SOME COMPARATIVE TESTS OF TIDAL ANALYTICAL PROCESSES

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Abstract

Tests have been conducted on five analytical processes and the results have been examined by statistical and power spectral techniques. Tide observations at Atlantic City (small range and large low-frequency noise), Swansea (large range and noise primarily in tidal frequencies), and San Francisco (small range and moderate low frequency noise) were used in the study. Residuals were obtained by subtracting the predicted hourly heights from observations and were evaluated for total energy (variance) and energy per frequency band. The latter calculation was found to be useful in comparing residual energy for particular portions of the frequency spectrum.

Introduction

The advent of electronic computers has made possible new approaches to tidal analysis. Numerous organizations have initiated least squares analytical approaches in which a set of fixed frequencies (speeds of tidal constituents) is proposed to fit the data so that the sum of the squares of the residuals is a minimum. This modified the traditional approach, as exemplified by Doodson (1928) and Schureman (1941), in which only one constituent is examined at a time and the results are modified for the interference effects of other constituents in the same species. Munk and Cartwright (1966) have recently developed a response method of prediction in which the input functions are the time-variable spherical harmonics of the gravitational potential and of radiant flux on the Earth's surface.

This study was initiated to permit comparative evaluations of some of the proposed analytical processes. Inasmuch as Munk and Hasslemann (1964) have shown that the ability to separate two frequencies depends on the noise level, it was decided to conduct the tests with real data. The tests have been conducted on the Coast and Geodetic Survey method (Schureman, 1941), the Harris-Pore-Cummings (1963) least squares method that has been accepted by the Coast and Geodetic Survey for series of one year, the DOODSON method as treated by LENNON (1965), the MURRAY (1963) least squares method, and the MUNK-CARTWRIGHT (1966) method. The data, hourly heights of tide, included Atlantic City (1939), Swansea (1961-1962), and San Francisco (1931 and 1939).

Atlantic City Comparisons

This is the only test in the study that involves all five methods. Table 1 shows the results obtained by analyzing the data, predicting for the same period by using the derived constants, subtracting the predictions from the observations, and examining the residuals. At first, only the total variance of the residuals was obtained, but it quickly became obvious that a more refined procedure was needed because the small differences between total variances appeared relatively insignificant. Therefore, the power spectra were calculated to permit comparisons of residual energy at various frequencies. A peak of energy in the low frequencies shows up clearly on figure 1 (the spectrum of residuals from the Doodson method), even though the energy scale (vertical) is logarithmic. Atlantic City is directly on the open coast and the shallow water fetch is short. Hence, the non-linear interaction terms (ordinarily manifested as compound tides in the high frequencies) are small. However, the station is wide open to the effect of all storms on the North Atlantic and the results confirm that the noise is due to storms having effective periods of greater than a day. Figure 2 shows the large daily fluctuations in mean sea level for the year analyzed.

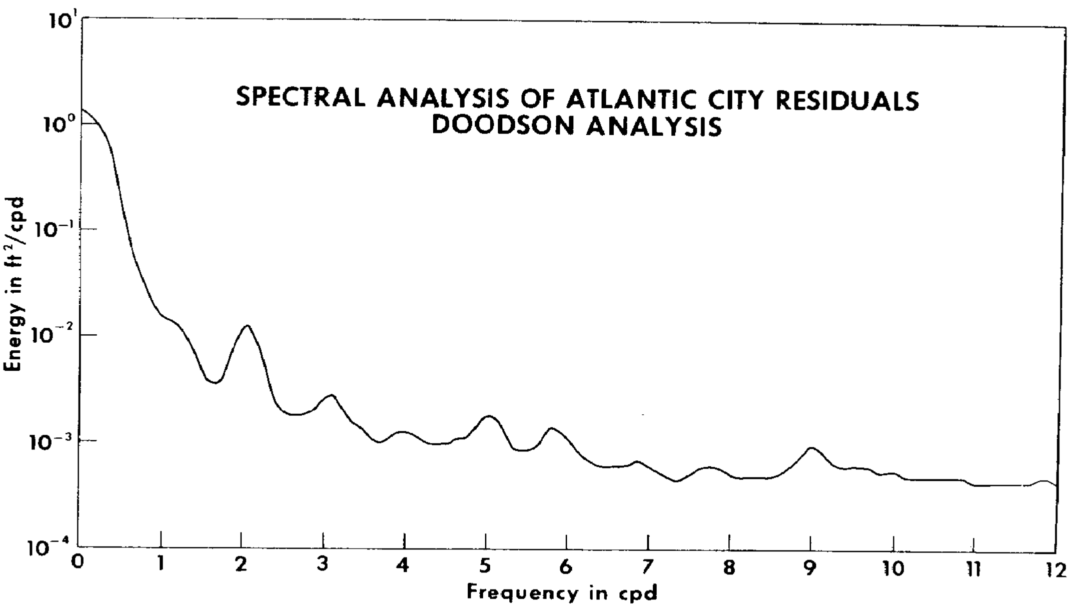

FIG. 1. — Spectrum of residuals from Doodson method for Atlantic City, 1939.

