THE SEMI-GRAPHIC METHOD OF ANALYSIS FOR 7 DAYS OF TIDAL OBSERVATIONS

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1. — INTRODUCTION

In the *Admiralty Tidal Handbook No. 3* Commander Suthons has just developed a semi-graphic method of harmonic analysis of tides for observation periods as short as 24 hours.

The author considers one week's hourly observations as in fact a set of groups of 24-hour observations whose analysis requires that the relations between the principal constituents for neighboring stations be known. The mean of the values for H and g found through these 24-hour analyses is then taken as the best answer obtainable.

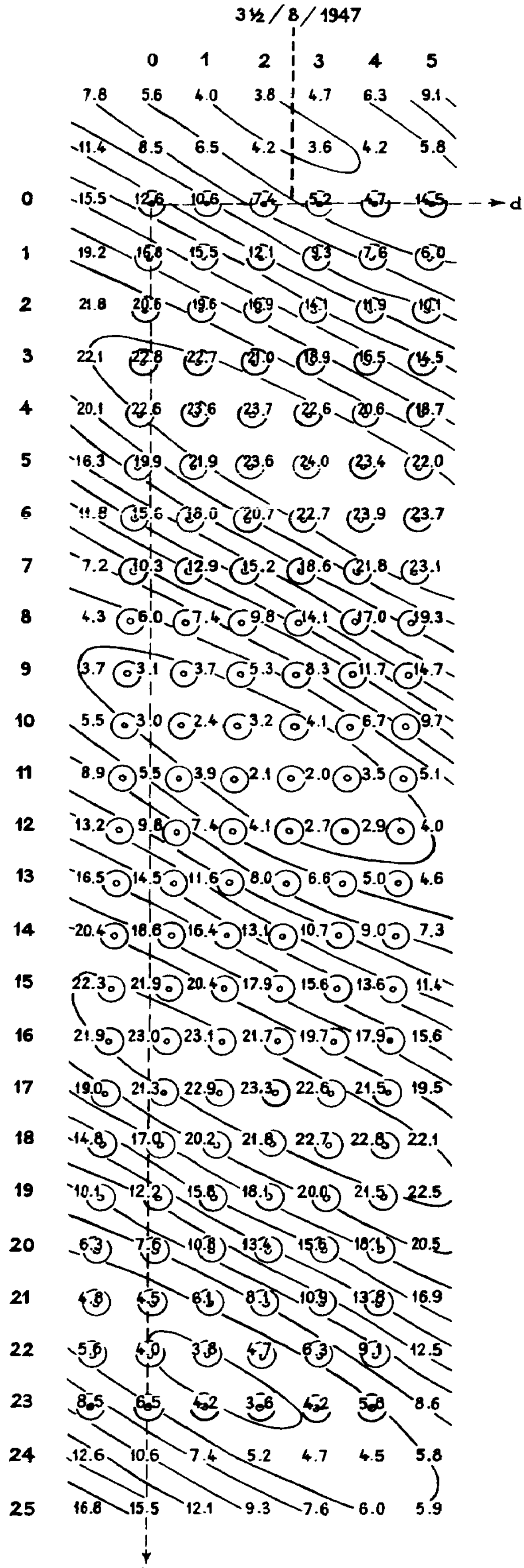
I believe, however, that if a continuous 7-day observation recording is available, it is better to analyse it directly, reducing to the utmost the number of hypotheses to be admitted. With this in mind I have developed another method of analysis, based also on the tidal surface, but having an entirely general character since the hypotheses to be accepted are the same as those which allowed me to establish the method I described in the *International Hydrographic Review,* July 1964 (Vol. XLI, No. 2).

Taking advantage of the properties of the tidal surface, I will show that the computations are simpler than those of the numerical method.

2. — THE DAILY PROCESS

If we take seven days of hourly observations, it will be possible to cut the tidal surface by six planes as shown in fig. 2 A. 24 points chosen along each of these sections, which we shall call "columns", will then give us 144 observations to analyse.

