PRACTICAL VALUE OF SIGHT REDUCTION TABLES

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It is a well-known fact that there are many tables published to assist navigators in finding a ship's position from observations of celestial bodies. In the I.H.B.'s *Hydrographic Review,* issued in August 1943, in the article " Chronological and Analytical List of various Tables or Treatises on Navigation intended to facilitate Nautical Computations and accelerate ship's position finding ", by Captain H. Bencker, there appeared an extensive account (80 pages) of the various means and methods published up to the year 1942 for navigational purposes and for finding a ship's position at sea. For the most important tables an explanation about the method and formulae on which they were based was given. A valuable and fairly extensive review (56 pages) of the various methods, tables, and graphical and mechanical solutions of the position line, as well as direct solutions for a celestial fix published before 1957 is given in chapter XXI of the H.O. Pub. No. 9 " American Practical Navigator " by N. Bowditch, 1958 edition. The question inevitably arising in this connection is : How may we find out which is the best method to use in actual practice ?

This paper deals with the method of determining the practical value of sight reduction tables. In addition, as an illustration of this method and its practical applications, an assessment has been made of the Tables K 1 (representing the short method of two-entry tables for the computation of the altitude and azimuth of celestial bodies) issued by the Hydrographic Institute of the Yugoslav Navy at the end of the year 1958.

Determining the Practical Value of Tables

All astronomical navigation problems consist essentially of solving spherical triangles, whose solution represents no difficulty to a mathematician. However we should realize that if many mariners navigate their ships around the world safely even although they are not experts in mathematics, this is because continuous efforts are being made to prepare methods, tables, etc., that are fool-proof. In this article, therefore, the principles upon which different tables are based will not be analysed for this would primarily be of historical and bibliographic interest. The principles of astronomical navigation, its improvements and its merits are of interest only to the extent in which practical tables (or diagrams) can be published and instruments manufactured.

The practical value of sight reduction tables is generally understood to be the brevity, simplicity and uniformity of the procedure of sight solution. By comparing the number of tasks (such as book openings, tables entries, additions and subtractions, rules about signs, interpolations, etc.) necessary when computing the celestial position line a certain factor will be obtained. This factor, taken into consideration together with other properties described below (schemes of computation, types of tables, and rates of tabulation for entering arguments, number of pages, size, etc.), gives the possibility of obtaining an approximate assessment of the practical value of the tables. This, primarily from the point of view of navigators who use the tables and who, in the real meaning of the word, " feel " at once when tables are simple to use (enabling computations to be made rapidly, in a uniform way and without risk of error because of the multitude of rules about signs, the non-uniformity of procedures, unusual tabulation methods, etc.).

When the answers to questions 1 to 8 enumerated below are given with the best possible objectivity, and the questions 9 to 18 are also taken into consideration, the practical value of the tables can be determined with considerable approximation.

In order to make illustration of the subject easier these questions, with their respective abbreviations and explanations, are listed below.

In the sight solution procedure what is the number of :

1. Book openings. BO.
2. Table entries. TE.
3. Additions and subtractions. AS.
4. Rules about signs. RS.
5. Interpolations. In.
6. Changes of tabulated entering arguments or tabulated results; or is the same table used several times with different entering arguments not marked in the table. Ch.
7. Special cases. SC.
8. Steps in diagrams. SD.

Other properties :

9. Scheme of computation according to the number of principal steps in the procedure (simple when not over 7, average when from 8 to 10, complicated over 10) and uniformity of the procedure. Scheme.
10. Type of diagram used : simple (with two entering arguments or with two lines to be followed) or more complex (with more than two entering arguments or lines to be followed). Diagrams.

11. Usual type of tables : one-entry, two-entry, combined (one-entry and two-entry), three-entry;
    Rate of tabulation of latitudes and hour angles : whole degrees, or whole and half degrees, or whole minutes of time. Type of tables; Lat and HA.
12. Reading of figures : easy, average or difficult (because figures are small in size, the print not clear, lack of differentiation of type in the columns of figures). Reading.
13. Tabulation with limited values of latitude or declination, or with unlimited values. Limited or unlimited Lat. and decl.
14. Table limited to the assumed position or applicable to both the D.R. position and the assumed position AP. Limited to AP, or not.
15. Table compiled for sea navigation, air navigation or both. For Sea or Air navig.
16. Size in centimetres.
17. Number of volumes and number of pages. Volumes; pages.
18. Price.

