DAILY MEAN SEA LEVEL AND SHORT-PERIOD SEICHES

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Abstract

Slow water-level variations and short-period seiches are found at the outer ends of the tidal spectrum. In order to isolate these types of oscillation from the body of the tide it is necessary to filter the observations. In this paper the filter theory is briefly set out first and then two simple and practical filters are discussed. The one consists of symbol products of arithmetical means and can be used to detect the true variations of the daily mean sea level. The other, which is a product of differences, isolates short-period seiches fairly well.

Introduction

The tidal spectrum can be divided into three regions :

1) The low frequency band, defined over an interval extending from 0 to 1 cycle per day;
2) The central band, limited to 1 to 6 cycles per day;
3) Finally, the high frequency band containing all the frequencies above 6 cycles per day.

The relation between period T and frequency σ is :

$$T = 1/\sigma \tag{1}$$

The central band contains the diurnal, semi-diurnal tides, etc., and is the most important part of the spectrum. This region is investigated by ordinary tidal analysis, and a large number of methods have been formulated with this object.

We shall therefore give our attention to the outer ends of the tidal spectrum. The low frequency band contains the annual, seasonal and secular variations as well as certain weak waves such as M_m, M_f, and MS_f; some meteorologic disturbances also contribute to the spectrum. In order to define this band it is necessary to have available a sequence covering a fairly long interval — of about several months, or even several years. The other outer band of the spectrum, the high frequency band, contains seiches and all the short-period oscillations. The study of this band necessitates samples being taken very closely in time, but only several hours or several days of observation are needed. Moreover the inertia of measuring instruments ensures that the spectrum becomes zero for every frequency above, say, σ_M; σ_M being of the order of 12 cycles per hour.

Before analysing the contribution of these outer bands it will be good to isolate them first from the central band. In fact since this band contains a whole body of waves of unknown periods, it is necessary first of all to follow their development in time in order to recognise the predominating waves. We shall then be in a better position to attempt an analysis.

Certain areas of the spectrum are isolated by means of a *filter*. For example, the *arithmetical mean* is a low-pass filter and it is used for computing the mean level. It is often thought that the mean level change from day to day reflects the contribution in time of the low frequency band. Unfortunately this is not true, since the arithmetical mean is a filter of fairly poor quality; a certain proportion of the central band is allowed through, and the daily sampling of the arithmetical mean, which is not in this case permissible, creates fictitious waves.

However we cannot reject the idea leading us to calculate the mean level. It is by this computation that we wish to follow the variation in water level when this is freed from the diurnal and semi-diurnal tides, as well as other tides of higher species. It is most desirable, both from the scientific and the practical point of view to know this mean level, but it must be sought with the help of a good filter.

Of this filter we require that it eliminate the central band and that it let through the annual, semi-annual, monthly and weekly waves. It would also be useful if this filter let through all the waves of a period higher than 1 day without amplitude distortion, but we shall see that this is not always possible.

This is also the case for seiches : here we also need a filter which eliminates the central band, but this filter must allow the high frequencies through without much amplitude distortion, above all when we do not know precisely the period of the seiche.

THE MATHEMATICAL THEORY OF FILTERS

A filter is a sequence of numbers

$$\{ f_j \} \qquad j = 0, \pm 1, \pm 2, \ldots \tag{2}$$

that are *convoluted* to a sequence of observations

$$\{ z(k\Delta t) \} \qquad k = 0, \pm 1, \pm 2, \ldots \tag{3}$$

The result of the convolution is written in the following way :

$$\{ z_f(k\Delta t) \} \equiv \left\{ \sum_{j=-\infty}^{\infty} f_j z\,[(k-j)\,\Delta t] \right\} \qquad k = 0, \pm 1, \pm 2, ... \qquad (4)$$

Δt is the time interval which must always be lower than $1/2\ \sigma_M$, σ_M being the highest frequency contributing to $z(k\Delta t)$. z_f indicates the result of the filtering of the sequence of observations $\{ z(k\Delta t) \}$ by sequence $\{ f_j \}$. This last may be finite or infinite. In general we shall designate by the letter F the filter having the temporal representation (2) and the representation (3) in the frequency space.

In order to understand the effect of a filter on the spectrum of observations we note that any sequence of numbers defines a function of σ, called the *spectrum of the sequence.* Thus $\{ f_j \}$ defines the spectrum

$$F(\sigma) = \sum_{j=-\infty}^{\infty} f_j e^{2\pi i j \Delta t \sigma} \qquad (5)$$

Similarly the sequence of observations $\{ z(k\Delta t) \}$ defines the spectrum

$$Z(\sigma) = \sum_{k=-\infty}^{\infty} z(k\Delta t)\, e^{-2\pi i k\,\Delta t \sigma} \qquad (6)$$

Spectrum $Z(\sigma)$ is unknown since only a finite portion of observations is available, and if the sequence $\{ z(k\Delta t) \}$ consists of a consecutive sequence of observations on the water level then $Z(\sigma)$ *is the tidal spectrum.* Moreover the spectrum $F(\sigma)$ of filter F is *exactly* known since the definition of the f_j's automatically defines spectrum $F(\sigma)$.

