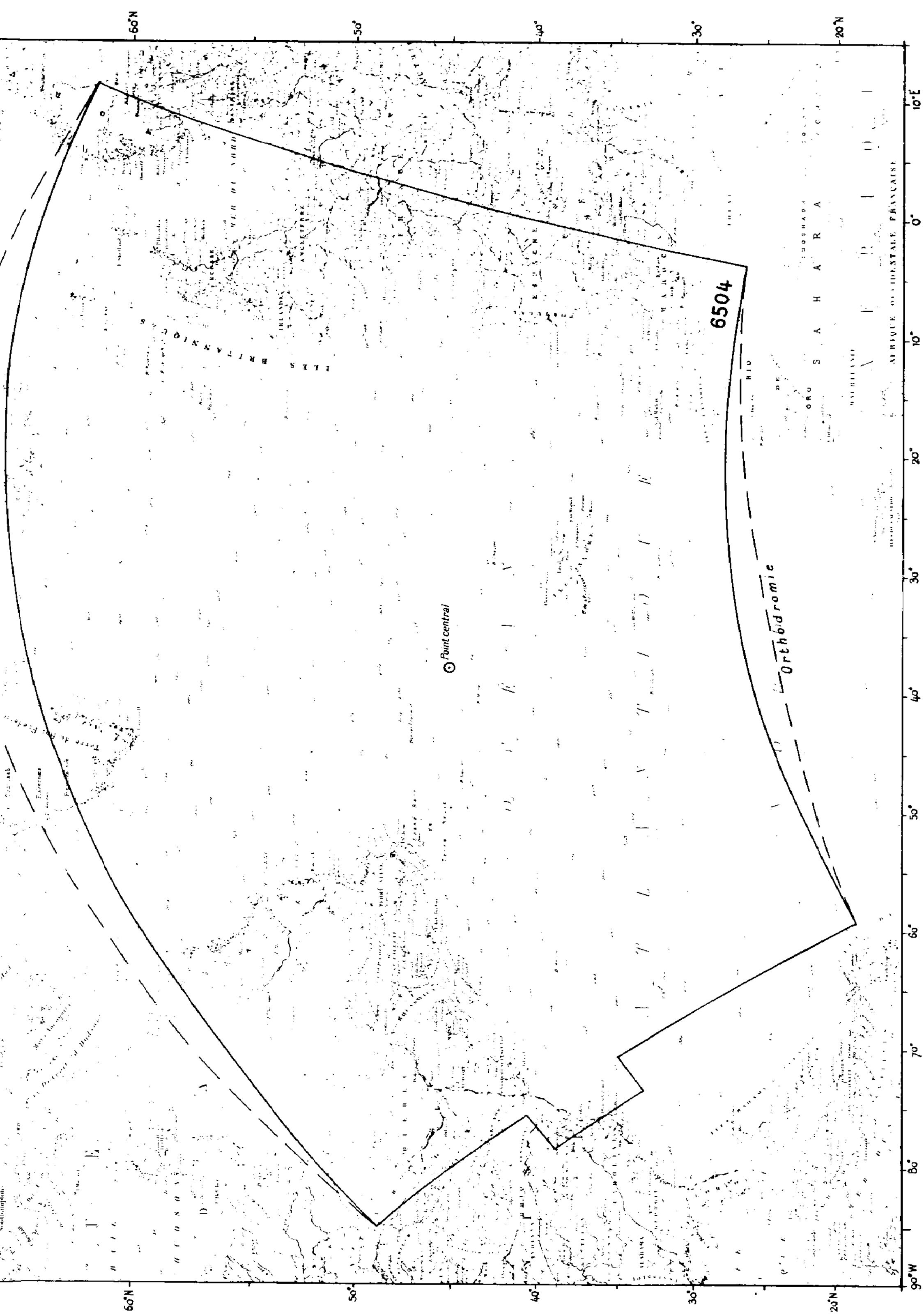

FIG. 3. — Limits of the oblique conformal chart shown on a Mercator projection chart.

orthodrome and the border of the oblique chart, since this border is equivalent to a loxodrome plotted at 18° of auxiliary latitude.

But as regards the upper border, for which the arc of a great circle runs between 49° and 66° of geographic latitude, the deviations between the orthodrome and the loxodrome become considerable, whereas those between the orthodrome and the upper border are identical with those existing on the lower border.