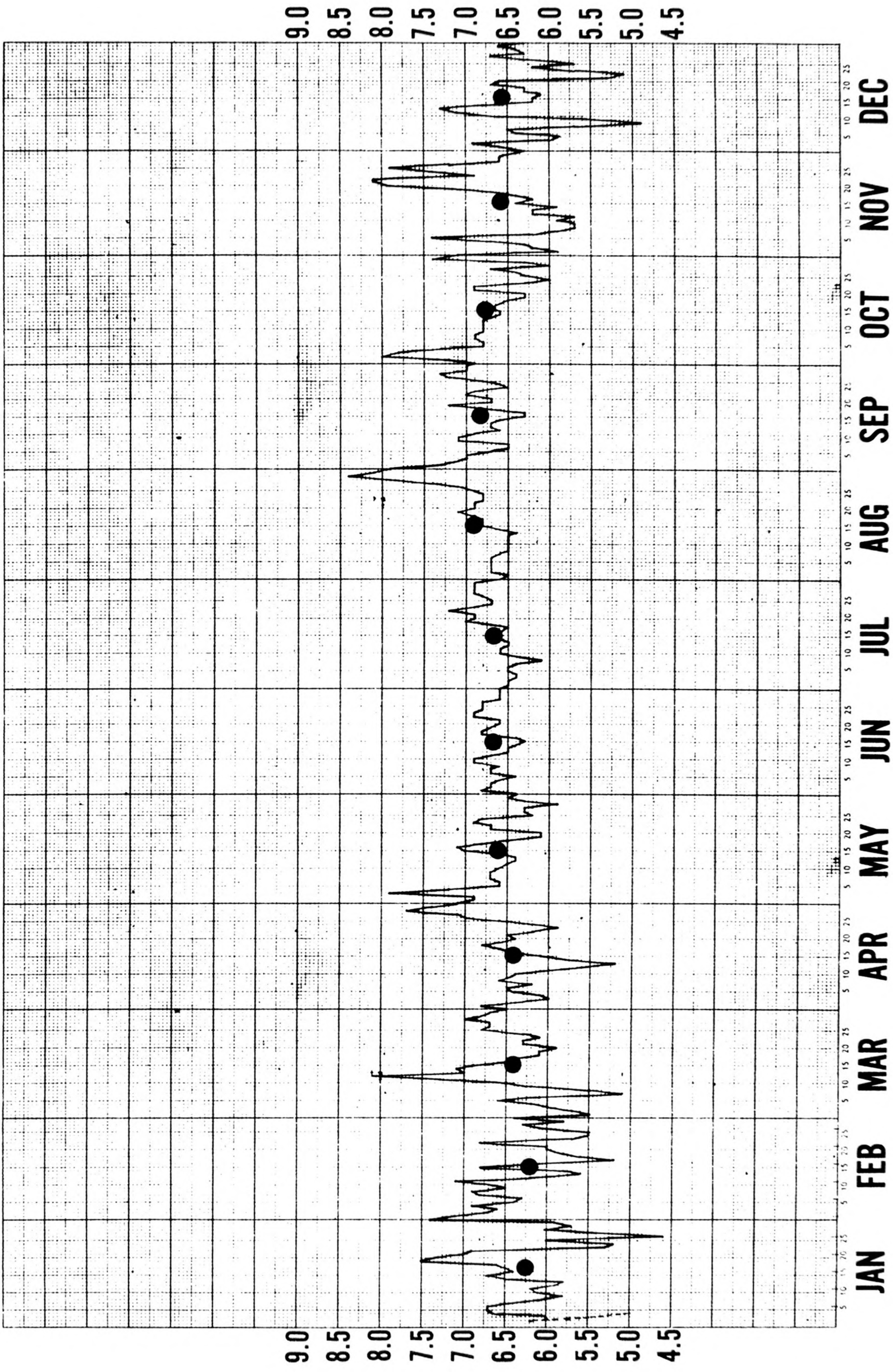

Fig. 2. — Daily sea level at Atlantic City, 1939.