When answering the above questions the number of the various tasks which may be encountered in sight solution should be given; easier cases, when certain rules about signs, interpolations, etc., are avoided, should not be cited.

In this article the words " interpolation " and " correction " are differentiated, although in other articles these expressions may sometimes be interchangeable. Accordingly, if the interpolation is made by means of the tabulated multiplier and the multiplication table it is not considered as an interpolation but as a correction. Thus for instance, in principle, in the well-known American tables H.O. 214, the interpolation of the tabulated altitude for the minutes of arc of declination lying between two tabulated declinations is considered as the altitude correction due to minutes of arc of declination because this interpolation is made by means of tabulated multiplier Δd and the multiplication table and not by means of mental interpolation between tabulated results, as in the case of the interpolation of azimuth angle in the same tables, which is treated as a true interpolation.

Regarding the data given below, which show the assessment of the practical value of the tables, the number given before the abbreviation indicates the total number of tasks when the assumed position (AP) is used. The additional number of tasks when the dead reckoning position (DR) is used is shown between brackets and preceded by the sign +.

Assessment of Tables K 1

Principal stages of procedure	Number of tasks by AP + DR	Description of the operations
I.	1 BO	For extracting from Table I the first part of azimuth ω + F, the auxiliary values M and C, and the sign of M with the entering arguments hour angle *s* and latitude φ. See example No. 1 given below.
	1 TE	— Idem —
II.	1 AS	For the algebraic addition of M and declinaison (M + δ).
III.	1 BO	For extracting from Table II the second part of azimuth F, altitude V, and indices of corrections *i*M δ and *i*C together with their signs with entering arguments M + δ and C.
	1 TE	— Idem —
	1 In	For F. This interpolation is not necessary in practical navigation.
IV.	1 BO + (1)	For taking altitude corrections for neglected minutes (over 00′.0 or 30′.0) of M + δ and C (plus corrections for latitude φ and hour angle *s* when DR is used) from Table IV for the M + δ, C and φ corrections, and from Table III for the *s* correction. See example No. 2.
	2 TE + (3)	— Idem —
V.	1 AS + (2)	For applying altitude corrections. (When a DR position is used a distinction is made between positive and negative corrections in order to obtain the computed altitude more easily).
VI.	1 AS	For subtracting F from ω + F to obtain azimuth ω.
Total	11 + (6)	

From the above data and the other properties of the Tables we shall answer the following questions in order to obtain the main characteristics for the assessment of these Tables and to make a comparison with other tables.

	Using AP	Using DR
1. Book openings	3	4
2. Table entries	4	7
3. Additions or subtractions	3	5
4. Rules about signs	—	—
5. Interpolations	1*	1*
6. Changes of the entering arguments	—	—
7. Special cases	—	—
8. Steps in diagrams	—	—
Total	11	17
or		or
* In practical navigation (see Note 4 below)	10	16

9. Scheme	Simple and uniform
10. Diagrams	None
11. Type of tables; Lat. and HA	Two-entry; at every whole and half degree
12. Reading	Easy
13. Lat. and decl. limited, or not limited	Not
14. Limited to AP, or not	Not
15. For sea or air navigation	Both
16. Size in inches	9.2 × 12.6
17. Volume; pages	1; 238
18. Price	$ 2.00

NOTE : 1. One general note regarding entering arguments applies to the main tables (Tables I and II). When the given values of the entering arguments are not equal to those tabulated, the nearest lower value of whole degrees or half degrees of the tabulated entering arguments should be used.

2. There are no rules about signs; all the signs (for M and the altitude corrections) are tabulated at the head of the Tables, and the user need only rewrite the sign.

3. The azimuth angle obtained is reckoned from the elevated pole of the observer through 180°, and should be labelled in the usual way : N or S to agree with latitude, and E or W to agree with the hour angle.

4. For very high altitudes, when the difference between neighbouring tabulated values of F is greater than 1°, F may be interpolated by inspection of the remainder of minutes of arc of the entering arguments (i.e. over 00'.0 or 30'.0) if greater accuracy of azimuth is desired. In the usual practice of navigation this interpolation is not necessary.