The angular error in adopting a rectilinear course on the chart instead of the true curvilinear plotting of the orthodrome can easily be taken into account. In fact this error, which is the angle of contact at one end of the path, is nearly equal to half the convergency of the meridians between the two extremities, which at the first approximation is equal to the range in auxiliary longitude multiplied by the sine of the mean auxiliary latitude. Since this latitude is rather low — always less than 18° — we may consider that the convergency of the meridians, i.e. the angular error, is proportional to the area included between the path and the auxiliary equator. We have previously established that in the extreme case of the border of the chart, that is to say for an area equal to half that of the chart, the angular error between the orthodrome and its chord is 8°8. It suffices therefore to compute roughly the ratio of the area under consideration to that of half the chart to obtain immediately the size of the expected angular error. If the two extremities of the path are situated on either side of the chart's longitudinal axis, the area to be computed is the difference of the areas of the two triangles included between the path and the auxiliary equator.

III. — COMPUTATIONAL PROCESS

The computations required for the establishment of the chart's grid having been made by an electronic computer, we shall develop in detail the formulae used on this occasion and obtained by the conversion of the usual spherical trigonometrical formulae. Particularly for the deduction of arcs defined by a trigonometric line it is convenient that this line should be a circular tangent or a hyperbolic sine; furthermore the computations were made by expressing the angles in degrees and decimal fractions of a degree.

The ellipsoid adopted is the international ellipsoid (Madrid, 1924) characterized by the values of its flattening $\alpha = 1/297$ and of its semi-major axis $a = 6\ 378\ 388$ m. Excentricity e, which frequently appears in the formulae is given by $e = (2\alpha - \alpha^2)^{\frac{1}{2}}$.

It should be remembered that the great normal N to the ellipsoid at the point with latitude L is expressed by :

$$N = a\,(1 - e^2 \sin^2 L)^{-\frac{1}{2}}$$

and that the meridional part, or Mercator variable, is defined on the ellipsoid by :

$$\mathcal{L} = \text{Log} \tan\left(\frac{\pi}{4} + \frac{L}{2}\right) + \frac{e}{2} \text{Log} \frac{1 - e \sin L}{1 + e \sin L}$$

a relation in which $\mathcal{L}$ is expressed in radians.

Conformal representation of the ellipsoid on the sphere

In the system used the relation between the spherical latitudes φ and the ellipsoidal latitudes L is :

$$\Phi - \Phi_0 = \mathcal{L} - \mathcal{L}_0 \quad \text{or} \quad \Phi = \mathcal{L} + \Phi_0 - \mathcal{L}_0$$

Φ being the meridional part on the sphere and the subscript $_0$ being assigned to elements relating to the central point.

Since moreover $\varphi_0 = L_0$, it follows that :

$$\Phi = \mathcal{L} - \frac{e}{2} \text{ Log } \frac{1 - e \sin L_0}{1 + e \sin L_0}$$

or :

$$\text{Log tan}\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = \text{Log tan}\left(\frac{\pi}{4} + \frac{L}{2}\right) + \frac{e}{2} \text{ Log sin } \frac{1 - e \sin L}{1 + e \sin L} - \frac{e}{2} \text{ Log } \frac{1 - e \sin L_0}{1 + e \sin L_0}$$

For machine computing, we then have :

$$\varphi = -90° + 2 \tan^{-1}\left[\left(\frac{1 - e \sin L}{1 + e \sin L} \cdot \frac{1 + e \sin L_0}{1 - e \sin L_0}\right)^{\frac{e}{2}} \tan\left(45° + \frac{L}{2}\right)\right]$$

For manual computation it is possible to employ with sufficient approximation the restricted development :

$$\varphi = L + e^2 \cos L \,(\sin L_0 - \sin L)$$

The similarity ratio at any point on the chart is strictly defined by :

$$\frac{\text{Sphere}}{\text{Ellipsoid}} = \frac{N_0 \cos \varphi}{N \cos L}$$

At the first approximation it may be written :

$$1 + \frac{e^2}{2} (\sin L_0 - \sin L)^2$$

Scale and extent of the chart

(a) Let p be the chart's length and q its width, expressed in metres, μ the range of the auxiliary equator in degrees. The value of this range in radians is :

$$\Delta = \frac{\mu \pi}{180}$$

Its length in metres is consequently $N_0 \Delta$ and the scale on the auxiliary equator is written $E = \frac{p}{N_0 \Delta}$, neglecting the alteration introduced by the conformal representation of the ellipsoid on the sphere.

