TIDE AND TIDAL CURRENT PREDICTIONS BY DIGITAL COMPUTER IBM 7090

by Tito A. CIRIANI
Centro Studi IBM, Pisa
and Carlo STOCCHINO
Centro nazionale per la Fisica dell'Atmosfera
e la Meteorologia del Centro Nazionale delle Ricerche

PREFACE

During the last decade the use of electronic computers has had a great impulse and there have been numerous applications in the various fields of scientific research.

Since immediately after the second World War the use of electronic computers has developed to such an extent that they successfully replace many of the old analogue computers used in the different branches of technical work.

Consequently even machines used for tidal predictions were no longer able to be compared to the new computers and in general went out of use because the new machines could be adapted to many uses whereas the machines commonly used in the field of tides were only useful for the summation of harmonic terms.

Even although electronic computers have now replaced mechanical analogue computers with advantage, at the beginning there remained many problems concerning their use in the field of tidal prediction. Such problems were above all of a technical and financial order.

In fact, the computer developed by Lord KELVIN and E. ROBERTS for tidal predictions gave, and is still giving, precious service in this field.

The machine realised by Lord KELVIN has, since is inception, always retained intact its fundamental principles though it has undergone great improvements in the course of years. Even though new apparatuses have been built according to the latest modern techniques the original idea has always remained the same.

As to the cost of the use of the new computers, this may have appeared prohibitive at the beginning in relation to the quantity of work required, which could easily have continued to be performed by mechanical machines. Subsequently, however, this no longer appeared as an obstacle when the high performances offered by electronic computers are considered as these control the results and suitably prepare them for publication.

Moreover the adopted methods of prediction permit the resolution easily and without risk of errors of the two fundamental problems in mechanical prediction, i.e. the calculation of hourly heights and of the times and heights of high and low tide, and the calculation of shallow water corrections.

It is even possible nowadays to extend the calculation until the maximum accuracy is obtained, and to resolve particular problems of tidal prediction in those harbours where, due to specific meteorological conditions, the tide curve shows relatively flattened portions.

With the arrival of electronic computers an entirely new organisation for the prediction of tides came into being, resulting in great reduction in calculation time, high accuracy, lower cost and less personnel needed for that problem, releasing personnel who could thus devote themselves to duties of equal importance.

At the Hydrographic Institute of the Italian Navy, in view of the reorganisation of the tide gauge network at present being undertaken by the National Council of Research, a long programme of work has been scheduled which envisages the use of electronic computers.

This programme besides testing the actual conditions in all tidal observatories along the coasts of the peninsula also includes the calculation of tidal harmonic constants for those ports which would otherwise be without such information, as well as predictions for the times and heights of high and low tides and of tidal current velocity for some of the main tidal observatories along the Tyrrhenian, Adriatic and Ionian coasts.

Up to the present the tidal harmonic constants have been calculated for the port of Genoa, using tidal observations made over a period of 18 years.

The calculation was made on an IBM 1620 electronic computer using a programme based on already known methods of analysis.

Furthermore, with regard to tidal prediction, times and heights of high and low tides have been calculated for the ports of Genoa, Venice, Trieste and Porto Corsini (Ravenna), and predictions of tidal current made for Venice (Canal of Lido harbour) and for two localities in the Straits of Messina (near Ganzirri and Punta Pezzo).

Both the programme of prediction and analysis and the corresponding calculations have been carried out with a financial contribution from the National Council of Research.

GENERALITIES ON THE PREDICTION PROGRAMME

Both for the prediction of tides and of tidal currents the maximum and minimum daily measurements are found by searching for the zeros of a function of sinusoidal character.

The latter is none other than the derivative of the expression providing the heights of the tide with respect to time in relation to a fixed time.

The regularity of tides and tidal currents observed in the readings carried out in some of the aforementioned ports allows phases of major interest to be identified with remarkable accuracy. These are the time and height of high and low tides, the time and the maximum values of current velocity in phases of ebb and of flow and also the time of zero velocity (slack water).

