ERRORS IN THE DETERMINATION OF DEPTH

BY PRESSURE GAUGES UTILIZING A LINEAR PRESSURE-DEPTH RELATIONSHIP

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Although depth is a lineal measurement, it has been found feasible to determine depth by taking advantage of the relationship between hydrostatic pressure and depth. At present this determination is accomplished by measuring pressure and converting it to a depth value using a constant pressure-depth relationship. To calculate depth with greater precision, however, the non-linear pressure-depth relationship must be considered.

Present day oceanographic techniques require measurements to be made to the bottom of the sea floor, necessitating instrumentation with a pressure measuring capability of about 1 055 kg/cm^2.

In this paper the error involved in calibrating pressure gauges with a depth constant of 0.10244 kg/cm^2 per metre is examined, and corrections for various oceans are calculated.

The increase in hydrostatic pressure with depth in the sea varies from about 0.1019 to 0.1034 kg/cm^2 per metre. Pressure measuring instrumentation such as strain gauges and vibrating strings used by the U.S. Naval Oceanographic Office are calibrated at 0.10244 kg/cm^2 per metre of depth. In other words, for every pressure increase of 0.10244 kg/cm^2 on the gauge, one metre of depth is assumed.

The corrections calculated in this paper are based upon density values found in the U.S. Naval Oceanographic Office publication H.O. 607 (Instruction Manual for Oceanographic Observations), Table 17, Mean Density of Sea Water Column above Estimated Depth.

A table of mean pressure per metre of depth from the surface to the indicated depth for various oceans (Table I) was constructed, utilizing these mean density values in the hydrostatic equation :

$$P = pgh$$

where P is the pressure exerted by water of constant density p, and of height h, at a nominal gravity value of g. The value of g used in calibrating instruments is 980.665. In nature at sea level g varies from 978.0 at the equator to 983.2 at the poles. Thus an additional uncertainty of up to perhaps 12 metres in 5000 can be introduced, depending on whether a correct-

TABLE I

MEAN PRESSURE
(kg/cm²)
PER METRE OF DEPTH BELOW SURFACE

Depth (metres)	Mediterranean	Arctic & Antarctic	Northeast Pacific	North Atlantic
0	0.10282	0.10279	0.10246	0.10262
100	0.10286	0.10281	0.10248	0.10264
200	0.10289	0.10283	0.10255	0.10267
300	0.10293	0.10285	0.10261	0.10270
400	0.10296	0.10288	0.10267	0.10274
500	0.10200	0.10290	0.10272	0.10278
600	0.10302	0.10292	0.10276	0.10281
700	0.10304	0.10295	0.10280	0.10285
800	0.10307	0.10297	0.10283	0.10288
900	0.10310	0.10299	0.10286	0.10291
1 000	0.10312	0.10302	0.10289	0.10294
1 500	0.10324	0.10314	0.10304	0.10308
2 000	0.10335	0.10326	0.10318	0.10321
2 500	0.10346	0.10338	0.10331	0.10334
3 000	0.10358	0.10351	0.10344	0.10346
3 500		0.10363	0.10356	0.10358
4 000		0.10375	0.10369	0.10370
4 500		0.10387		0.10383
5 000		0.10400		0.10395

TABLE II

TABLE OF PRESSURE GAUGE CORRECTIONS
MEDITERRANEAN

	Col. 1	Col. 2	Col. 3	Col. 4
Depth (metres)	kg/cm² Based on H.O. 607	Gauge Calibrated to read (kg/cm²)	Col. 1 — Col. 2 (kg/cm²)	Col. 3 (metres)
0	0.0	0.0	0.0	0.0
50	5.14	5.13	0.01	0.1
100	10.28	10.25	0.03	0.3
150	15.43	15.38	0.05	0.5
200	20.57	20.50	0.07	0.7
300	30.86	30.75	0.11	1.1
400	41.16	41.01	0.15	1.5
500	51.46	51.26	0.20	2.0
600	61.76	61.51	0.25	2.4
700	72.06	71.76	0.30	2.9
800	82.36	82.02	0.34	3.3
900	92.67	92.27	0.40	4.0
1 000	102.98	102.52	0.46	4.5
1 500	154.54	153.77	0.77	7.5
2 000	206.17	205.05	1.12	10.9
2 500	257.84	256.29	1.55	15.1
3 000	309.58	307.54	2.04	19.9