5. It is interesting to note that the U.S.N. Hydrographic Office publication " American Practical Navigator—Bowditch" (H.O. Pub. No. 9) 1958 edition, pages 539 and 540, gives nearly the same assessment of KOTLARIĆ's Tables K 1 as this article. It is there said : " With the assumed position so selected that latitude and meridian angle are the nearest whole or half degrees, the method requires only 4 table entries and 4 mathematical steps ". This means 8 operations in all and if 3 book openings are added the total factor will be 11, i.e. the same as the author of this article has assessed.

The formulae on which the Tables K 1 are based are not given here, as this has not been considered necessary; the method was explained in the May 1956 edition of the *I.H. Review* in the article " New Methods of Ship Position Finding from Celestial Observations ", and extracts of the Tables were there given.

Example 1 (using Assumed Position)

Find the computed altitude Hc and azimuth Az if latitude (φ) is 41° N, meridian angle (s) is 10°30′ E and declination (δ) is 0°34′.1 N.

Solution with Tables K 1

With three book openings the problem is solved.

First		δ	+ 0°34′.1		
ω + F	173°.1	...... M	+ 48°31′.2	.... C 82°05′.7	
		M + δ	49°05′.3		
Second					
— F	9.1	.... Altitude (V)		48°21′.8	
Az (ω)	N 164°.0 E	.... iMδ	98.3 , 5′.3 corr. +	5.2	Third
		 iC	15.3 , 5′.7 corr. +	0.9	
			Hc	48°27′.9	

Checking by Haversine method

using the following formulae for altitude (Hc) and azimuth angle (Z) :

$$\text{hav } z = \text{hav }(L - d) + \cos d \cos L \text{ hav } t$$
$$\text{hav } \theta = \cos d \cos L \text{ hav } t$$
$$\text{hav } z = \text{hav }(L - d) + \text{hav } \theta \;;\; z = 90^\circ - Hc$$
$$\sin Z = \sin t \cos d \sec Hc$$

t	10°30′.0 E	.... log hav	7.92 286	.... log sin	9.26 063
d	0°34′.1 N	.... log cos	9.99 998	.... log cos	9.99 998
L	41°00′.0 N	.... log cos	9.87 778		
		log hav θ	7.80 062		
		nat hav θ	0.00 631		
L — *d*	40°25′.9	.... nat hav	0.11 941		
z	41°32′.1	.... nat hav	0.12 572		
Hc	48°27′.9	....		log sec	0.17 842
Z	16°.0	....		log sin	9.43 903
Az	N 164°.0 E				

NOTE : Checking showed that the results for Hc and Az, obtained by the above two methods, are the same. However, the haversine method, in fact, represents a logarithmic solution of the basic formulae and this method does not compare favourably with the so-called short-method tables. Solution by the haversine method involves 9 book openings, 9 table entries, 9 interpolations, 5 additions or subtractions and one complex rule for the azimuth sign, which makes in all 33 operations for a DR position, or 30 for an assumed position. This means three times more than for Tables K 1 when the assumed position is used.

Example 2 (using Dead Reckoning position)

Find the computed altitude Hc and azimuth Az if latitude (φ) is 19°25′ N, meridian angle (*s*) is 3°38′ E, and declination (δ) is 5°48′ N.

Solution using Tables K 1

With only four book openings the problem is solved.

		δ	5°48′.0 N			
ω + F	178°.9	M	70 58′.0 N	C	86°41′.5	
		M + δ	76°46′.0			
— F	14°.3	Altitude (V)			76°03′.7	
Az (ω)	N 164°.6 E	*i*Mδ	96.7, 16′.0 corr.	+	15′.5	
		*i*C	23.0, 11′.5 corr.	+	2′.6	
		L (φ)	25′.0 corr.			— 24′.0
		t (*s*)	8′.0 corr.			— 2′.0
			Sum		76°21′.8	— 26′.0
			Hc		75°55′.8	

Checking by logarithmic solution of the basic formulae

$$\cos z = \cos (L - d) - 2 \sin^2 \frac{t}{2} \cos L \cos d$$

$$z = 90° - Hc$$

$$\sin Z = \sin t \cos d \sec Hc$$

		log 2	0.30 103		
t	3°38'.0 E	log hav	7.00 216	log sin	8.80 189
d	5°48'.0 N	log cos	9.99 777	log cos	9.99 777
L	19°25'.0 N	log cos	9.97 457		
		log hav Σ	7.27 553		
		— nat hav Σ	0.00 189		
L — d	13°37'.0	+ nat cos	0.97 189		
z	14°04'.1	nat cos	0.97 000		
Hc	75°55'.9	....		log sec	0.61 425
Z	15°	....		log sin	9.41 391
Az	N 165° E				