(b) Half the smaller border of the chart, whose length is $\frac{q}{2}$, has an aperture of :

$$\frac{q}{2\,N_0\,E} = \frac{\Delta}{2}\,\frac{q}{p} \text{ radians.}$$

Since the chart is in Mercator projection referred to the auxiliary equator, this quantity represents the auxiliary meridional part λ_e of the chart borders; then their auxiliary latitude l_e is represented by :

$$\text{Log tan}\left(\frac{\pi}{4} + \frac{l_e}{2}\right) = \lambda_e = \frac{\Delta}{2}\,\frac{q}{p}$$

or :

$$l_e = 2\ \tan^{-1}\left(\text{th}\,\frac{\lambda_e}{2}\right)$$

(*c*) The scale on the borders of the chart, which is the maximum scale encountered in the whole area of the chart, is :

$$E/\cos l_e \quad \text{or} \quad E\ \text{ch}\ \lambda_e$$

It is interesting to determine the auxiliary parallels on which the scale is exactly half-way between the scale on the equator and the scale on the borders. Their latitude is given by :

$$\cos l_m = \sqrt{\cos l_e}$$

or again by :

$$l_m = \tan^{-1}\left(\sqrt{2}\ \text{sh}\,\frac{\lambda_e}{2}\right)$$

(*d*) For the Hydrographic Office's chart :

$$p = 0.96 \qquad q = 0.65 \qquad \mu = 53^\circ\!.6$$

we find :

equatorial scale : E = 1/6 226 069 ;
auxiliary latitude of the borders of the chart : $l_e = 17^\circ\!.8498 = 17^\circ 50'\!.9$;
scale on the chart borders : 1/5 926 364 ;
parallels of the intermediate scale : $l_m = 12^\circ\!.6738 = 12^\circ 40'\!.43$;
scale on these parallels : 1/6 074 368 .

Computation of spherical auxiliary coordinates

(*a*) In what follows we assume that the longitudes are counted positively towards the east. We shall call M_0 the longitude of the central point, and φ and M the geographic latitude and longitude of a point A on the sphere having geographic pole P (figure 4).

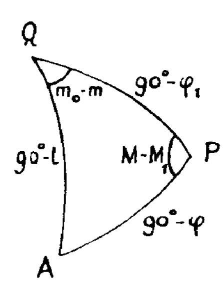

FIG. 4

To obtain auxiliary latitude l and auxiliary longitude m of this point, referred to the auxiliary equator and to the auxiliary meridian of the central point C, it is necessary to determine beforehand, on the one hand the geographic coordinates φ_1 and M_1 of the pole Q of the auxiliary equator, and on the other hand the auxiliary longitude m_0 of the geographic pole, a longitude which is moreover equal to the distance from the central point to the geographic vertex of the auxiliary equator (figure 5).

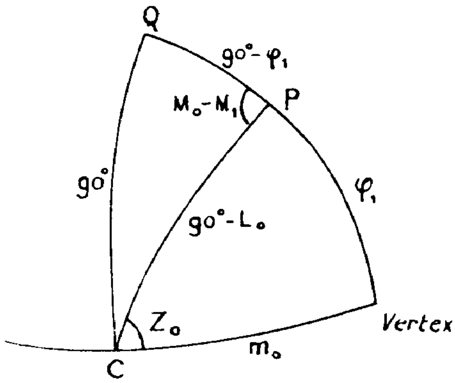

FIG. 5

Since the auxiliary pole Q is situated on the meridian of this vertex and at 90° from this point, the coordinates of the vertex are expressed by $90° - \varphi_1$ and $M_1 - 180°$, so that we may immediately obtain :

$$m_0 = \tan^{-1} (\cos Z_0 \cot L_0) \qquad (m_0 = 10^{\circ}\!.9087 = 10°54'\!.52)$$

$$M_1 = M_0 + \tan^{-1} \left(\frac{\cot Z_0}{\sin L_0}\right) + 180° \qquad (M_1 = 157^{\circ}\!.8158 = 157°48'\!.95)$$

$$\varphi_1 = \tan^{-1} (\sin m_0 \tan Z_0) \qquad (\varphi_1 = 43^{\circ}\!.9010 = 43°54'\!.06)$$

(*b*) The conversion to auxiliary coordinates is then made by resolving the spherical triangle QPA for which angle P and the two adjacent sides are known. The standard formulae of spherical trigonometry give :

$$\sin l = \sin \varphi \sin \varphi_1 + \cos \varphi \cos \varphi_1 \cos (M_1 - M)$$

$$\sin (m - m_0) = \frac{\cos \varphi \sin (M_1 - M)}{\cos l}$$

But to obtain l and m by trigonometric tangents the computation is a little more complicated; it is necessary to have recourse to the Neper formulae :

$$\tan \frac{A + Q}{2} = \frac{\sin \dfrac{\varphi - \varphi_1}{2}}{\cos \dfrac{\varphi + \varphi_1}{2}} \cot \frac{M - M_1}{2}$$

$$\tan \frac{A + Q}{2} = \frac{\cos \dfrac{\varphi - \varphi_1}{2}}{\sin \dfrac{\varphi + \varphi_1}{2}} \cot \frac{M - M_1}{2}$$