Let us now see the effect of the convolution of F with Z on spectrum $Z(\sigma)$. Definitions (5) and (6) being quite general, we deduce that sequence $\{ z_f(k\Delta t) \}$ also possesses a spectrum that we call $Z'(\sigma)$ for the moment, and which we can compute with the help of definition (4) :

$$\begin{aligned} Z'(\sigma) &= \sum_{k=-\infty}^{\infty} z_f(k\Delta t)\, e^{-2\pi i k\,\Delta t \sigma} = \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} f_j z\,[(k-j)\,\Delta t]\, e^{-2\pi i k\,\Delta t \sigma} \\ &= \sum_{j'=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} f_j e^{-2\pi i j\,\Delta t \sigma}\, z(j'\Delta t)\, e^{-2\pi i j'\,\Delta t \sigma} \\ &= \left(\sum_{j=-\infty}^{\infty} f_j e^{-2\pi i j\,\Delta t \sigma} \right) \left(\sum_{j'=-\infty}^{\infty} z(j'\Delta t)\, e^{-2\pi i j'\,\Delta t \sigma} \right) \\ &= F(\sigma)\, Z(\sigma) \end{aligned} \qquad (7)$$

where we have written :

$$j' = k - j$$

Thus we have

$$Z'(\sigma) = F(\sigma)\, Z(\sigma)$$

which indicates that when Z is convoluted by F in time, spectrum $Z(\sigma)$ is multiplied by $F(\sigma)$ for all the values of σ on which the spectra are defined.

If $F(\sigma)$ is zero on certain intervals of σ, it follows that $Z'(\sigma)$ is also zero on the same intervals, thus any band of $Z(\sigma)$ can be isolated through the judicious application of a filter F.

Equation (7) implies certain characteristics desirable for a filter :

1) $F(\sigma)$ should have a value close to unity in the band of concern, and be zero anywhere else.
2) Sequence $\{ f_j \}$ should be finite and as short as possible in order to use only a minimum number of observations.
3) Numbers f_j themselves should be as simple as possible in order to make the computation of the convolution easy.

These characteristics are mathematically contradictory : hence a compromise must be found. We prefer first to transgress 1) a little in order to comply with 2) and 3). Afterwards we shall use products of filters which will allow condition 1) to be satisfied to a high degree, as well as conditions 2) and 3).

We may convolute the sequence of observations $\{ z(k\Delta t) \}$ as many times as we wish by a sequence of filters

$$\{f_j^{(1)}\},\ \{f_j^{(2)}\},\ \dots,\ \{f_j^{(n)}\}$$

The result of this repeated convolution is the following :

$$\{z_{f_1 f_2 \dots f_n}(k\Delta t)\} = \sum_{j_1=-\infty}^{\infty} \sum_{j_2=-\infty}^{\infty} \dots \sum_{j_n=-\infty}^{\infty} f_{j_1}^{(1)} f_{j_2}^{(2)} \dots f_{j_n}^{(n)} \cdot \{z\,[(k-j_1-j_2- \dots -j_n)\,\Delta t]\} \qquad (8)$$

and the spectrum of the sequence is :

$$Z'' \cdots '(\sigma) = F_1(\sigma)\, F_2(\sigma) \dots F_n(\sigma)\, Z(\sigma) \qquad (9)$$

If the $F_j(\sigma)$'s are small outside the areas of concern their product is still smaller, and if their value is close to unity in the band sought for their product is also close to unity. Thus we can expect that a product of fairly rough filters will allow us to satisfy 1), as well as 2) and 3) which only need a simple temporal representation.

A SIMPLE AND PRACTICAL FILTER FOR COMPUTING THE DAILY MEAN LEVEL

The representation as a filter of the arithmetical mean is the following:

$$\{f_j\} \qquad j = 0, \pm 1, \pm 2, \dots$$

where

$$f_j = \begin{cases} 1/(2N+1) & j = 0, \pm 1, ..., \pm N \\ 0 & |j| > N \end{cases} \tag{10}$$

The convolution of (10) with the sequence of observations $\{ z(k\Delta t) \}$ gives the sequence

$$\{\bar{z}(k\Delta t)\} = \frac{1}{2N+1} \sum_{j=-N}^{N} z\,[(k-j)\,\Delta t] \tag{11}$$

which is a sequence of arithmetical means of 2N + 1 consecutive observations. (10) defines the spectrum

$$S_{2N+1}(\sigma) = \frac{\sin\,[(2N+1)\,\pi\,\Delta t\,\sigma]}{(2N+1)\sin \pi\,\Delta t\,\sigma} \tag{12}$$

which we have computed with the help of (5) and (10). The summation of the exponentials in (5) can be made analytically in our case. Now an even number of elements can be chosen also from the sequence (10); the spectrum is then

$$S_{2N}(\sigma) = \frac{\sin\,[2N\,\pi\,\Delta t\,\sigma]}{2N \sin \pi\,\Delta t\,\sigma} \tag{13}$$

If the arithmetical mean of 24 consecutive observations is computed the spectrum of this mean is $S_{24}(\sigma)$. If $\Delta t = 1$ hour as is frequently the case in tidal observations, spectrum $S_{24}(\sigma)$ is zero for all the frequencies which are a multiple of 1/12 cycle per day. This filter thus completely eliminates solar tides S_1, S_2, S_3, etc. Moreover, like any arithmetical mean, filter (13) is in fact a low-pass filter since it becomes small for high frequencies.