Such possibilities, however, depend upon the accuracy with which the sine and cosine functions are calculated and upon the number of times they must be calculated for each height (or speed) predicted.

A reduced number of sinusoidal constituents in the function considered is therefore advantageous for computational accuracy.

This is what occurs for the ports under consideration. Moreover, instead of using tables and then calculating the sine and cosine function by means of interpolations it has been preferred to make use of the serial developments already prepared in the Fortran IV language in the form of sub-programmes.

These sub-programmes compute the sine and cosine functions with a relative error of less than 0.34×10^{-8} and 0.73×10^{-10} respectively. Such a choice has led to really exceptional results being obtained in an extremely short computing time.

MATHEMATICAL DEVELOPMENT OF THE PROBLEM

The function used for predicting the instant and the height of the tide is the following :

$$h = H_0 + \sum_{n=1}^{9} f_n H_n \cos [\omega_n t + (E+u)_n - g_n] \qquad (1)$$

where :

h = height of the tide at instant t;
H_0 = reference water level used in the prediction;
H_n = amplitude of the n^{th} constituent;
f_n = reducing factor of H_{nth}, in relation to the year of prediction;
ω_n = hourly speed of the n^{th} constituent;
t = time, in hours, from the beginning of the year of the prediction;
$(E+u)_n$ = Greenwich equilibrium argument of the n^{th} constituent when $t = 0$;
g_n = phase lag of the n^{th} constituent.

For the ports for which predictions have been made nine components were used :

M_2, S_2, N_2, K_2, K_1, O_1, P_1, M_4, MS_4

The amplitude H_n and the phase lag g_n of each constituent are constant, depending upon the locality for which the prediction is required, and have been determined by means of harmonic analysis from a long series of tidal observations.

The nodal factor f_n, used to determine the true amplitude of each component, and the equilibrium argument $(E + u)_n$ of each component depend upon the year for which it is desired to calculate the prediction.

The angular speed ω_n of each constituent is constant.

Function (1) is also used for predicting currents. We may observe, in fact, that the same periodical constituents are used in predicting both the heights of the tide and tidal currents. Furthermore the nodal factors f_n and the equilibrium argument $(E + u)_n$ are the same as for tidal heights.

Since the time and maximum speed of the current are calculated in the same way as for tide heights this means that the programme for tide heights can also be used for currents, except for small modifications necessary when the zero velocity instant (slack water) is sought.

The amplitudes regarding tidal currents that appear in function (1) are given in knots, the hourly speeds are therefore also calculated in knots.

The constant H_0 in this case signifies a permanent current. In function (1) the impossibility of expressing it as a simple product of sinusoidal functions is immediately recognised. It is therefore impossible to find a function of the arguments for which the zeros, maximums or minimums are easier to determine. The only thing left to do is therefore to find by numerical methods the solution of the transcendental equation obtained by deriving function (1) and equating it to zero. Furthermore, the need to know the time of slack water for tidal currents makes it necessary also to consider the solutions of the equation obtained by equating function (1) to zero.

The derivative of function (1) has the following sinusoidal form :

$$\frac{dh}{dt} = - \sum_{n=1}^{9} f_n H_n \omega_n \cos \left[\omega_n t + (E+u)_n - g_n\right] \qquad (2)$$

The result is that the same method can be used for calculating both tides and tidal currents.

NUMERICAL SOLUTION

The methods for the numerical solution of transcendental equations fall into two fundamental categories :

a) Methods that make use of the behaviour of the derivative;

b) Methods that find zero only according to the function's sign.

Type *a*), which belongs to the Newton-Raphson method, cannot be used unless the derivative of the function is far from zero and near the solution.

Therefore the *a*) type methods, although converging considerably more than the second category, cannot be taken into consideration because of the very nature of tidal problems.

In fact, the functions of sinusoidal type show a tangent which can be zero at an infinite number of points. Furthermore, these functions have an unlimited number of zeros and are not even easily found with a variable frequence, as is the case with function (1).