TABLE III

TABLE OF PRESSURE GAUGE CORRECTIONS
ARCTIC & ANTARCTIC

	Col. 1	Col. 2	Col. 3	Col. 4
Depth (metres)	kg/cm² Based on H.O. 607	Gauge Calibrated to read (kg/cm²)	Col. 1 — Col. 2 (kg/cm²)	Col. 3 (metres)
0	0.0	0.0	0.0	0.0
50	5.14	5.13	0.01	0.1
100	10.28	10.25	0.03	0.3
150	15.42	15.38	0.04	0.4
200	20.56	20.50	0.06	0.6
300	30.85	30.75	0.10	1.0
400	41.13	41.01	0.12	1.2
500	51.42	51.26	0.16	1.6
600	61.71	61.51	0.20	2.0
700	72.00	71.76	0.24	2.3
800	82.30	82.02	0.28	2.7
900	92.60	92.27	0.33	3.2
1 000	102.90	102.52	0.38	3.7
1 500	154.41	153.77	0.64	6.3
2 000	205.98	205.05	0.93	9.1
2 500	257.62	256.29	1.33	13.0
3 000	309.31	307.54	1.77	17.3
3 500	361.07	358.82	2.25	22.0
4 000	412.88	410.06	2.82	27.5
4 500	464.76	461.34	3.42	33.4
5 000	516.70	512.59	4.11	40.1

TABLE IV

TABLE OF PRESSURE GAUGE CORRECTIONS
NORTHEAST PACIFIC

	Col. 1	Col. 2	Col. 3	Col. 4
Depth (metres)	kg/cm² Based on H.O. 607	Gauge Calibrated to read (kg/cm²)	Col. 1 — Col. 2 (kg/cm²)	Col. 3 (metres)
0	0.0	0.00	0.00	0.0
50	5.12	5.13	— 0.01	— 0.1
100	10.25	10.25	— 0.00	— 0.0
150	15.37	15.38	— 0.01	— 0.1
200	20.50	20.50	— 0.00	— 0.0
300	30.76	30.75	0.01	0.1
400	41.02	41.01	0.01	0.1
500	51.29	51.26	0.03	0.3
600	61.56	61.51	0.05	0.5
700	71.84	71.76	0.08	0.8
800	82.12	82.02	0.10	1.0
900	92.40	92.27	0.13	1.3
1 000	102.69	102.52	0.17	1.7
1 500	154.14	153.77	0.37	3.6
2 000	205.66	205.05	0.61	6.0
2 500	257.25	256.29	0.96	9.4
3 000	308.91	307.54	1.37	13.4
3 500	360.64	358.82	1.82	17.8
4 000	412.42	410.06	2.36	23.0

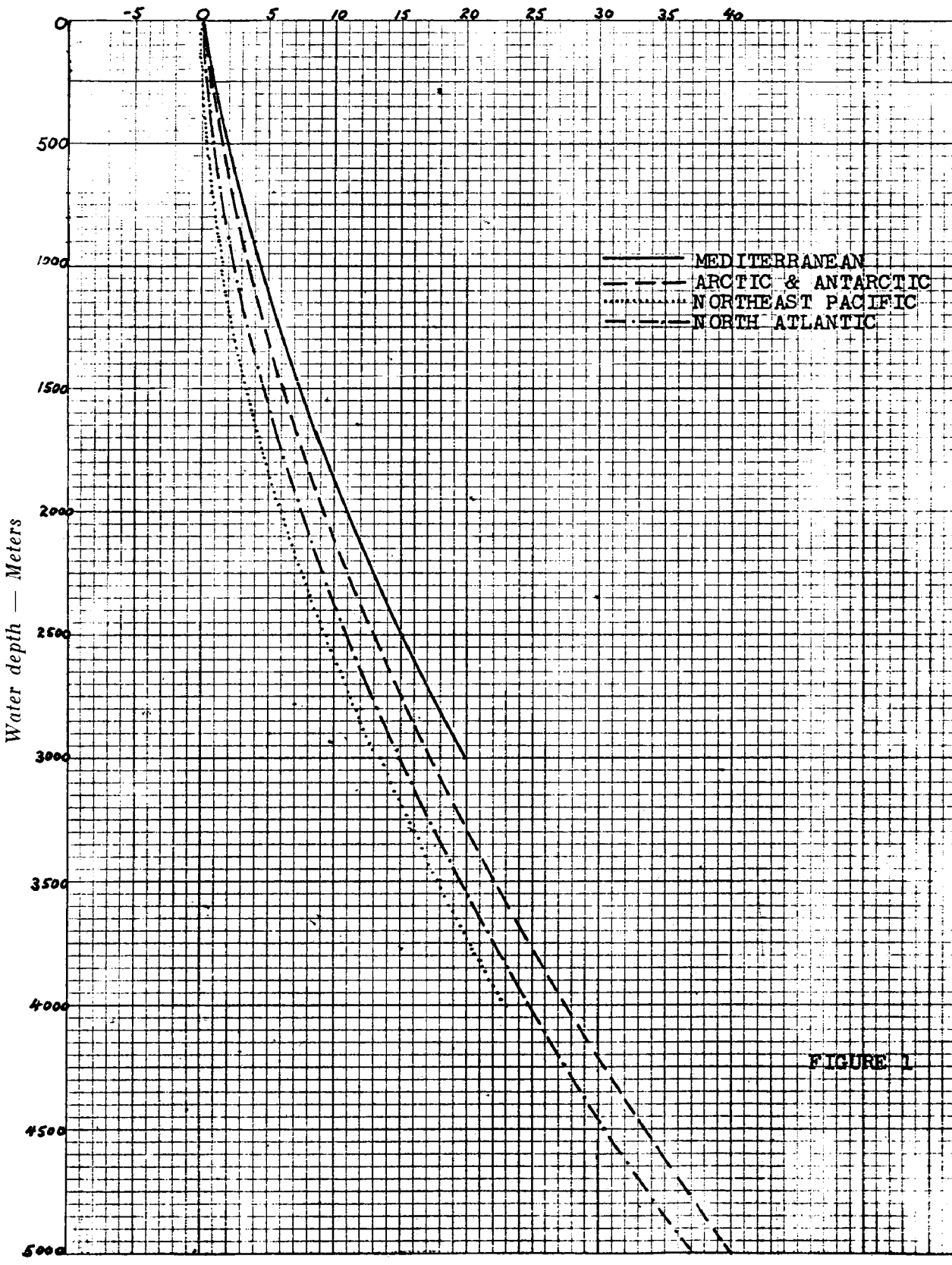
Correction to pressure gauge depth values — Meters
-5
0
5
10
15
20
25
30
35
40
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Water depth — Meters
MEDITERRANEAN
ARCTIC & ANTARCTIC
NORTHEAST PACIFIC
NORTH ATLANTIC