The degree of resolution in the power spectrum analysis was chosen to adequately separate the species, to minimize the computer effort, and to have sufficient degrees of freedom to provide reasonable confidence in the results. Inasmuch as the data are hourly, the Nyquist frequency is 12 cycles per day. Using 100 lags in the autocorrelations, we get a " delta f " of .12 cpd which essentially separates the species by about eight values, a satisfactory separation. The number of degrees of freedom in a Tukey analysis is roughly 2 N/M, where N is the number of data points and M is the number of lags used in the autocorrelations (MUNK *et al.,* 1959). With a year of data, there are about 170 degrees of freedom, a rather conservative and satisfactory number.

The results shown in table 1 show the Munk-Cartwright method to have a small advantage over the two least square methods. The small differences between the Murray and Harris-Pore-Cummings methods may be caused by other factors, such as the number of significant digits used in the calculation. Both least squares methods appear to do slightly better than the Doodson method. Considering the limitations on accuracy imposed by the minimizing of correction processes when hand computations were the order of the day, the Doodson method does remarkably well. The fact that the Harris-Pore-Cummings does as well as the Murray with 19 fewer constituents relates to the particular station being analyzed, the additional constituents (primarily compound tides) are essentially trivial (all are less than .02 foot) and therefore do not give the Murray (or Doodson) method any significant advantage for this particular station. The Harris-Pore-Cummings computer program is dimensioned for 41 constituents and this was considered a limiting characteristic at the time the computations in this study were made. Since that time the program has been used with as many as 114 constituents for Anchorage (ZETLER and CUMMINGS, 1966) simply by separating the constituents into groups and using the restriction that all constituents within any species had to be included in the same group. As a matter of fact, this approach reduces the computing effort. For example, it is far simpler to reduce three 40 × 40 matrices than one 120 × 120 matrix. The poorer results for the traditional C&GS method are clearly related to the small number of constituents in the solution and there is no doubt that it would have moved into the general area of the other results had a greater effort been considered warranted. The C&GS harmonic constants for Atlantic City were taken from the archives and were typical of the methods that were used twenty years ago when manual stencil summation for each constituent limited strongly the number of constituents sought. In more recent years, summations have been made on an electronic computer but the remainder of the process is continued by hand (SCHUREMAN, 1941). There is no question that the remainder of the process can be programmed for an electronic computer and it has recently been done for 15- and 29- day series. It has not been programmed for a year by the Coast and Geodetic Survey because the least square approach has been shown to be acceptable and to have greater flexibility in the choice of series length and by virtue of not necessarily requiring data equally spaced in time (ZETLER *et al.,* 1965).

Consideration was given to plotting in one illustration the spectra for all five sets of residuals. In general they are so close together that the composite graph would be confusing. Therefore only one is shown in figure 1 to more or less typify the station characteristics.

Swansea Comparisons

Although the test conditions for the Swansea data were comparable to those for Atlantic City, it is clear that the results in table 1 cannot be interpreted in terms of comparative evaluations of the Harris-Pore-Cummings and Doodson methods. Unlike the conclusions for Atlantic City, it is obvious that some of 23 additional constituents (60 compared to 37) improve the prediction and thereby reduce the residual variance.

In species 2 (two cycles per day) there are six constituents in the Doodson analysis that are not included in the standard Coast and Geodetic Survey analysis. The total energy in these six (using energy equals one half amplitude squared) is .0256, somewhat greater than the .0173 difference obtained from the table 1 values for residual variance in species 2. Inasmuch as the calculated values for one constituent can change if another constituent in the same species is added to the model in the least square analysis, no rigorous comparisons can be made from the results. On the other hand, it seems obvious that if the same model (set of constituents) were used with both methods, it is unlikely the results would differ significantly. Therefore the principal interest in these results is in Figure 3 where the contribution of the additional constituents is demonstrated in the various species. Even here, in considering these comparisons, it is important to keep in mind that the vertical scale is logarithmic.