We realise, however, that (13) is not a perfect filter and that it does not become zero for all the points of the central band. Some contribution from M_2, N_2, O_1, etc. will be found again in the filtered set (11). As, for example, the amplitude ratio of $\frac{M_2}{M_f}$ or $\frac{M_2}{M_m}$ may be of the order of 100/1, sequence (11) can be disturbed by M_2 in the same proportion as by M_m or M_f. If, at worst, the contribution of M_2 and other constituents in the central band is sampled in varying phase conditions this creates a fictitious wave which does not exist in reality, and this wave masks the actual waves arising from long-period constituents.

If we wish to eliminate nearly all the waves having an angular speed that is a multiple of M_1 we can use the arithmetical mean of 25 hourly data. In this case the spectral function is $S_{25}(\sigma)$ and it is zero for all the waves whose angular speed is a multiple of one cycle per 25 hours. On the other hand this filter lets in a certain proportion of other waves.

Thus neither one of the filters we have considered is by itself perfect, but a combination of both may certainly be very useful. Consider, for example

$$S_{25}^2 S_{24} \text{ or } S_{24}^2 S_{25} \tag{14}$$

where we have used the symbolic notation for filter F. Figure 1 illustrates the spectra $S_{24}(\sigma)$, $S_{25}(\sigma)$, $X_0(\sigma)$, $S_{25}^2(\sigma)\,S_{24}(\sigma)$ and $S_{24}^2(\sigma)\,S_{25}(\sigma)$ whilst