The *b*) type methods have a simplicity of formulation, but unfortunately a slow convergence.

If the case of the secant method is excluded as it is applicable only in surroundings where the zero of the function exists, we see that the *b*) type methods are very general.

The *inversion of sign* method guarantees that all the zeros of the transcendental equation in the interval considered are found, upon condition that the minimum interval in which a single solution exists be known. Another advantage of the inversion of sign method is that the accuracy depends only on the number of significant figures used inside the computer. In fact, it would be possible to arrive at an accuracy of up to the last significant figure if the rounding-off errors and the increase in calculating time were to permit it.

INVERSION OF SIGN METHOD

The finding of the zeros of a function is obtained by beginning from the left hand limit of the selected field of definition and moving the independent variable towards the increasing values. At each increment of the independent variable, the function (for instance, the height of the tide) is calculated. If its sign is equal to the one taken on the previous point then we continue to augment the independent variable. Otherwise — in the case of unequal signs — it is necessary to return, proceeding again by using half the increment of the independent variable.

We continue thus until the desired accuracy for the independent variable is reached, knowing that the transcendental function being examined becomes zero in these circumstances.

On account of the slowness of such a method and the number of points for which it is necessary to calculate the sine or cosine function, for the present case we stopped when a change of sign was found.

At this stage, in view of the poor precision with which the results would have been obtained, the secant method, also of the *b*) type, was then iterated.

When the value of high or low water time is found, the time variable (determined by practical observations at the ports being examined) is increased by a constant amount. This is continued until the signs are inverted, and then the procedure is repeated.

FORTRAN PROGRAMME

Block Diagram

The block diagram refers to tidal calculations.

For the case of tidal current speeds an addition has been made to the sub-programmes operating with the " inversion of sign " method since the zeros are not only sought for in the derivative (2) but also in (1), the instants when the speed is zero being important.

The part illustrated in diagram 1 concerns the control and handling of input data referring to several ports for more than one year.

Diagram 2 shows the logical flow of instructions used in the calculation of high and low tides. For a better comprehension of this second block diagram it is well to show the meaning of some of the symbols used :

SINUS ...... represents the value of the derivative of function (1) calculated at TIME;
TIME ...... time measured in hours and hundredths of an hour, starting from zero hour each day;
SINPR. ..... value of the derivative at the previous increment;
EXACT ..... instant of high (or low) tide, i.e. the unknown;
NDAY ...... successive day number in the month;
PR .... time (in hours and hundredths of an hour) starting from the time origin;
H .... height of tide;
M .... number of high and low tides during one day;
0 .... zero value.