Some explanations regarding the assessments made for Tables K 1

1. In Tables K 1 the system of using the next lower whole or half degree is applied for the entering arguments (except in multiplication tables III and IV where the nearest values are used). For instance, if latitude is 32°47'.6, Table I should be entered with 32°30' and the multiplication table with 17'.6, so that mental addition of 30 + 17.6 = 47.6 is so easily done that it does not represent as important an operation as the rule about signs operations. The remaining minutes of arc above 00' or 30' are found very easily indeed, so that this operation made by inspection is considered insignificant and it has not been counted in the analysis of the number of tasks necessary for taking out a value from the multiplication table. But in some other short-method tables where the entering arguments have to be rounded off to the nearest tabulated value, the situation is different. In such a case, for the above example and using the nearest tabulated argument, we should have to make the normal operation of subtraction, i.e. 33°00'.0 — 32°47'.6 = 12'.4, in order to find the remainder of minutes of arc to be used in the multiplication table.

2. In the sight solution procedure the subtraction $(\omega + F) - F = \omega$ and the addition $M + \delta = (M + \delta)$ are only counted as two operations. Instructions that F has to be subtracted from $(\omega + F)$ in order to obtain ω, and M added to δ for obtaining $(M + \delta)$ are not counted as rules. This is, however, due to the fact that, in the practical use of Tables K 1, it was not necessary to write the special rules on either the top or the bottom of the page, to indicate to the users how to obtain the azimuth ω and the value $M + \delta$, because the abbreviations themselves show this in a very clear way that is understandable to the mariners of all nations. It is quite certain

that there is no navigator, regardless of the language he speaks, who could not understand this on seeing in the sight solution procedure the following :

$$\begin{array}{lll} \omega + F = 125^\circ & \delta = +12^\circ & \text{or} \quad \delta = 22^\circ\ S \\ \quad - F = 25^\circ & M = +32^\circ & \qquad M = 52^\circ\ N \\ \hline \omega \quad = 100^\circ & M + \delta = 44^\circ & \qquad M + \delta = 30^\circ \end{array}$$

This means that the abbreviations used in this way were so suitably selected that at the first glance they themselves indicated what to do, eliminating the necessity for a special rule.

3. It is the same for the rule for the value of M. It is a principle of the method K 1 that there is a rule that M carries the same sign as latitude if the meridian angle is less than 90°, and a contrary sign to the latitude if the meridian angle is greater than 90°. In Tables K 1, however, M has been so suitably tabulated, and its signs already printed at the head of the Table I, that there is no need to insert the rule on all the pages of Table I, because when taking out the value M the user has only to rewrite the sign. For the columns with a two-fold entering argument the signs for M are separated by a limiting line showing clearly the sign belonging to each entering argument. The abbreviations there printed (" ist. φ " for a meridian angle less than 90°, and " raz. φ " for a meridian angle greater than 90° have the meaning " same name as latitude " and " contrary name to latitude ") are as clearly understandable to Yugoslav navigators as, for example, the signs " + " and " — ", or " N " and " S ". For foreign navigators, in order to reduce turning of leaves and to facilitate the use of the Tables, the author has added separate instructions for use, printed in English on the fly-sheet of soft yellow cardboard, where all the main Yugoslav abbreviations are also explained. Shortened instructions in English, consisting of only three sentences, with corresponding colour sketches, together with an explanation of the abbreviations, are given separately on the white cardboard fly-sheet. In this way foreigners may use these Tables with ease, and in a short time they will become familiar with these few Yugoslav abbreviations. Of course, it would be better if the Yugoslav abbreviations in the Tables were replaced by abbreviations in the language of the users, but if foreign issuing authorities are interested in this question this can be arranged with the author.

To conclude this discussion, it is important to point out that the principle of solution could sometimes be simplified by suitable tabulation. The table-maker must know the users' requirements and be able to find out the most suitable way to meet these requirements and to avoid those factors which the users consider tedious.