In my article in the IHR (July 1963) the expression ($2j$) represents the intersection of plane ($2g$) with the tidal surface assumed to be generated by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0 1 2 3 4 5
7.8 5.6 4.0 3.8 4.7 6.3 9.1
11.4 8.5 6.5 4.2 3.6 4.2 5.8
0 15.5 12.6 10.6 7.4 5.2 4.7 4.5
d
1 19.2 16.8 15.5 12.1 9.3 7.6 6.0
2 21.8 20.6 19.6 16.9 14.1 11.9 10.1
3 22.1 22.8 22.7 21.0 18.9 16.5 14.5
4 20.1 22.6 23.6 23.7 22.6 20.6 18.7
5 16.3 19.9 21.9 23.6 24.0 23.4 22.0
6 11.8 15.6 18.0 20.7 22.7 23.9 23.7
7 7.2 10.3 12.9 15.2 18.6 21.8 23.1
8 4.3 6.0 7.4 9.8 14.1 17.0 19.3
9 3.7 3.1 3.7 5.3 8.3 11.7 14.7
10 5.5 3.0 2.4 3.2 4.1 6.7 9.7
11 8.9 5.5 3.9 2.1 2.0 3.5 5.1
12 13.2 9.8 7.4 4.1 2.7 2.9 4.0
13 16.5 14.5 11.6 8.0 6.6 5.0 4.6
14 20.4 18.6 16.4 13.1 10.7 9.0 7.3
15 22.3 21.9 20.4 17.9 15.6 13.6 11.4
16 21.9 23.0 23.1 21.7 19.7 17.9 15.6
17 19.0 21.3 22.9 23.3 22.6 21.5 19.5
18 14.8 17.0 20.2 21.8 22.7 22.8 22.1
19 10.1 12.2 15.8 18.1 20.0 21.5 22.5
20 6.3 7.9 10.8 13.4 15.6 18.1 20.5
21 4.8 4.5 6.1 8.1 10.9 13.8 16.9
22 5.6 4.0 3.8 4.7 6.3 9.1 12.5
23 8.5 6.5 4.2 3.6 4.2 5.8 8.6
24 12.6 10.6 7.4 5.2 4.7 4.5 5.8
25 16.8 15.5 12.1 9.3 7.6 6.0 5.9
t

FIG. 2A

one single constituent. But that article showed that if the sections are chosen to satisfy the condition

$$a/b = 1/24$$

we shall have

$$q - \rho\, a/b = 15^\circ\, n$$

n being the constituent's suffix. That being the case, if S_0 is considered a constituent for which $q = \rho = r = 0$, the contributions from all the constituents whose suffix is n could be written in the form

$$y'_n = \sum_{cn} R \cos (15^\circ\, nt + \rho\, c/b - r_0)$$

If the distance between the columns, measured along the d axis, equals one day, we will have $c/b = d$, d being the number of days from a suitably chosen origin. Thus the preceding expression may be written in the form

$$y'_n = \sum_{cn} R \cos (15^\circ\, nt + \rho\, d - r_0)$$

or, by developing,

$$y'_n = \sum_{cn} R \cos (\rho\, d - r_0) \cos 15^\circ\, nt$$
$$- \sum_{cn} R\, \mathit{sin}\, (\rho\, d - r_0) \sin 15^\circ\, nt$$

or, by putting

$$\sum_{cn} R \cos (\rho\, d - r_0) = X'_n \qquad (2a)$$

and

$$- \sum_{cn} R \sin (\rho\, d - r_0) = Y'_n \qquad (2b)$$

we will have

$$y'_n = X'_n \cos 15^\circ\, nt + Y'_n \sin 15^\circ\, nt$$

Consequently, if we consider all the constituents, the height y of the tidal surface at hour t will be expressed by

$$y = \sum_{c} (X'_n \cos 15^\circ\, nt + Y'_n \sin 15^\circ\, nt) \quad (n = 0, 1, 2, 4) \qquad (2c)$$

If we assign y suffixes indicating the hours corresponding to the heights y, we shall see that expression ($2c$) can be expressed in matrix form by the redundant system in table 2-I. If we call M the matrix of this system and $\{y\}$ and $\{Z\}$ the column vectors of the known and unknown terms respectively, the system of normal equations will be expressed by

$$M^T \{y\} = M^T . M . \{Z\} \qquad (2d)$$

But the product of each row vector of M^T and the column vectors of M will always be expressed by one of the four following expressions :

$$\sum_{t=0}^{23} \cos 15^\circ\, nt \cos 15^\circ\, n't = \frac{1}{2} \sum_{t=0}^{23} [\cos 15^\circ\, (n+n')t + \cos 15^\circ\, (n-n')t] \qquad (2e)$$

$$\sum_{t=0}^{23} \cos 15^\circ\, nt \sin 15^\circ\, n't = \frac{1}{2} \sum_{t=0}^{23} [\sin 15^\circ\, (n+n')t - \sin 15^\circ\, (n-n')t] \qquad (2f)$$