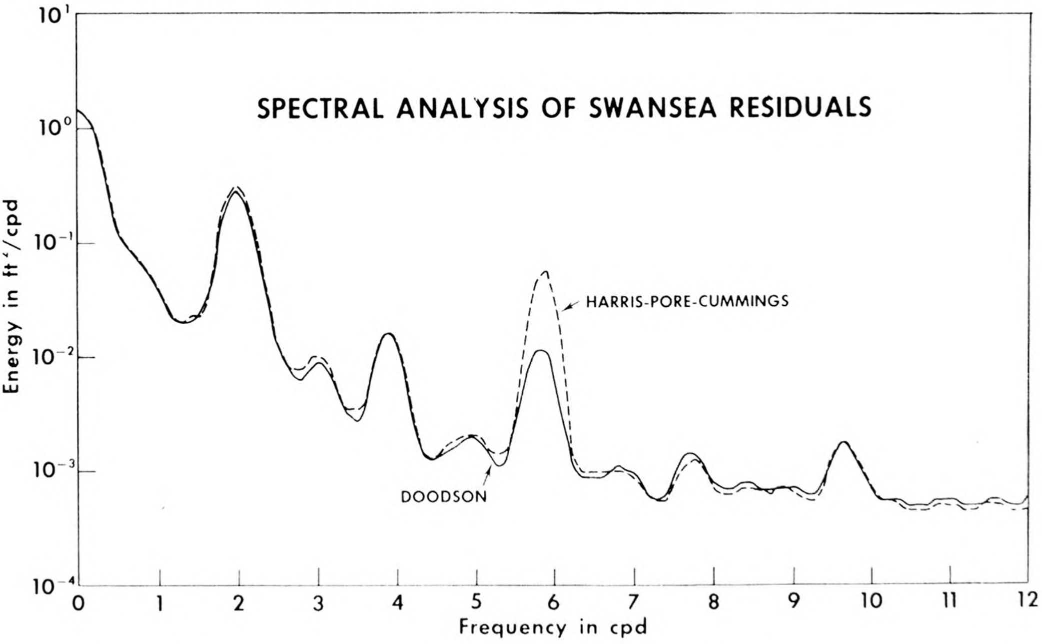

FIG. 3. — Spectra of residuals from Doodson and Harris-Pore-Cummings methods for Swansea, one year (1961-1962).

TABLE 1

Residuals from Analysis of Tide Data
(energy in feet^2)

Method of Analysis	Number of Constituents	Observed Energy	Residual Energy	
			Total	Near 2 cpd
	Atlantic City, 1939			
Murray Least Square	60	2.6236	.3135	.0046
Doodson	59	2.6236	.3158	.0047
Munk - Cartwright Response	59*	2.6236	.3118	.0041
Harris - Pore - Cummings Least Square	41	2.6236	.3139	.0044
Coast & Geodetic Survey (Schureman)	14	2.6236	.3224	.0069
	Swansea, 1961-1962			
Doodson	60	66.8968	.4462	.1065
Harris - Pore - Cummings	37	66.8968	.4872	.1238
	San Francisco, 1931			
Coast & Geodetic Survey	18	3.2649	.0680**	—
Munk - Cartwright	59*	3.2649	.0490***	—
	San Francisco, 1939			
Coast & Geodetic Survey	18	2.9660	.0913**	.0076**
Munk - Cartwright	59*	2.9660	.1082***	.0023***

* Number of weights used in Munk-Cartwright prediction. Harmonic predictions have two variables per constituent.

** C&GS predictions based on the mean of 11 one-year analyses; Sa and Ssa on 19 years.

*** Munk-Cartwright predictions based on analysis of 1931 only.

San Francisco Comparisons

The results shown in table 1 for the Coast and Geodetic Survey and the Munk-Cartwright methods for San Francisco are derived from different basic data and therefore they are not directly comparable. Nevertheless, some of the results are quite interesting.

The C&GS predictions for San Francisco for both 1931 and 1939 are based on mean harmonic constants from 11 one-year analyses and the annual and semi-annual constituents (Sa and Ssa) are based on 19 years of monthly mean sea level. The 1931 Munk-Cartwright predictions were based on an analysis of the same year and therefore it was to be expected that the residual variance would be less. Spectra were not computed for the 1931 residuals.

The 1939 Munk-Cartwright predictions were made with weights derived from just one year, 1931. Table 1 shows the C&GS ahead on total variance but behind in species 2. Figure 4 shows comparisons in each species.