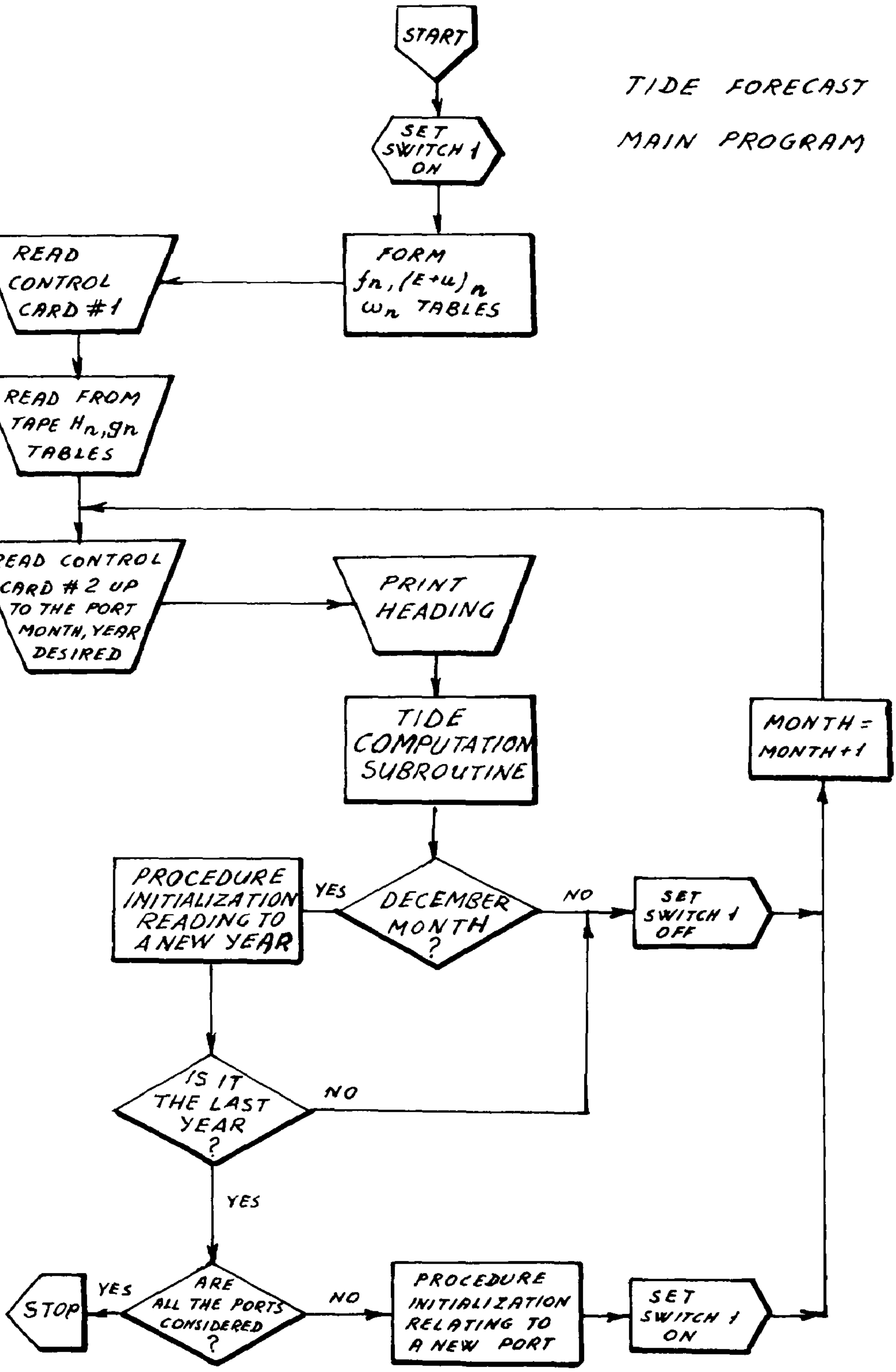
The coding

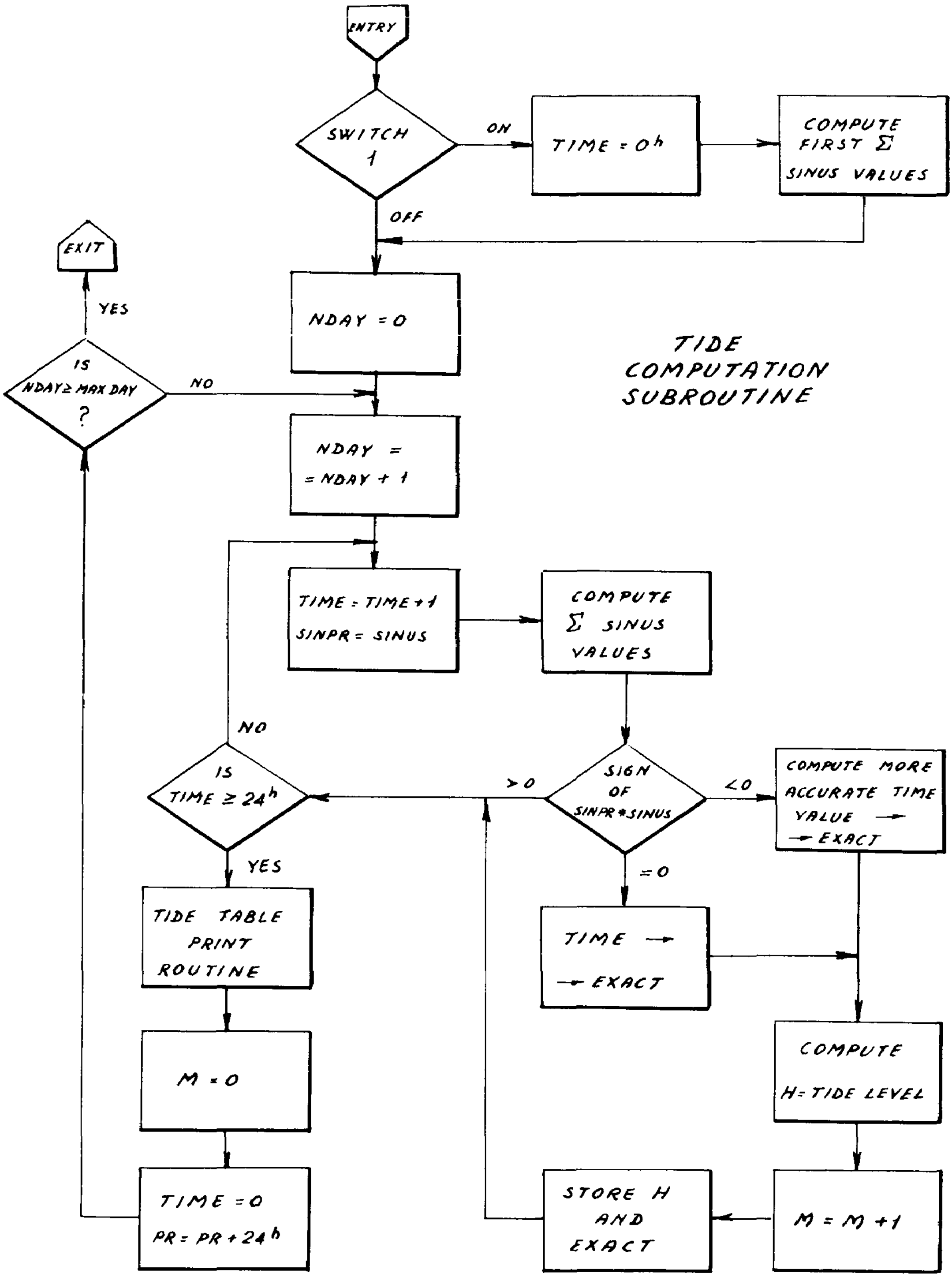
The Fortran programme makes use of the symbols indicated in the above-mentioned diagrams. Therefore those who already know the Fortran language will find the comprehension of the program easier.

After preliminaries, formed by various instructions of the input (READ) and including two control cards for use at different times in the programme, the zeros of the derivative (2) are sought, using the " inversion of sign " method.

Predictions for the required year are made, taking one port at a time. The computation is carried out by repeating the use of the sine subroutine nine times since we are dealing with nine tide constituents.

It must be noted that each argument of the sine function, besides being calculated in radians, is reduced to the first 360 degrees so as to eliminate possible error in serial developments.


TIDE FORECAST
MAIN PROGRAM
START
SET SWITCH 1 ON
FORM f_n, $(V+u)_n$ ω_n TABLES
READ CONTROL CARD #1
READ FROM TAPE H_n, g_n TABLES
READ CONTROL CARD #2 UP TO THE PORT MONTH, YEAR DESIRED
PRINT HEADING
TIDE COMPUTATION SUBROUTINE
MONTH = MONTH + 1
DECEMBER MONTH ?
YES
NO
PROCEDURE INITIALIZATION READING TO A NEW YEAR
SET SWITCH 1 OFF
IS IT THE LAST YEAR ?
NO
YES
ARE ALL THE PORTS CONSIDERED ?
YES
NO
STOP
PROCEDURE INITIALIZATION RELATING TO A NEW PORT
SET SWITCH 1 ON