$$\sum_{t=0}^{23} \sin 15^\circ\, nt \cos 15^\circ\, n't = \frac{1}{2} \sum_{t=0}^{23} [\sin 15^\circ\, (n+n')t + \sin 15^\circ\, (n-n')t] \qquad (2g)$$

$$- \sum_{t=0}^{23} \sin 15^\circ\, nt \sin 15^\circ\, n't = \frac{1}{2} \sum_{t=0}^{23} [\cos 15^\circ\, (n+n')t - \cos 15^\circ\, (n-n')t] \qquad (2h)$$

where n' allows us to identify the rows (0, 1, 2, 4) of the transposed matrix.

TABLE 2-I

		n = 0	n = 1		n = 2		n = 4		
		cos 0°	cos 15°t	sin 15°t	cos 30°t	sin 30°t	cos 60°t	sin 60°t	
y_0		1	1,000	0,000	1,000	0,000	1,000	0,000	
y_1		1	0,966	0,259	0,866	0,500	0,500	0,866	X'_0
y_2		1	0,866	0,500	0,500	0,866	−0,500	0,866	
y_3		1	0,707	0,707	0,000	1,000	−1,000	0,000	X'_1
y_4		1	0,500	0,866	−0,500	0,866	−0,500	−0,866	
y_5		1	0,259	0,966	−0,866	0,500	0,500	−0,866	Y'_1
y_6		1	0,000	1,000	−1,000	0,000	1,000	0,000	
y_7	=	1	−0,259	0,966	−0,866	−0,500	0,500	0,866	X'_2
y_8		1	−0,500	0,866	−0,500	−0,866	−0,500	0,866	
y_9		1	−0,707	0,707	0,000	−1,000	−1,000	0,000	Y'_2
y_{10}		1	−0,866	0,500	0,500	−0,866	−0,500	−0,866	
y_{11}		1	−0,966	0,259	0,866	−0,500	0,500	−0,866	X'_4
y_{12}		(a)	(b)	(b)	(a)	(a)	(a)	(a)	
à to y_{23}		De t = 12 à t = 23 les valeurs se répètent :				(a) avec même signe (b) avec signes contraires			Y'_4

From t = 12 to t = 23 the values are repeated : (a) with the same sign (b) with contrary signs

Applying the general expression

$$\sum_{t=0}^{T} \cos(a+15°\,Nt) = \frac{\sin 15°\,(T+1)\,N/2}{\sin 15°\,N/2} \cos(a+15°\,NT)$$

to the second members of expressions (2*e*) through (2*h*), it is easy to conclude that the results of the sums for $T = 23$ and $N = n \pm n' \neq 0$ are equal to zero, but if $n = n'$, $N = n - n' = 0$, and we shall have :

$$\sum_{t=0}^{23} \cos(a+15°\,Nt) = 24 \cos a$$

Under these circumstances expressions (2*f*) and (2*g*), where $a = \pm 90°$, will always be zero, while expressions (2*e*) and (2*h*), where $a = 0°$, will equal 12. Then noting that expressions (2*e*) and (2*h*) for $n = n'$ correspond to the diagonal of matrix $M^T \cdot M$, we logically deduce that the product is a scalar matrix whose expression is

$$M^T \cdot M = 12\,U$$

where U is the unit matrix. Expression (2*d*) may then take the form

$$2M^T \cdot \{\,y\,\} = 24\,U\,\{Z\} \qquad (2i)$$

This expression represents the rigorous solution to the problem and, as expression (2*i*) shows, this solution requires multiplying the column vector made up of the values of *y* by matrix $2M^T$ whose elements are 2 cos 15°*nt* and 2 sin 15°*nt*. Under these circumstances we would have to make quite lengthy computations with the help of a standard desk computer. Thus we must look for a less rigorous solution but one whose results are acceptable in practice and whose computations are not very lengthy. A reasonably satisfying solution is found by replacing matrix $2M^T$ by another matrix whose elements will be 0, ± 1 and ± 2, corresponding to the best approximations of the values of 2 cos 15°nt and 2 sin 15°nt. If we call this new matrix K^T, we would normally find matrix K by rounding off values of 2 cos 15°nt and 2 sin 15°nt taken from matrix 2M. Table 2-II comprises the elements of matrix K where the column vectors corresponding to $n = 0$ have not been multiplied by two. Actually this duplication was not necessary since all the cosines have, in this case, the value $+1$ and all the sines the value 0. In table 2-II we see in addition that symbols s_n and c_n have been used to designate the column vectors derived respectively from the values of the sines and of the cosines.