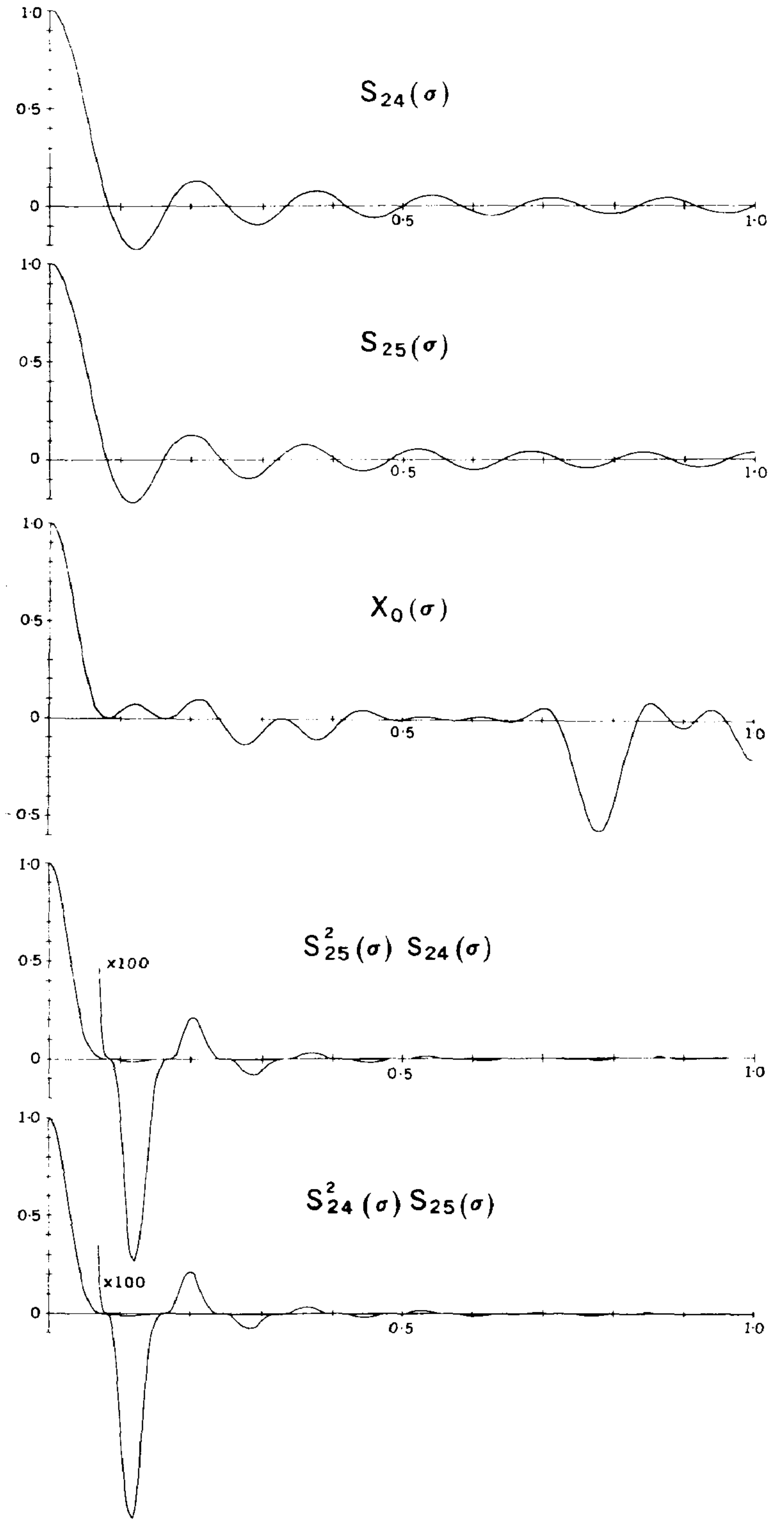


FIG. 1. — Graphs of the spectra

$S_{24}(\sigma)$, $S_{25}(\sigma)$, $X_0(\sigma)$, $S^2_{24}(\sigma)S_{25}(\sigma)$ and $S^2_{25}(\sigma)S_{24}(\sigma)$ as functions of $2\Delta t\sigma$. All these functions are standardized and $2\Delta t\sigma$ varies between the values 0 and 1. The functions $S^2_{24}(\sigma)\ S_{25}(\sigma)$ and $S^2_{25}(\sigma)\ S_{24}(\sigma)$ have been multiplied by the factor 100 in the interval (.07, 1.00) so that their variation in this interval may be followed. $X_0(\sigma)$ is the spectrum of the Doodson X_0 filter and it is given by the expression

$$X_0(\sigma) = \frac{1}{15} \cos 2\pi\Delta t\sigma \frac{\sin 25\,\pi\Delta t\sigma}{\sin 5\,\pi\Delta t\sigma} \cdot \frac{\sin 24\,\pi\Delta t\sigma}{\sin 8\,\pi\Delta t\sigma}$$

table 1 gives their numerical values versus $2\,\Delta t\sigma$. The maximum value of $2\,\Delta t\sigma$ can only be unity or else the law of sampling would be violated. It is seen that either one or other of the filters (14) has an almost equivalent selectivity, and that they actually cut the central band as well as the high frequency band. $X_0(\sigma)$, which is Doodson's filter, cuts the tidal waves of the central band fairly well, but it has disquieting peaks almost everywhere else and its use is not recommended.

It would therefore seem that one or other of the filters (14) could be employed for the efficient computation of the daily mean level. If we select $S_{25}^2\,S_{24}$ we need 72 consecutive observations. A sequence of means is first computed for 24 observations, then a series of means for 25 of these means, and finally the mean of this last series which gives the daily mean level as normally understood. This value holds good for time 1130 of the central day if the first observation is taken at 0000 on the preceding day, and the last at 2300 on the following day. The computation of the mean level with $S_{24}^2\,S_{25}$ requires 71 consecutive observations and the result represents the mean level at 1200 on the central day. We give below an example of the process to be followed for $S_{25}^2\,S_{24}$. Let the observational sequence be :

preceding day :	central day :	following day :
$z(00),\ z(01),\ ...,\ z(23)$	$z(00),\ z(01),\ ...,\ z(23)$	$z(00),\ z(01),\ ...,\ z(23)$

which will be rewritten in the form

$$z(1),\ z(2),\ z(3),\ ...,\ z(72)$$

We first compute the sequence of means

$$\{X_i\} \qquad i = 1, 2, 3, ..., 48$$

where

$$X_i = \sum_{k=0}^{24} z(k+i)$$

then the sequence

$$\{Y_j\} \qquad j = 1, 2, 3, ..., 24$$

where

$$Y_j = \sum_{i=0}^{24} X_{i+j}$$

TABLE 1

Value of spectra $S_{24}(\sigma)$, $S_{25}(\sigma)$, $X_0(\sigma)$, $S_{25}^2(\sigma)\,S_{24}(\sigma)$, $S_{25}(\sigma)\,S_{24}^2(\sigma)$ *in terms of* $2\,\Delta t\sigma$

$2\,\Delta t\sigma$	S_{24}	S_{25}	X_0	$S_{25}^2\,S_{24}$	$S_{24}^2\,S_{25}$
0.000	1.000	1.000	1.000	1.000	1.000
.01	.977	.975	.955	.927	.929
.02	.908	.900	.829	.736	.743
.03	.800	.785	.646	.493	.503

$2\,\Delta t\sigma$	S_{24}	S_{25}	X_0	$S_{25}^2\,S_{24}$	$S_{24}^2\,S_{25}$
.04	.662	.637	.443	.269	.279
.05	.505	.471	.257	.112	.120
.06	.341	.300	.115	.031	.035
.07	.189	.139	.030	.004	.005
.08	.042	.000	.000	.000	.000
.09	—.074	—.109	.010	—.001	—.001
.10	—.157	—.181	.039	—.005	—.004
.11	—.205	—.215	.066	—.009	—.009
.12	—.218	—.213	.075	—.010	—.010
.13	—.202	—.182	.065	—.007	—.007
.14	—.161	—.130	.041	—.003	—.003
.15	—.105	—.066	.015	—.000	—.001
.16	.042	.000	.000	.000	.000
.17	.020	.058	.003	.000	.000
.18	.072	.101	.025	.001	.001
.19	.109	.126	.057	.002	.002
.20	.128	.129	.087	.002	.002
.21	.128	.114	.101	.002	.002
.22	.111	.084	.090	.001	.001
.23	.081	.043	.054	.000	.002
.24	.042	.000	.000	.000	.000
.25	.000	—.040	—.059	.000	.000
.26	—.039	—.071	—.106	.000	.000
.27	—.069	—.090	—.132	—.001	.000
.28	—.089	—.094	—.129	—.001	—.001
.29	—.095	—.084	—.103	—.001	—.001
.30	—.087	—.062	—.063	.000	.000
.31	—.069	—.033	—.025	.000	.000
.32	—.042	.000	.000	.000	.000
.33	—.010	.031	.004	.000	.000
.34	.020	.056	—.014	.000	.000
.35	.047	.071	—.045	.000	.000
.36	.066	.075	—.079	.000	.000
.37	.075	.067	—.103	.000	.000
.38	.073	.050	—.109	.000	.000
.39	.061	.027	—.095	.000	.000
.40	.042	.000	—.064	.000	.000
.41	.017	—.025	—.025	.000	.000
.42	—.009	—.046	.011	.000	.000
.43	—.032	—.059	.036	.000	.000
.44	—.050	—.063	.046	.000	.000
.45	—.061	—.057	.041	.000	.000
.46	—.063	—.043	.027	.000	.000
.47	—.056	—.023	.011	.000	.000
.48	—.042	.000	.000	.000	.000
.49	—.022	.022	—.004	.000	.000
.50	.000	.040	.000	.000	.000
.60	—.030	—.049	.013	.000	.000
.70	.044	.032	.063	.000	.000
.80	—.042	.000	—.436	.000	.000
.90	.025	—.029	—.039	.000	.000
1.00	.000	.040	—.200	.000	.000