And, since $Q = m_0 - m$, by eliminating A we obtain for the auxiliary longitude :

$$m = m_0 + \tan^{-1}\left[\frac{\sin\frac{\varphi_1 - \varphi}{2}}{\cos\frac{\varphi_1 + \varphi}{2}} \cot\frac{M_1 - M}{2}\right]$$

$$+ \tan^{-1}\left[\frac{\cos\frac{\varphi_1 - \varphi}{2}}{\sin\frac{\varphi_1 + \varphi}{2}} \cot\frac{M_1 - M}{2}\right]$$

The auxiliary latitude l is computed in a similar way by using the formulae :

$$\tan\frac{l - \varphi}{2} = \frac{\sin\frac{1}{2}\,[(m_0 - m) - (M - M_1)]}{\sin\frac{1}{2}\,[(m_0 - m) + (M - M_1)]}\tan\left(45^\circ - \frac{\varphi_1}{2}\right)$$

$$\cot\frac{l - \varphi}{2} = \frac{\cos\frac{1}{2}\,[(m_0 - m) - (M - M_1)]}{\cos\frac{1}{2}\,[(m_0 - m) + (M - M_1)]}\tan\left(45^\circ - \frac{\varphi_1}{2}\right)$$

Whence, by the elimation of φ :

$$l = 90^\circ - \tan^{-1}\left[\frac{\cos\frac{1}{2}\,[(M_1 - M) - (m - m_0)]}{\cos\frac{1}{2}\,[(M_1 - M) + (m - m_0)]}\tan\left(45^\circ - \frac{\varphi_1}{2}\right)\right]$$

$$- \tan^{-1}\left[\frac{\sin\frac{1}{2}\,[(M_1 - M) - (m - m_0)]}{\sin\frac{1}{2}\,[(M_1 - M) + (m - m_0)]}\tan\left(45^\circ - \frac{\varphi_1}{2}\right)\right]$$

Computation of plane rectangular coordinates

The plane rectangular coordinates X and Y are expressed in millimetres and are referred to two axes of rectangular coordinates having point O, corresponding to the chart's central point C, as origin. The axis of the abscissae OX represents the auxiliary equator which at C makes an angle Z_0 with the meridian. Abscissae are counted parallel to this axis in the direction of increasing longitude, that is approximately towards the east. At point O the OY axis is perpendicular to OX and the ordinates are counted from the auxiliary equator in the direction of increasing latitudes, that is approximately towards the north.

We expressed a, consequently N, in metres and m in degrees. In consequence the Mercator projection formulae give :

$$X = 1\,000\,N_0\,E\,\frac{\pi}{180}\,m$$

$$Y = 1\,000\, N_0 \text{ E Log } \tan\left(45° + \frac{l}{2}\right)$$

But earlier on we wrote :

$$E = \frac{p}{N_0\, \Delta} = \frac{p}{N_0} \frac{180}{\mu\, \pi}$$

whence :

$$X = 1\,000\, p \frac{m}{\mu}$$

$$Y = \frac{180\,000}{\mu\,\pi} \text{ Log } \tan\left(45° + \frac{l}{2}\right)$$

If the chart's length p is also expressed in millimetres :

$$X = p \frac{m}{\mu}$$

$$Y = p \frac{180}{\mu\,\pi} \text{ Log } \tan\left(45° + \frac{l}{2}\right)$$

The accuracy of 0.1 millimetre is sufficient in the determination of X and Y.

Specific points

For the construction of the chart it is useful to determine the coordinates of a certain number of specific points; this is easily done by the standard computations of spherical trigonometry which need not be developed here.

We have already mentioned the geographical vertex of the auxiliary equator; its geographic coordinates on the sphere are :

$$\varphi_v = 90° - \varphi_1 \qquad M_v = M_1 - 180°$$

The places of maximum latitude on the upper and lower borders of the chart are situated on the same meridian as the vertex and have as respective latitudes :

$$\varphi_v + l_e \quad \text{and} \quad \varphi_v - l_e$$

The simplest way to obtain each of the east and west extremities of the central axis is to resolve the rectangular spherical triangle which has as its vertices the geographic pole, the aforementioned vertex and the extremity under consideration, this latter being separated from the vertex by an arc equal to $\frac{''}{2} \pm m_0$. The azimuth of the central axis at these points is obtained from these same triangles.

The geographic coordinates of the corners of the chart are determined with the help of spherical triangles defined by the geographic pole, the corner under consideration and the extremity of the central axis nearer to this corner. The value of the arc of great circle joining these is l_e. This arc is orthogonal to the central axis.