TABLE 2-II

$\{c_0\}$	$\{c_1\}$	$\{s_1\}$	$\{c_2\}$	$\{s_2\}$	$\{c_4\}$	$\{s_4\}$
1	2	0	2	0	2	0
1	2	1	2	1	1	2
1	2	1	1	2	—1	2
1	1	1	0	2	—2	0
1	1	2	—1	2	—1	—2
1	1	2	—2	1	1	—2
1	0	2	—2	0	2	0
1	—1	2	—2	—1	1	2
1	—1	2	—1	—2	—1	2
1	—1	1	0	—2	—2	0
1	—2	1	1	—2	—1	—2
1	—2	1	2	—1	1	—2
1	—2	0	2	0	2	0
1	—2	—1	2	1	1	2
1	—2	—1	1	2	—1	2
1	—1	—1	0	2	—2	0
1	—1	—2	—1	2	—1	—2
1	—1	—2	—2	1	1	—2
1	0	—2	—2	0	2	0
1	1	—2	—2	—1	1	2
1	1	—2	—1	—2	—1	2
1	1	—1	0	—2	—2	0
1	2	—1	1	—2	—1	—2
1	2	—1	2	—1	1	—2

By writing the system in table 2-I in the form

$$\{y\} = M \cdot \{Z\}$$

we can write

$$K^T \cdot \{y\} = K^T \cdot M \{Z\}$$

Introducing into this expression the values for the elements of matrix K (table 2-II) and those for matrix M (table 2-I) we obtain the following final system :

$$\begin{Bmatrix} \{c_0\}^T \cdot \{y\} \\ \{c_1\}^T \cdot \{y\} \\ \{s_1\}^T \cdot \{y\} \\ \{c_2\}^T \cdot \{y\} \\ \{s_2\}^T \cdot \{y\} \\ \{c_4\}^T \cdot \{y\} \\ \{s_4\}^T \cdot \{y\} \end{Bmatrix} = \begin{Vmatrix} 24.000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 24.250 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24.250 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 25.856 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25.856 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 24.000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 27.712 \end{Vmatrix} \begin{Bmatrix} X'_0 \\ X'_1 \\ Y'_1 \\ X'_2 \\ Y'_2 \\ X'_4 \\ Y'_4 \end{Bmatrix} \quad (2j)$$

This expression shows that we have arrived in the end at a system of equations whose matrix is diagonal, which means that the use of the table 2-II multipliers permits us to isolate completely the groups with the same suffix. If we had also considered the third-diurnal and sixth-diurnal species, we would have seen that they also would have been isolated.

Designating by C any one of the elements of the diagonal matrix we can express the equations in (2*j*) by :

$$\{c_n\}^{\mathrm{T}} \cdot \{y\} = \mathrm{C}\mathrm{X}'_n \tag{2k}$$

or

$$\{s_n\}^{\mathrm{T}} \cdot \{y\} = \mathrm{C}\mathrm{Y}'_n \tag{2l}$$

as the case may be. But if we make

$$\{c_n\}^{\mathrm{T}} \cdot \{y\} = \mathrm{X}_n \tag{2m}$$

and

$$\{s_n\}^{\mathrm{T}} \cdot \{y\} = \mathrm{Y}_n \tag{2n}$$

(2*k*) and (2*l*) will become

$$\mathrm{X}'_n = \mathrm{X}_n/\mathrm{C} \tag{2o}$$

and

$$\mathrm{Y}'_n = \mathrm{Y}_n/\mathrm{C} \tag{2p}$$

Expression (2*j*) also shows that, except for X'_4 and Y'_4, the values of C are equal for each pair of functions X'_n and Y'_n.

At this point it will be easy to explain the operations in the daily process. We first take the values of y in fig. 2 A and with them construct the matrix seen under (*a*) in the form given at the end of this article. We perform the operations indicated in expressions (2*m*) and (2*n*) using the multipliers in table 2-II and record the results under (*b*) of the form mentioned. The daily process is then at an end.

3. — THE MONTHLY PROCESS

As in the development of the numerical method of harmonic analysis for seven days of hourly observations, we will use the term monthly process for the combining of daily values of X_n and Y_n, the first step in isolating the constituents.