Fine structure in the vicinity of the tidal constituents

Constituents	$2\,\Delta t\sigma$		S_{24}	S_{25}	X_0	$S_{25}^2\,S_{24}$	$S_{24}^2\,S_{25}$
SLOW	0.000		1.00000	1.00000	1.00000	1.00000	1.00000
	.001		.99976	.99974	.99954	.99925	.99927
	.002		.99905	.99897	.99815	.99701	.99971
	.003		.99787	.99769	.99585	.99327	.99345
	.004		.99622	.99590	.99263	.98807	.98839
	.005		.99410	.99360	.98850	.98141	.98191
	.006		.99151	.99079	.98347	.97332	.97403
	.007		.98845	.98747	.97755	.96384	.96480
	.008		.98494	.98366	.97075	.95301	.95424
	.009		.98096	.97934	.96310	.94085	.94240
DIURNAL	.071		.16787	.12440	.02449	.00260	.00351
	.072		.15299	.10953	.01975	.00184	.00256
	.073		.13830	.09490	.01554	.00125	.00182
	.074		.12382	.08051	.01186	.00080	.00123
	.075		.10955	.06639	.00869	.00048	.00080
	.076		.09549	.05254	.00603	.00026	.00000
	.077	[O_1]	.08167	.03897	.00384	.00012	.00026
	.078		.06809	.02568	.00212	.00004	.00012
	.079		.05475	.01269	.00085	.00001	.00004
	.080		.04167	.00000	.00000	.00000	.00000
	.081		.02885	—.01238	—.00044	.00000	—.00001
	.082		.01630	—.02443	—.00049	—.00001	—.00001
	.083	[K_1]	.00403	—.03616	—.00018	—.00001	.00000
	.084		—.00796	—.04756	.00047	—.00002	.00000
	.085		—.01965	—.05862	.00145	—.00007	—.00002
	.086		—.03105	—.06933	.00273	—.00015	—.00007
	.087		—.04214	—.0797	.00428	—.00027	—.00014
	.088		—.05293	—.08971	.00609	—.00043	—.00025
	.089		—.06339	—.00935	.00812	—.00063	—.00040
SEMI DIURNAL	.151		— .09875	— .05892	.01302	— .00034	— .00057
	.152		— .09253	— .05227	.01096	— .00025	— .00045
	.153		— .08625	— .04562	.00903	— .00018	— .00034
	.154		— .07994	— .03898	.00724	— .00012	— .00025
	.155		— .07359	— .03237	.00561	— .00008	— .00018
	.156		— .06722	— .02579	.00414	— .00004	— .00012
	.157		— .06083	— .01926	.00283	— .00002	— .00007
	.158	[N_2]	— .05444	— .01278	.00170	— .00001	— .00004
	.159		— .04805	— .00635	.00076	.00000	— .00001
	.160		— .04267	.00000	.00000	.00000	.00000
	.161	[M_2]	— .03530	.00628	—.00057	.00000	.00001
	.162		— .02897	.01247	—.00094	.00000	.00001
	.163		— .02267	.01856	—.00111	—.00001	.00001
	.164		— .01641	.02456	—.00107	—.00001	.00001
	.165		— .01021	.03045	—.00084	—.00001	.00000
	.166	[S_2]	— .00406	.03622	—.00040	—.00001	.00000
	.167	[K_2]	.00202	.04187	.00024	.00000	.00000
	.168		.00803	.04739	.00108	.00002	.00000
	.169		.01395	.05277	.00212	.00004	.00001

and finally

$$Z_0 = \sum_{j=1}^{24} Y_j$$

We obtain

$$\bar{z}_0(1130) = Z_0/15000 \qquad (15)$$

which is the mean level at 1130 on the central day. This computation is very simple to programme and fast with an electronic computer.

One or other of the (14) filters allows through the Z_0, S_a, S_{sa}, M_m, M_f and MS_f constituents almost entirely and becomes zero on the central band. However these filters reduce by about 50 % the amplitude of the waves having a period of about a ½ cycle per day, which is rather unfortunate. It is, however, impossible to imagine a simple filter which is flat everywhere in the low frequency interval and which becomes zero suddenly for any frequency above or equal to 1 cycle per day. Table 2 gives the amplitude of long-period constituents in percentages after a filtering by one of the (14) filters.