FIGURE 1

TABLE V

TABLE OF PRESSURE GAUGE CORRECTIONS
NORTH ATLANTIC

Depth (metres)	Col. 1 kg/cm² Based on H.O. 607	Col. 2 Gauge Calibrated to read (kg/cm²)	Col. 3 Col. 1 — Col. 2 (kg/cm²)	Col. 4 Col. 3 (metres)
0	0.0	0.0	0.00	0.0
50	5.13	5.13	0.00	0.0
100	10.26	10.25	0.01	0.1
150	15.40	15.38	0.02	0.2
200	20.53	20.50	0.03	0.3
300	30.80	30.75	0.05	0.5
400	41.07	41.01	0.06	0.6
500	51.35	51.26	0.09	0.9
600	61.62	61.51	0.11	1.1
700	71.90	71.76	0.14	1.4
800	82.19	82.02	0.17	1.7
900	92.48	92.27	0.21	2.1
1 000	102.77	102.52	0.25	2.5
1 500	154.24	153.77	0.47	4.6
2 000	205.79	205.05	0.74	7.2
2 500	257.40	256.26	1.11	10.8
3 000	309.07	307.54	1.53	14.9
3 500	360.80	358.82	1.98	19.3
4 000	412.59	410.06	2.53	24.7
4 500	464.45	461.34	3.11	30.4
5 000	516.37	512.59	3.78	36.9

ion is applied for variation in g. There is a further higher-order correction resulting from the change in gravity with depth below the surface. Inspection of Table I shows that the change in pressure with depth is not a constant of 0.10244 kg/cm² per metre but varies with water depth and water masses. When these calculated pressure changes are multiplied by the depth interval over which they occur and are added cumulatively, the total pressure at depth is obtained (column 1 of Tables II through V). Since the pressure gauges are calibrated to read a metre of depth for each 0.10244 kg/cm² increase in pressure, it is possible to obtain the gauge reading at any assumed depth by multiplying the depth by 0.10244 kg/cm² (column 2 of Tables II through V). The difference between the gauge pressure and the calculated pressure is shown in column 3. Column 4 is the correction to be applied to the meter reading and is obtained by dividing the pressure difference shown in column 3 by 0.10244 kg/cm² per metre. These values are expressed graphically in Figure 1.

Corrections to the pressure gauge from the surface to 200 metres in depth are less than one metre for all the areas in this study. The error in the northeast Pacific is negative (i.e., the meter reads too deep a value) and has a magnitude of about 0.1 metre to depths of 200 metres.

In general, the least depth error occurs in the northeast Pacific and increases in the North Atlantic, Arctic-Antarctic, and Mediterranean, respectively. Errors at the maximum depth of calculation are as follows : Mediterranean, 20 metres at a depth of 3000 metres; Arctic-Antarctic, 40

metres at a depth of 5000 metres; North Atlantic, 37 metres at a depth of 5000 metres; and northeast Pacific, 23 metres at a depth of 4000 metres.

Using the mean density values found in H.O. 607 as the bases for depth calculations, pressure gauges calibrated at 0.10244 kg/cm^2 per metre can be expected to have a maximum error resulting from calibration of about + 0.7 %. The inherent error of the most accurate pressure measuring instrument available commercially is ± 0.25 %.

These calculations are based on mean density values for the various water masses and serve as a guide to the errors that exist in the determination of depth from pressure measurements. Density variations within the water masses undoubtedly lead to pressure corrections different from those indicated in Figure 1. More specific corrections can be determined by utilizing oceanographic station data for the calculation of density.