From expressions (2*a*), (2*b*), (2*o*) and (2*p*) we can write :

$$\mathrm{X}_n = \mathrm{C} \sum_{cn} \mathrm{R} \cos (\rho d - r_0)$$

and

$$\mathrm{Y}_n = - \mathrm{C} \sum_{cn} \mathrm{R} \sin (\rho d - r_0)$$

If we take as initial day ($d = 0$) the day which corresponds to the intersection of the first "column" (fig. 2 A) with the d axis, the value of the central day will be 2.5 and the preceding expressions can be transformed into

$$\mathrm{X}_n = \mathrm{C} \sum_{cn} \mathrm{R} \cos [\rho (d - 2.5) + 2.5 \rho - r_0]$$

and

$$\mathrm{Y}_n = - \mathrm{C} \sum_{cn} \mathrm{R} \sin [\rho (d - 2.5) + 2.5 \rho - r_0]$$

or, by making

$$-r_0 + 2{,}5\,\rho = -r \tag{3a}$$

and by developing, we obtain

$$X_n = C \sum_{cn} [R \cos r \cos \rho (d - 2.5) + R \sin r \sin \rho (d - 2.5)]$$

and

$$Y_n = -C \sum_{cn} [R \cos r \sin \rho (d - 2.5) - R \sin r \cos \rho (d - 2.5)]$$

These two expressions may be written in the general form :

$$F_n = C \sum_{cn} [\alpha \cos \rho (d - 2.5) + \beta \sin \rho (d - 2.5)] \tag{3b}$$

where

$$\alpha = R \cos r \tag{3c}$$

and

$$\beta = R \sin r \tag{3d}$$

for $F_n = X_n$, and

$$\alpha = R \sin r \tag{3e}$$

and

$$\beta = -R \cos r \tag{3f}$$

for $F_n = Y_n$.

As in the numerical method for seven-day analysis (see IHR July 1964), we have in the present case daily values of F_n to combine. But the six values we have here have only the contributions of the constituents of suffix n. This allows us to consider each species separately which considerably facilitates the isolation of each constituent. Let us take for instance the diurnal constituents K_1 and O_1, taking into account also the disturbing effect of Q_1 on these constituents. By introducing into expression (3*b*) the ρ values for constituents K_1, O_1 and Q_1 and the successive values $d = 0$, 1, 2, 3, 4, 5, we shall have a group of expressions which can be written as the following single matrix :

$$\frac{1}{C}\{F_1\} = \left\| \begin{array}{ccc} 0.999 & 0.448 & \vdots\ 0.106 \\ 1.000 & 0.787 & \vdots\ 0.536 \\ 1.000 & 0.796 & \vdots\ 0.944 \\ \multicolumn{3}{c}{\text{These three lines}} \\ \multicolumn{3}{c}{\text{are repeated in}} \\ \multicolumn{3}{c}{\text{inverse order and}} \\ \multicolumn{3}{c}{\text{with the same sign}} \end{array} \right\| \begin{Bmatrix} \alpha_{K_1} \\ \alpha_{O_1} \\ \cdots \\ \alpha_{Q_1} \end{Bmatrix} + \left\| \begin{array}{ccc} -0.044 & 0.894 & \vdots\ 0.994 \\ -0.026 & 0.617 & \vdots\ 0.844 \\ -0.009 & 0.220 & \vdots\ 0.329 \\ \multicolumn{3}{c}{\text{These three lines}} \\ \multicolumn{3}{c}{\text{are repeated in}} \\ \multicolumn{3}{c}{\text{inverse order and}} \\ \multicolumn{3}{c}{\text{with the opposite sign}} \end{array} \right\| \begin{Bmatrix} \beta_{K_1} \\ \beta_{O_1} \\ \cdots \\ \beta_{Q_1} \end{Bmatrix} \tag{3g}$$

In the matrix which multiplies column vector $\{\alpha\}$ the lines are the values of $\cos \rho\, (d - 2.5)$ from $d = 0$ to $d = 5$, while the lines of the matrix which multiplies column vector $\{\beta\}$ are the values of $\sin \rho\, (d - 2.5)$ from $d = 0$ to $d = 5$. We are now in a position to find the multipliers ± 1 whose signs are those of $\cos \rho_{K_1} (d - 2.5)$ for combination $\{0\}$ of table 3-I, and those of $\sin \rho_{O1}\, (d - 2.5)$ for combination $\{b\}$ of this table, as we have already explained for the numerical method (see IHR, July 1964). These same multipliers will be used to isolate the semi-diurnal constituents. As for isolating the quarter-diurnal constituents, it has been necessary to try several filters, the most effective being that indicated by $\{2\}$ in table 3-I. In this special case these multipliers have been established by using the signs and values of $\cos \rho_{MS4}\, (d - 2.5)$.