TABLE 2

Quotient of the amplitude of the long-period constituents before and after filtering

$S_{25}^2 S_{24}$ or $S_{24}^2 S_{25}$

$$\frac{A_{\text{FILTERED}}}{A}$$

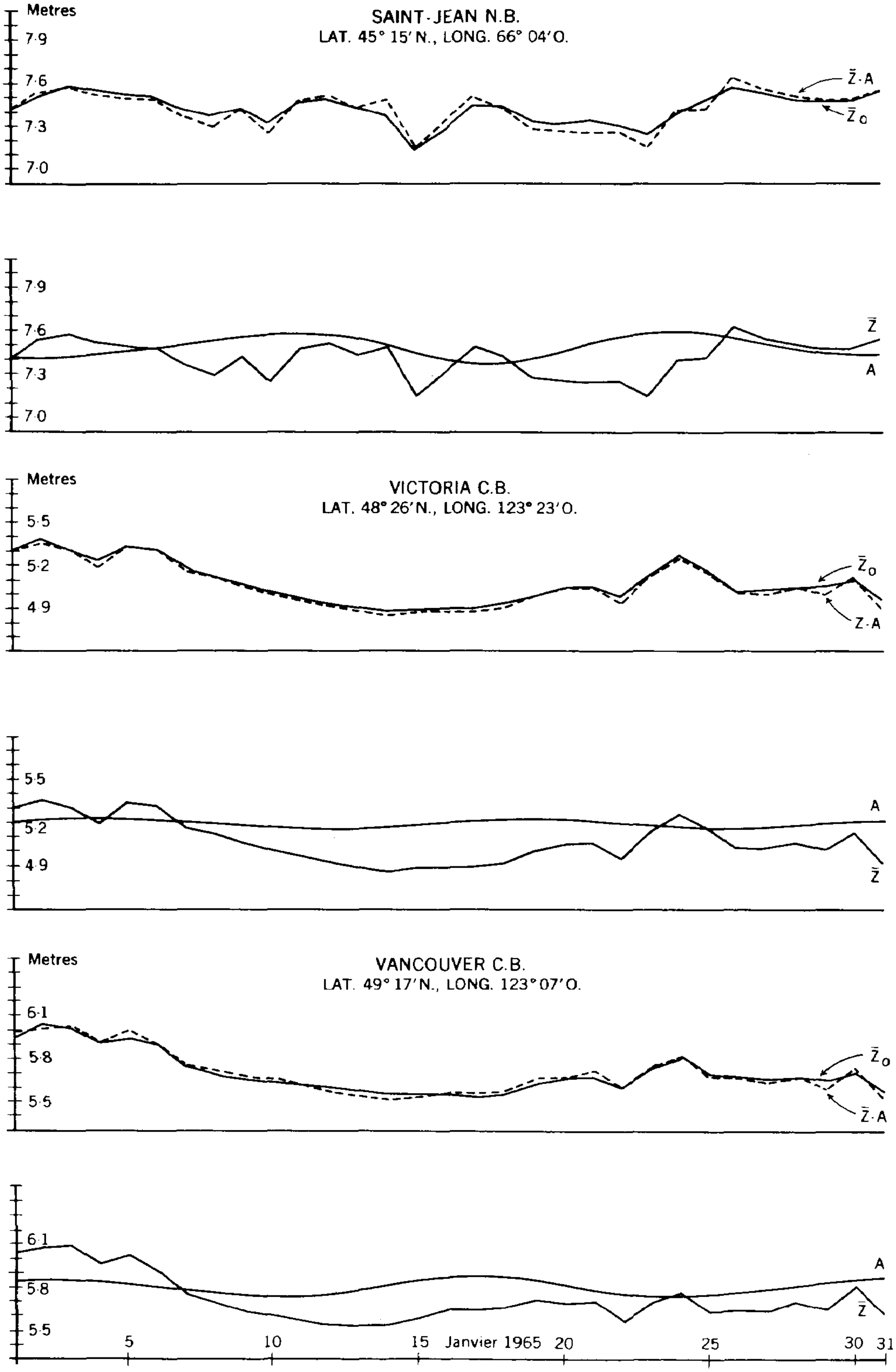
Constituent	$S_{25}^2 S_{24}$ %	$S_{24}^2 S_{25}$ %
Z_0	100.0	100.0
S_a	100.0	100.0
S_{sa}	100.0	100.0
M_m	99.3	99.3
MS_f	97.7	97.7
M_f	97.2	97.4

Example

We have computed the daily mean value for three Canadian ports with the help of $S_{25}^2 S_{24}$; we shall call this mean level $\bar{z}_0$ as in (15). We have compared $\bar{z}_0$ with $\bar{z}$, the mean level computed with the help of the arithmetical mean. At the same time we have reckoned the fictitious wave $A(j)$ created by the faulty sampling of $\bar{z}$.