4. In Table II the indices of altitude corrections *i*Mδ, *i*C are tabulated with a decimal figure although in the instructions for use it is clearly pointed out that the multiplication table (Table IV) is entered using the nearest tabulated value of *i*Mδ and *i*C, without any intermediary interpolation. Why then are the decimal figures printed ? There are several reasons, e.g. :

(*a*) Although the interpolation for *i*Mδ and *i*C in the multiplication table is not necessary, there are older and conservative navigators who prefer working out the sight from the D.R. position, and if they see that the intercept obtained with a short table method is not equal to that obtained with the old logarithmic method with which they are familiar, they will usually hesitate to accept the new short-table method. In order to reach such accuracy, and to avoid possible incorrectness of 0′.1 for each entry in the multiplication tables (Tables IV and III) without interpolation (i.e. using the nearest entering argument to the left or the right margin), those navigators could then make the interpolation, but gradually they will be convinced that this is not necessary as is stressed in the instruction itself. This is why it is sometimes also necessary to take into consideration this factor if the author wishes to satisfy older and conservative navigators as well as the new and modern ones.

(*b*) Such detailed tabulation of the values *i*Mδ and *i*C enables easy checking of the tabulated altitudes, and gives more certainty of printing these numerical data without error.

(*c*) Oceanographic measurements obtained at the present time are of special importance, and during these operations more accurate celestial position lines are necessary than is the case for ordinary navigation, therefore the use of interpolation of *i*Mδ and *i*C could be of interest in these special cases.

5. Regarding the signs for altitude corrections, it is to be noted that working with Tables K 1 it is not necessary to compare the tabulated altitudes in order to find out whether they increase or decrease as the tabulated entering arguments approach the exact value, neither it is necessary to apply special rules to determine the sign of the altitude correction due to the neglected minutes of arc of latitude. All signs for altitude corrections in Tables K 1 are printed in the headings of the Tables.

6. For the tabulated entering arguments it was said that in Tables K 1 these are rounded off to whole and half degrees. This kind of tabulation has been preferred to the other tables having entering arguments (hour angle and latitude) tabulated only to every whole degree for the following two reasons. Firstly, the assumed position is closer to the actual position of the observer, and it is unnecessary to rectify the line of position, due to a large intercept, leading to a lack of coincidence of the circle of position and the usual line of position laid off perpendicular to the bearing of the celestial body. Secondly, in practice, there is no necessity for interpolation of the azimuth angle because these values are tabulated with 4 times closer intervals than in the tables having entering arguments (latitude and hour angle) rounded off to whole degrees only.

In spite of their more detailed tabulation of the entering arguments, Tables K 1 are relatively small in bulk; only 238 pages for a single volume comprising all latitudes for the observer and all declinations of celestial bodies. Of course, the small number of pages is reflected in the low price of the tables, not only for the customer but also for the publisher. The explanations in English contained in the tables give them the possibility of being used by navigators of different nations.

Conclusion

This method of determining the practical value of sight reduction tables could be of help to mariners in making both the analysis of other types of tables and the approximate assessment of their practical value. The comparison of the shortcomings and the advantages would give them more chance to select those tables which meet their practical requirements in the best way. The makers of the tables might also interest themselves in this question, because the practical value of tables depends firstly on their ingenuity.

The assessment which I have given in this paper on Tables K 1 shows a practical example of the application of this method. It will enable some other navigator or table maker to make a similar assessment for other tables, and by solving the same examples of sight solution a comparison of the tables could be easily made. However, the checking of the computed altitude and azimuth by some logarithmic method is also advisable in order to verify the accuracy of the tables. This is also one of the factors which should be used in assessing the tables.

Finally I would like to use this opportunity to give the answer to a possible criticism of this paper. It could be said that I have drawn it up to show my own tables in a more favourable light than the other tables. In this connection I should say that the main reason for studying this subject, however, was neither the desire to write a paper, nor to review my own or other tables, but rather the necessity which I encountered of having to make a very detailed analysis of the practical value of a large number of methods and tables of sight reduction, because after I solved theoretically the mathematical basis for my Tables K 1 and completed the computation work on the manuscript, I was obliged to find out whether my Tables were mature for printing or whether they should still be improved before sending them to print. Under the term "mature for printing" I have considered that the Tables represent not only a new method of solution of the astronomical triangle, but also that they offer certain advantages (simplicity and shortness) in the procedure of computation of altitude and azimuth (position line). In such an analysis and comparison I have noted various remarks and found shortcomings in the different types of tables. I have avoided quoting them, because by application of this method not only experts in table-making but also ordinary navigators will be able to ascertain the advantages and shortcomings of the tables they use or have the intention to publish or to buy.

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