TABLE 3-I

Daily multipliers

d	$\{0\}$	$\{b\}$	$\{2\}$
0	1	1	1
1	1	1	2
2	1	1	1
3	1	—1	1
4	1	—1	2
5	1	—1	1

In practice the products

$$\begin{array}{lll} \{0\}^T \cdot \{X_0\} = X_{00} & & \\ \{0\}^T \cdot \{X_1\} = X_{10} & \{b\}^T \cdot \{Y_1\} = Y_{1b} & \\ \{b\}^T \cdot \{X_1\} = X_{1b} & \{0\}^T \cdot \{Y_1\} = Y_{10} & \\ \{0\}^T \cdot \{X_2\} = X_{20} & \{b\}^T \cdot \{Y_2\} = Y_{2b} & (3h) \\ \{b\}^T \cdot \{X_2\} = X_{2b} & \{0\}^T \cdot \{Y_2\} = Y_{20} & \\ \{2\}^T \cdot \{X_4\} = X_{42} & \{b\}^T \cdot \{Y_4\} = Y_{4b} & \\ \{b\}^T \cdot \{X_4\} = X_{4b} & \{2\}^T \cdot \{Y_4\} = Y_{42} & \end{array}$$

would be arrived at by using Table 3-I and the values of X_n and Y_n taken from the form under (*b*). The results, inscribed under (*c*), are already the known terms of the equations to be solved.

4. — FORMATION AND SOLUTION OF THE EQUATIONS

Let us continue with the example of isolating constituents K_1 and O_1. By pre-multiplying (3*g*) by $\{0\}^T$ and $\{b\}^T$ we have :

$$\frac{1}{C} \{0\}^T \cdot \{F_1\} = (5.998 \quad 4.062 \;\vdots\; 3.162) \{\alpha\} \qquad (4a)$$

and

$$\frac{1}{C} \{b\}^T \cdot \{F_1\} = (—0.158 \quad 3.462 \;\vdots\; 4.334) \{\beta\} \qquad (4b)$$

If in the first of these equations we replace F_1 by X_1 and by Y_1 in the second equation, then from (3*c*), (3*f*) and (3*h*) we obtain

$$\frac{1}{C} X_{10} = (\; 5.998 \quad 4.062 \;\vdots\; 3.162) \{R \cos r\} \qquad (4c)$$

and

$$—\frac{1}{C} Y_{1b} = (—0.158 \quad 3.462 \;\vdots\; 4.334) \{R \cos r\} \qquad (4d)$$

Now making $F_1 = Y_1$ in (4*a*) and $F_1 = X_1$ in (4*b*) we will have, from (3*d*), (3*e*) and (3*h*) :

$$\frac{1}{C} Y_{10} = (5.998 \quad 4.062 \;\vdots\; 3.162) \{R \sin r\} \qquad (4e)$$

and

$$\frac{1}{C} X_{1b} = (—0.158 \quad 3.462 \;\vdots\; 4.334) \{R \sin r\} \qquad (4f)$$

Thus if we designate by l'/C the first member of either (4*c*) or (4*e*), and by l''/C the first member of either (4*d*) or (4*f*), and if further we designate by z', z'' and z''' either the elements of $\{ R \cos r \}$, or those of $\{ R \sin r \}$, we can write the general system of equations :

$$\frac{1}{C}\begin{Bmatrix} l' \\ l'' \end{Bmatrix} = \left\| \begin{array}{cc:c} 5.998 & 4.062 & 3.162 \\ -0.158 & 3.462 & 4.334 \end{array} \right\| \begin{Bmatrix} z' \\ z'' \\ z''' \end{Bmatrix}$$

C being the diagonal element of matrix (2*j*) which corresponds to X_1 and which is equal to the one corresponding to Y_1. Since its value is 24.250 we can write the preceding equations in the following way :

$$\frac{1}{24.250}\begin{Bmatrix} l' \\ l'' \end{Bmatrix} - z''' \begin{Bmatrix} 3.162 \\ 4.334 \end{Bmatrix} = \left\| \begin{array}{cc} 5.998 & 4.062 \\ -0.158 & 3.462 \end{array} \right\| \begin{Bmatrix} z' \\ z'' \end{Bmatrix}$$

