TIDAL PREDICTIONS
ON A MEDIUM SIZED DIGITAL COMPUTER

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INTRODUCTION

The University of Cape Town recently acquired an I.C.T. 1301 Computer, and the author undertook to write a Tidal Prediction Programme for the use of the Hydrographic Office of the South African Navy. The 1301 is not a very large or fast machine, and has an Immediate Access Store with storage locations for only 200 main variables, so that is would be unrealistic to expect it to turn out results at anything like the speed achieved by the Liverpool Tidal Institute, using an IBM 7094 or a KDF 9.

A further incentive for writing our own programme was that the 1301 has to be programmed in Manchester Auto-Code, and as far as we have been able to ascertain, tidal prediction programmes in this code have up to now not been available, although programmes have been written in the better-known Fortran and Algol source languages. Only a general description of the programme is given here, as anyone wishing to use it may obtain it on punched cards by applying direct to the author.

The programme in its present form can utilize up to fifty constituents, and could be modified to handle more. It turns out complete predictions for a port in approximately 75 minutes, as opposed to the roughly ten days needed on the analogue machine previously used. This includes manual operation of the predictor, plus typing of results and proof reading.

DESCRIPTION OF METHOD

Using the usual notation, the height of the sea at any instant is

$$Z(t) = Z_0 + \sum_{n=1}^{N} H_n f_n \cos \{S_n t + (V_0+U_0)_n - g_n\} \qquad (1)$$

and the problem to be solved is the determination of all the roots of the equation :

$$Z'(t) = -\sum_{n=1}^{N} S_n H_n f_n \sin \{S_n t + (V_0+U_0)_n - g_n\} \qquad (2)$$

The prime, of course, denotes differentiation with respect to time, and all the roots for a period of one year are required.

M. T. MURRAY, in his interesting account (Ref. 1) of his solution of equation (2), explains that to save machine time, he fed a table of cosines into the computer's immediate access store. However, on our machine, storage locations are simply not sufficient in number for this to be done, and the machine's built-in sub-routine was used instead. This was faster than storing the cosines on the magnetic drum and making numerous drum transfers. Similarly the use of repeated bisection of an interval containing a root, described by him, could not be used for the same reason. The method adopted was straightforward iteration based on Newton's method. It was found that a root could be located in most instances after three iterations, and in all instances after four iterations. We have :

$$Z'(t) = -\sum_{n=1}^{N} S_n f_n H_n \sin \{S_n t + (V_0+U_0)_n - g_n\} \tag{3}$$

$$Z''(t) = -\sum_{n=1}^{N} S_n^2 f_n H_n \cos \{S_n t + (V_0+U_0)_n - g_n\} \tag{4}$$

Let t_0 be an approximation to the smallest root of $Z(t) = 0$.
Then

$$t_0 - \frac{Z'(t_0)}{Z''(t_0)} \tag{5}$$

is a better approximation. Using (5) we set a loop which we jump out of when successive values of t_0 differ by a sufficiently small amount. (In this case 0.005 hour or 0.3 minute). Having obtained t_0, it is then substituted in equation (1) to obtain the corresponding height.

Initially a guess is made for the value of the time of the first low or high water and the true value found by iterating. In practice it was found that taking $t_0 = 0$ was good enough to find the correct value after three or four iterations.

Having found the first root, the next one is obtained by increasing the value of t_0 by six hours, this period being suitable for the consistently semi-diurnal nature of the tides around the Republic, and using the result as the initial point for a fresh cycle of iteration, culminating in the true value of t_1, the next root. The programme thus took the form of a loop within a loop, control jumping out of the inner loop when a root had been successfully computed, and out of the outer one when all the roots for the year had been found. The value of a root, when found, was the actual time in hours, from the time origin at the beginning of the year. The programme then converted this to time in hours and minutes on the particular day of the particular month concerned, and printed the resulting times and sea levels out with an " L " in the case of low water and an " H " in the case of high water. The format, of which an example is reproduced, gave one month's predictions per page. Since the transverse perforations on the computer paper were not far enough apart to accommodate conveniently a month's results between perforations, paper without transverse perforations was used.

TIDAL PREDICTIONS FOR KNYSNA - JANUARY 1966

TIME		Ht.	TIME		Ht.	TIME		Ht.	TIME		Ht.
4.10	L	1.9	10.57	H	4.2	17.26	L	2.0	23.27	H	3.7
5.26	L	1.8	12.06	H	4.5	18.34	L	1.7			
0.42	H	3.9	6.32	L	1.5	13.09	H	4.8	19.29	L	1.3
1.45	H	4.3	7.28	L	1.2	14.01	H	5.2	20.16	L	0.9
2.35	H	4.6	8.19	L	0.9	14.49	H	5.5	21.01	L	0.5
3.19	H	5.0	9.06	L	0.6	15.34	H	5.7	21.43	L	0.2
4.03	H	5.3	9.52	L	0.5	16.17	H	5.9	22.24	L	0.1
4.46	H	5.5	10.37	L	0.4	16.59	H	5.8	23.04	L	0.0
5.29	H	5.6	11.22	L	0.4	17.42	H	5.7	23.45	L	0.1
8.13	H	5.6	12.08	L	0.6	18.25	H	5.4			
0.25	L	0.3	6.57	H	5.5	12.56	L	0.9	19.09	H	5.0
1.10	L	0.6	7.45	H	5.3	13.43	L	1.2	19.56	H	4.6
1.58	L	0.9	8.37	H	5.0	14.50	L	1.6	20.51	H	4.2
2.55	L	1.3	9.39	H	4.7	16.05	L	1.8	22.01	H	3.9
4.07	L	1.6	10.52	H	4.5	17.29	L	1.9	23.28	H	3.7
5.31	L	1.8	12.12	H	4.5	18.46	L	1.7			
0.54	H	3.9	6.49	L	1.7	13.22	H	4.6	19.46	L	1.5
1.59	H	4.2	7.51	L	1.5	14.17	H	4.8	20.33	L	1.2
2.50	H	4.5	8.41	L	1.4	15.01	H	4.9	21.12	L	1.0
3.31	H	4.8	9.22	L	1.2	15.40	H	5.1	21.47	L	0.8
4.08	H	5.0	9.58	L	1.1	16.14	H	5.2	22.18	L	0.6
4.42	H	5.1	10.30	L	1.0	16.46	H	5.2	22.48	L	0.6
5.14	H	5.1	10.59	L	1.0	17.17	H	5.2	23.16	L	0.6
5.44	H	5.1	11.29	L	1.1	17.46	H	5.1	23.44	L	0.6
6.13	H	5.0	11.58	L	1.1	18.15	H	4.9			
0.12	L	0.8	6.42	H	4.9	12.30	L	1.3	18.45	H	4.7
0.43	L	1.0	7.14	H	4.8	13.06	L	1.4	19.18	H	4.4
1.17	L	1.2	7.51	H	4.6	13.50	L	1.7	19.58	H	4.1
1.59	L	1.4	8.39	H	4.4	14.49	L	1.9	20.52	H	3.8
2.56	L	1.7	9.44	H	4.3	16.14	L	2.0	22.15	H	3.6
4.21	L	1.9	11.14	H	4.3	17.54	L	1.9			

We hope that in due course we will be able to photograph the computer sheets and reproduce them by a photo-litho process. At the moment, the standard of the print-out leaves a certain amount to be desired, and we may have to resort to off-line printing.

The programme has now been extensively tested and consistently comes up with the right answer, so that we feel confident that it has been successfully " de-bugged ".

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