The ports we chose are the following :

St. John, N.B. (45°15′ N, 66°04′ W)
Victoria, B.C. (48°26′ N, 123°23′ W)
Vancouver, B.C. (49°17′ N, 123°07′ W)



Curve A is not to scale

FIG. 2. — Comparison between variation of $\bar{z}$, the daily mean level computed with the help of the arithmetical mean S_{24}, and $\bar{z}_0$, the level computed with the help of the filter $S_{25}^2 S_{24}$.

and we reckoned $\bar{z}_0(j)$, $\bar{z}(j)$ and A(j) for the month of January 1965 (j is the day variable). To compute A(j) we took into account the contribution of M_2, N_2 and O_1 to the arithmetical mean of 24 consecutive hourly data. In the case of St. John, N.B., the contribution of A(j) can at certain times have an amplitude of 14 cm which is far from negligible. It we compute $\bar{z}$ — A, we are removing the largest imperfection from $\bar{z}$, and the variation of $\bar{z}(j)$ — A(j) must be very close to that of $\bar{z}_0(j)$. Such is the case, as we can see in figure 2, but there are oscillations in $\bar{z}$ — A which we do not find in $\bar{z}_0$. These are oscillations of periods of the order of a $\frac{1}{2}$ cycle per day which are reduced more strongly by $S_{25}^2 S_{24}$ than by S_{24}. Table 3 gives the components of the A(j) curve for the three ports in the month of January.

TABLE 3

Principal components of the fictitious wave A(j)

$$A(j) = a_1 \cos (24.38_j - b_1) + a_2 \cos (25.37_j - b_2) + a_3 \cos (37.45_j - b_3)$$
where

	a_1 cm	b_1 o	a_2 cm	b_2 o	a_3 cm	b_3 o
St. John, N.B.	— 10.7	034.4	00.9	086.0	— 03.4	257.8
Victoria, B.C.	— 01.3	291.4	02.8	068.7	— 00.5	151.1
Vancouver, B.C. ...	— 03.3	207.8	03.6	043.6	— 01.0	064.4

These values hold for January 1965. The suffixes 1, 2 and 3 refer to the M_2, O_1 and N_2 waves. j varies from 0 to 30 during this month.

SHORT-PERIOD SEICHES

It is not easy to separate seiches whose periods overlap those of the tidal constituents. Moreover, short-period seiches of, say, one hour or less are to be found in the high frequency band and they can be isolated by simple filters.

The simplest filter cutting off the long-period waves is the following :

$$f_j = \begin{cases} \pm\frac{1}{2} & j=\pm 1 \\ 0 & |j|>1 \end{cases} \quad (16)$$

If this filter is applied to two consecutive observations, separated by a time interval Δt, the spectrum of (16) is

$$H(\sigma) = \frac{1}{2}(-e^{-2\pi i\sigma \Delta t\, 1/2} + e^{2\pi i \sigma \Delta t\, 1/2}) = i \sin \pi \Delta t \sigma \quad (17)$$

Filter H is therefore a dephasing filter, and its spectrum H(σ) has its first maximum when

$$\Delta t \sigma_0 = \frac{1}{2} \quad (18)$$

Figure 3 shows the $-iH(\sigma)$ curve. We see that H is a filter of rather poor selectivity, but it will suffice for the case of short-period seiches. We shall now convince ourselves that this is so.

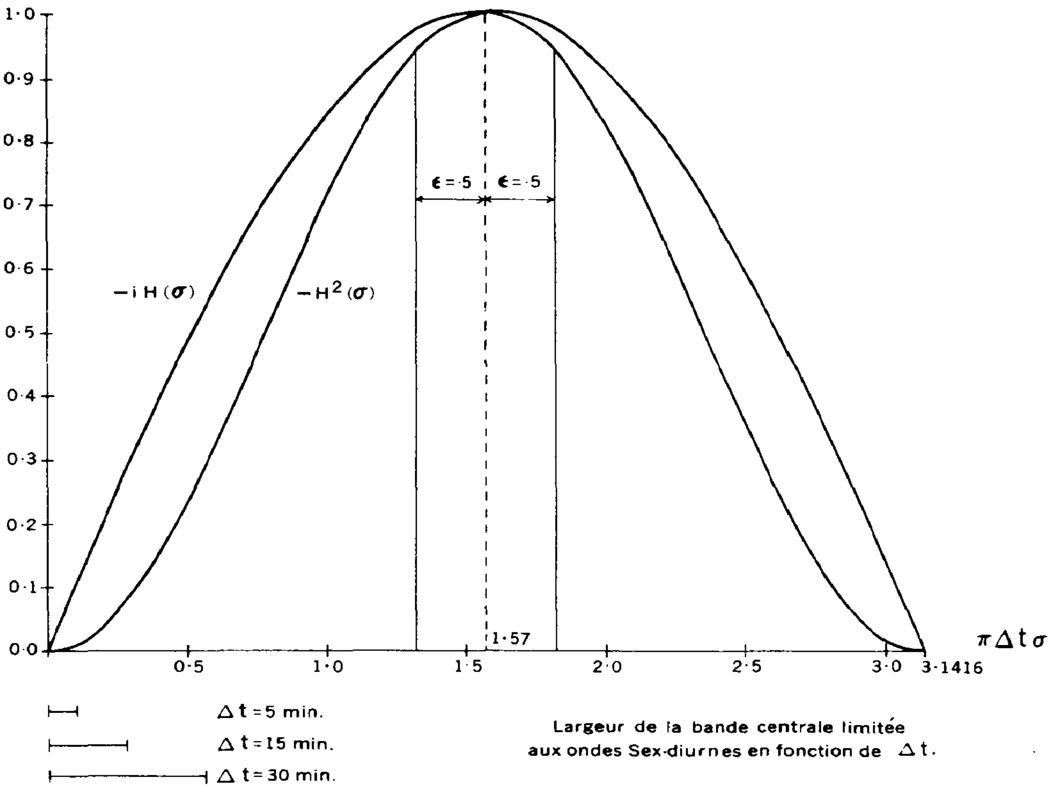


FIG. 3. — The spectra $-iH(\sigma)$ and $-H^2(\sigma)$ as functions of $\Delta t\sigma$.