Multiplying both members of this expression by the inverse matrix of the second member and transposing, we obtain :

$$\begin{Bmatrix} z' \\ z'' \end{Bmatrix} = 10^{-6} \left\| \begin{array}{cc} 6\,670 & -7\,825 \\ 304 & 11\,555 \end{array} \right\| \begin{Bmatrix} l' \\ l'' \end{Bmatrix} + z''' \begin{Bmatrix} 0.309 \\ -1.328 \end{Bmatrix} \qquad (4g)$$

In the form under (*d*) we again find the figures for the matrix of the first term of the second member of this expression which is used for isolating K_1 and O_1. It is then easy to verify that we have $l' = X_{10}$ and $l'' = -Y_{1b}$ for the computation of the values of R cos *r*, and that $l' = Y_{10}$ and $l'' = X_{1b}$ when we are dealing with the R sin *r* computation. The values found by using these expressions in the form give us the provisional values $\overline{R \cos r}$ and $\overline{R \sin r}$ which are recorded under (*g*). For constituent Q_1 they must be corrected for the second term of the second member of (4*g*) where z''' equals $R' \cos r'$ or $R' \sin r'$, as the case may be. My description of the numerical method already mentioned gives the theory which allows us to obtain the approximate values of z''' and we find under (*e*) and (*f*) in the form the details of this computation. Having found the values of $R' \cos r'$ and $R' \sin r'$ we compute the corrections to $\overline{R \cos r}$ and $\overline{R \sin r}$ by the formula under (*h*). These corrections are procured by following (4*g*).

The problem is resolved in exactly the same way for the semi-diurnal constituents as for the diurnal ones. Thus we need only clarify the isolation process of the quarter-diurnal constituents. Here we are considering only constituents M_4 and MS_4, and that being so, it is not necessary to take the corrections into account. In addition we must note that according to expression (2*j*) the values of the diagonal elements corresponding to X_4 and Y_4 are different. Consequently when we form expressions similar to (4*c*) and (4*f*), the values of C will be different and we will not have the same repetition of figures in the formulae under (*d*) for M_4 and MS_4.

Having found the values for R cos *r* and R sin *r* for the chosen constituents, the rest of the computation presents no particular interest. However one must note the fact that the central day is expressed by a fractional number and that the decimal part of this fraction is introduced into the computation of *g* in the form 0.5 ρ. As a matter of fact the quantity 6°.5 which is seen under (*e*) when σ is being computed derives also from the multiplication of the fraction of a day by the difference in daily speed of M_2 and N_2.

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(a)

t \ d	0	1	2	3	4	5
0	126	106	74	52	47	45
1	168	155	121	93	76	60
2	207	198	170	142	120	102
3	228	228	212	192	168	147
4	223	234	238	230	209	190
5	190	216	234	238	238	223
6	142	172	200	220	239	239
7	90	120	147	180	204	230
8	50	70	94	122	160	186
9	33	34	46	68	99	140
10	42	27	28	37	57	82
11	73	46	30	20	27	47
12	115	83	58	34	28	32
13	152	132	100	74	60	50
14	196	175	153	121	100	80
15	222	210	199	172	146	124
16	223	233	225	210	190	170
17	200	218	232	232	224	208
18	153	182	204	220	228	226
19	105	130	162	184	204	220
20	65	81	110	136	160	184
21	47	48	63	83	113	140
22	54	40	38	47	65	93
23	85	65	42	36	42	58

(b)

X_0	X_1	Y_1	X_2	Y_2	X_4	Y_4
3 189	164	-107	- 384	2 373	15	-114
3 203	254	- 46	- 1 080	2 316	70	- 36
3 180	228	- 28	- 1 786	1 942	44	- 20
3 143	247	35	- 2 281	1 365	34	4
3 204	225	115	- 2 491	679	46	64
3 276	180	153	- 2 480	- 38	- 9	82

(c)

	00	10	1b	20	2b	42	4b
X	19 195	1 298	- 6	-10 502	4 002	316	58
Y	–	-484	122	4 625	8 637	-320	8
		1b	10	2b	20	4b	42

(d)