The big difference is in the low frequency portion of the spectrum. Although Sa and Ssa fall within the first frequency band (0 to .06 cpd), one effect of tapering the autocorrelations is to widen the main pass band to include the two bands on either side, in this case to 0 to .30 cpd (Munk *et al.*, 1959). Inasmuch as Sa and Ssa are due to thermal variations (the astronomical input to Ssa is small), they are only quasi-stationary and a mean of 19 years of data can be expected to furnish a more reliable predictor than just one year. Therefore, it is not surprising to find the sum

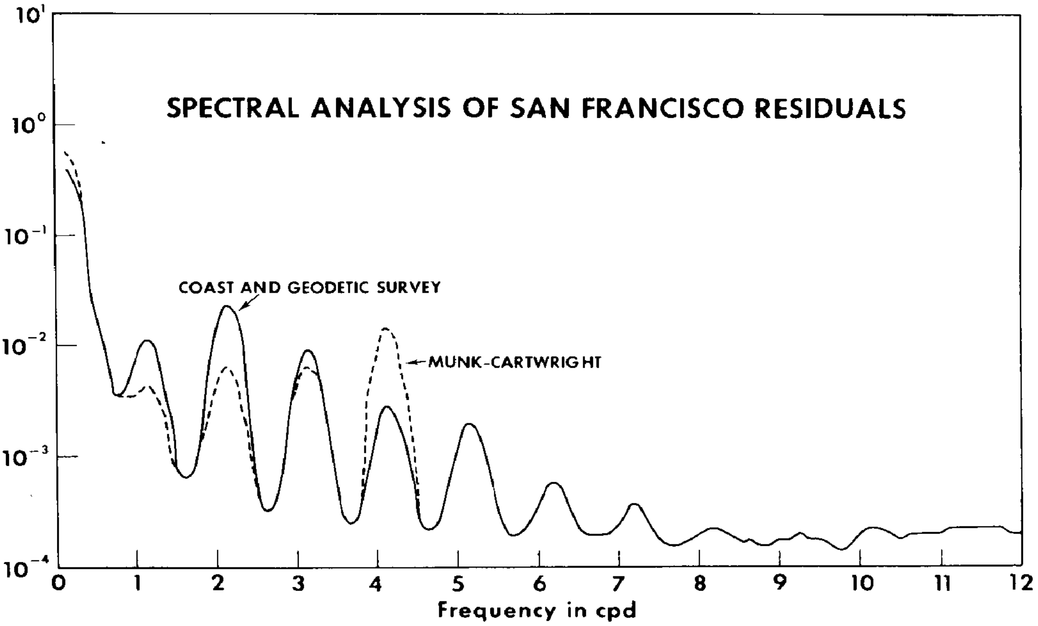

Fig. 4. — Spectra of residuals from Munk-Cartwright and Coast and Geodetic Survey methods for San Francisco, 1939.

of residual energy from 0 to .30 cpd is .0898 for the Munk-Cartwright method and .0683 for the C&GS method.

The Munk-Cartwright method does remarkably well with stationary phenomena in species 1, 2, and 3. Part of its apparent advantage is tempered by the smaller number of variables in the C&GS method ($2 \times 18 = 36$ compared to 59) but nevertheless it is quite an achievement to do so much better with the one year of input compared to 11 years for the C&GS method.

The C&GS method appears to have done better in species 4 but actually there was no contest. The Munk-Cartwright prediction did not include an input for bilinear interaction of species 2 with itself which would have produced a species 4 prediction. A bilinear input for the interaction of species 1 with species 3 could have been used also but this would have been considerably less important.

Summary

Objective test procedures have been developed for comparing tidal analytical processes. The use of power spectral techniques for evaluating residual energy for particular species has been found to be a useful tool. As a direct result of these tests, the Coast and Geodetic Survey has accepted the Harris-Pore-Cummings least squares procedure for the routine analysis of series of one year in length.

In retrospect, it is unfortunate that the only direct comparison of the five analytic methods was done with the least complicated set of data. This deprives the more sophisticated methods of a chance to demonstrate competence to cope with additional complications. Perhaps the study can be extended in the future to include various methods for predicting shallow water tides.

During the tests, it was found that the residual variances from the Harris-Pore-Cummings method did not match the claimed reduction in variance computed as part of the program. The resulting investigation found a small error in the program and this was corrected. The data shown in this paper are from the corrected program and therefore some values do not match the data in a preliminary report (ZETLER and LENNON, 1966). The detection of the discrepancy was an unanticipated by-product of the study.

An initial objective of this study was to compare computer cost as well as accuracy of analysis. This becomes confusing for various reasons, particularly since the same computers at different installations may have significantly different costs. With one exception, the Munk-Cartwright method, the computer costs are so low that differences are not very significant. Unlike the other methods which were developed for routine production usage, the Munk-Cartwright method was devised for research purposes. It would be unfair to compare it on the basis of cost until some effort has been made to make the program more efficient in a cost sense.

Possibly the greatest importance of the study is that representatives of tidal offices in two countries have agreed to use the same data to evaluate their analytical processes and have proposed criteria for the comparisons.

The authors hope that this is the beginning of a broader international program along similar lines.

Acknowledgement

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