Since H is dephasing and gives a filtered value falling between two observations we decide to use

$$-H^2 \tag{19}$$

which has as a spectrum

$$-H^2(\sigma) = \sin^2 \pi \Delta t \sigma \tag{20}$$

Thus, we eliminate the dephasing factor, and we reduce the filtered data to the instant of observation and we reinforce the selectivity. $-H^2(\sigma)$ is shown in figure 3.

The procedure to follow for applying $-H^2$ in time is the following. We have three data separated by an interval of time Δt

$$z(-\Delta t),\ z(0),\ z(\Delta t)$$

We compute

$$X_+ = z(\Delta t) - z(0)$$

$$X_- = z(0) - z(-\Delta t)$$

then

$$z_s(0) = \frac{1}{4}(X_- - X_+) \tag{21}$$

which is the value filtered by $-H^2$ at time 0.

TABLE 4

Values of σ_0 and T_0 corresponding to the different choices of time interval Δt

Δt min	σ_0 cycles/min	T_0 min
1	1/2	2
2	1/4	4
5	1/10	10
10	1/20	20
15	1/30	30
30	1/60	60

The rate of sampling in time must obviously be small, and its value depends on the period of the seiche, which may be estimated by eye. Table 4 gives the values of σ_0 and T_0 corresponding to the various choices of Δt. Having estimated the magnitude of the period of the seiche to be studied, we decide with the help of table 4 on the time interval Δt, then we carry out the filtering with the help of $-H^2$ which gives the sequence

$$\{z_s(k\Delta t)\} \qquad k = 0, \pm 1, \pm 2, \ldots \tag{22}$$

This sequence is almost entirely free from the contributions of the central band, as table 5 will convince us. Since $-H^2(\sigma)$ has no oscillations we show it in table 5 for a few values of σ only.

TABLE 5

Spectrum $-H^2(\sigma)$

		$\Delta t =$	1	2	5	10	15	30 min.
Species	cycles /day	cycles /min			$-H^2(\sigma)$			
diurnal	1	1/1440	0.0000	0.0000	0.0001	0.0005	0.0011	0.0043
semi-diurnal	2	1/720	.0000	.0001	.0005	.0019	.0043	.0170
quarter-diurnal	4	1/360	.0001	.0003	.0019	.0076	.0170	.0670
sixth-diurnal	6	1/240	.0002	.0007	.0043	.0170	.0381	.1464
twelfth-diurnal	12	1/120	.0007	.0027	.0171	.0670	.1464	.5000

This table indicates that it is not advisable to use the filter $-H^2$ for seiches of periods higher than 30 minutes, unless the seiche has an appreciable amplitude; in this case $-H^2$ can be used to isolate the seiches which have a period of as much as several hours.

Since the period of a seiche is first of all estimated by eye, and since $-H^2(\sigma)$ is far from being flat, we must know the behaviour of $-H^2(\sigma)$ in the neighbourhood of σ_0. Table 6 has been computed with this aim.

The true frequency of the seiche is probably not exactly $\sigma_0 (= 1/2\,\Delta t)$. We therefore write

$$\sigma = \sigma_0 + \Delta\sigma$$

TABLE 6

Spectrum — $H^2(\sigma)$ *in the vicinity of its peak, in percentage of its maximum value*

$$\pi \Delta t \sigma = \frac{1}{2} \pi \pm \varepsilon$$

$\frac{\vert\Delta\sigma\vert}{\sigma_0}$	ε	$-H(\varepsilon) = \cos^2\varepsilon$
$\times$ 100		$\times$ 100
0.64	0.01	99.99
1.27	.02	99.96
1.91	.03	99.91
2.55	.04	99.84
3.18	.05	99.75
3.82	.06	99.64
4.46	.07	99.51
5.09	.08	99.36
5.73	.09	99.19
6.37	.10	99.00
9.55	.15	97.70
12.73	.20	96.05
19.10	.30	91.27
25.46	.40	84.84
31.83	.50	77.01

The definition of σ_0 involves

$$\pi \Delta t (\sigma_0 + \Delta\sigma) = \frac{1}{2} \pi \pm \varepsilon \tag{23}$$

where ε is the error in non-dimensional unities. From (23) we deduce the relationship between $\Delta\sigma$ and ε which is the following

$$|\Delta\sigma| = \frac{2\varepsilon}{\pi} \sigma_0 \tag{24}$$

It can be seen from table 6 that an error of 32 % is permissible in the estimation of the seiche frequency without the filtering by $-H^2$ decreasing its amplitude appreciably.

In figure 3 we have indicated the width of the central band, that we shall here limit to the sixth-diurnal waves, in terms of certain values of Δt.