	$10^6\ \overline{R\ \cos\ r}$	$10^6\ \overline{R\ \sin\ r}$
K_1	$6\,670X_{10} + 7\,825Y_{1b}$	$-\ 7\,825X_{1b} + 6\,670Y_{10}$
O_1	$304X_{10} - 11\,555Y_{1b}$	$11\,555X_{1b} + 304Y_{10}$
M_2	$-\ 11\,496Y_{2b}$	$11\,496X_{2b}$
S_2	$6\,445X_{20} + 8\,679Y_{2b}$	$-\ 8\,679X_{2b} + 6\,445Y_{20}$
M_4	$-6\,758X_{42} - 10\,675Y_{4b}$	$12\,327X_{4b} - 5\,852Y_{42}$
MS_4	$8\,916X_{42} + 3\,357Y_{4b}$	$-\ 3\,877X_{4b} + 7\,721Y_{42}$

(e)

$V_{M2} = 335°8$
$-V_{S2} = -123,6$
$-w_{M2} = -\ \ 5,9$
$6°5$
$\sigma = \overline{212,8}$

$(g_{O_1} - g_{O_1})^* =$
$\sigma =$
$\theta_1 = \overline{212,8}$

$(g_{S2} - g_{M2})^* =$
$\sigma =$
$\theta_2 = \overline{212,8}$

$c_1 = k_1^*(1+W)_{M2} = 0,221$ $c_2 = k_2^*(1+W)_{M2} = 0,221$

(*) Si nous n'avons pas d'autres renseignements - If no other information exists : $g_{Q1} - g_{O1} = g_{S2} - g_{M2} = 0$; $k_1 = k_2 = 0{,}193$

(f)

	$\bar{R}^2$	$\bar{R}$	$R' = c\ \bar{R}$	$\tan \bar{r}$	$\bar{r}$	$r' = r + \theta$	$\cos\ r'$	$\sin\ r'$	$R'\ \cos\ r'$	$R'\ \sin\ r'$
O_1	37,83	6,2	1,37	0,235	13,2	226,0	-0,695	-0,719	- 0,952	-0,985
M_2	4 943,59	70,3	15,54	-0,865	139,1	351,9	0,990	-0,141	15,385	-2,191

(g)

	m	$\overline{R\ \cos\ r}$	$m\ R'\ \cos\ r'$	$R\ \cos\ r$	$\overline{R\ \sin\ r}$	$m'\ R'\ \sin\ r$	$R\ \sin\ r$	R^2	$\tan\ r$
K_1	0,309	4,870	- 0,294	4,576	0,861	-0,304	0,557	21,25	0,122
O_1	-1,238	5,987	1,179	7,166	-0,032	1,219	1,187	52,05	0,166
M_2	-1,272	-53,169	-19,570	-72,739	46,007	-2,787	43,220	7158,93	-0,594
S_2	0,436	-27,545	6,708	-20,837	20,932	-0,955	19,977	833,26	-0,959
M_4	...	...	...	1,280	...	...	0,668	2,08	0,522
MS_4	...	...	...	1,743	...	...	0,163	3,06	0,094

(h)

	S_0	K_1	O_1	M_2	S_2	M_4	MS_4	N_2
V 1/1	...	9,8	143,7	153,5	...	...	...	217,2
Corr. mois	...	209,3	22,2	231,1	...	...	...	341,3
Corr. jour	...	2,0	309,3	311,2	...	...	...	285,1
	...	221,1	115,2	335,8	...	311,6	335,8	123,6
u	...	7,0	8,1	- 1,8	...	- 3,6	- 1,8	
w	...	-15,5	...	...	-17,4	...	-17,4	
0,5 ρ	...	0,5	-11,4	-12,2	0,0	-24,4	-12,2	
r	...	7,0	9,4	149,3	136,2	27,6	5,4	
g	...	206,1	121,3	111,1	118,8	311,2	309,8	
f	...	1,069	1,111	0,981	...	0,962	0,981	
1 + W	...	1,116	...	...	...	...	0,862	
f (1 + W)	...	1,193	1,111	0,981	0,862	0,962	0,846	
R	...	4,6	7,2	84,6	28,9	1,4	1,7	
H	...	4	6	86	34	1	2	

$(2V + u)_{K1} = 76°6$
$(V + u)_{K1} = 214,8$
$3V_{M2} - 2V_{S2} = 40,2$
$f_{K1} = 1,069$
$f_{K2} = 1,170$

K_1 : $wf_{K1} = -16°6$ $Wf_{K1} = 0,124$
$w = -15,5$ $W = 0,116$

S_2 : $w/f_{K2} = -14°9$ $W/f_{K2} = -0,118$
$w = -17,4$ $W = -0,138$

N_2 : $w = 5°9$ $1 + W = 1,147$