ENTRY
SWITCH 1
ON
TIME = 0h
COMPUTE FIRST Σ SINUS VALUES
OFF
EXIT
YES
NDAY = 0
TIDE COMPUTATION SUBROUTINE
IS NDAY ≥ MAX DAY ?
NO
NDAY = = NDAY + 1
TIME = TIME + 1
SINPR = SINUS
COMPUTE Σ SINUS VALUES
NO
IS TIME ≥ 24h
> 0
SIGN OF SINPR * SINUS
< 0
COMPUTE MORE ACCURATE TIME VALUE → → EXACT
YES
= 0
TIDE TABLE PRINT ROUTINE
TIME → → EXACT
COMPUTE H = TIDE LEVEL
M = 0
TIME = 0
PR = PR + 24h
STORE H AND EXACT
M = M + 1

When the time of high (or low) tide has been found with the " inversion of sign " method we then proceed to determine H (the height of the tide) by means of (1), reducing in this case also the arguments of the cosine function to the first 360 degrees.

When such a calculation is finished the time is increased and a check is made to see whether 24 hours has been exceeded. Provided that the time is longer than 24 hours the values of the height obtained for high and low tide for the day can be printed. The subsequent day is then considered and so on until the end of the month is reached.

Proceeding from month to month, printing the data each time, the end of the year is reached, it being necessary to vary some parameters if the calculation for the port being examined is not finished.

When the calculation is finished another port is considered, and some very simple modifications are made to values tied to constants H_n and g_n, depending on the port.

The work ends when the predictions for all the ports under consideration have been calculated.

CONCLUSION

The tidal extremes calculated using the above-described method are in perfect accordance with those calculated by the standard tidal prediction machine.

By many statistical examinations carried out on both the predicted and the observed values, differences of from five to eight minutes were noticed for the instant of high and low tides in the case where the tidal curve has a very regular aspect, and such differences of time reach a maximum value of from ten to fifteen minutes in cases where the curve appears flattened under the effect of meteorological interferences.

Very small differences are obtained for the heights in view of the fact that the heights of predicted tides are obviously only valid for normal meteorological conditions and that the causes of interference which could alter to a greater or a lesser degree the calculated data are not taken into account.

Regarding the presentation of the result, figure 1 gives an example of the calculation for tides.

On the first line the name of the place, the month, and the year for which the prediction has been made are shown. The following lines are placed in such a way that the first number represents the day of the month and the other eight numbers the tidal predictions (times in hours and minutes) heights in metres). Although there are five or more extremes in a day, only the first four are printed.

G E N O V A GENNAIO 1967

GIORNO	ALTEZZA	ORA	ALTEZZA	ORA	ALTEZZA	ORA	ALTEZZA
1	0.01	5.29	0.30	11.58	0.08	19.02	0.19
2	0.03	6.21	0.29	12.52	0.07	19.57	
3	0.18	0.51	0.05	7.19	0.28	13.51	0.07
4	0.18	2.22	0.07	8.25	0.26	14.56	0.06
5	0.19	3.58	0.09	9.40	0.25	15.60	0.05
6	0.21	5.16	0.09	11.02	0.25	16.59	0.03
7	0.23	6.17	0.09	12.19	0.24	17.51	
8	0.02	0.42	0.26	7.07	0.09	13.24	0.23
9	0.01	1.27	0.28	7.52	0.08	14.20	0.23
10	0.00	2.09	0.29	8.35	0.08	15.10	0.22
11	-0.00	2.47	0.30	9.13	0.08	15.54	0.21
12	0.00	3.24	0.30	9.48	0.08	16.37	0.20
13	0.01	4.01	0.30	10.24	0.08	17.19	0.19
14	0.02	4.32	0.29	10.55	0.09	17.59	0.19
15	0.03	5.07	0.28	11.30	0.09	18.39	0.18
16	0.05	5.39	0.27	12.02	0.10	19.21	0.17
17	0.07	6.16	0.25	12.41	0.10	20.05	
18	0.16	0.33	0.09	7.02	0.24	13.24	0.10
19	0.16	1.53	0.11	8.02	0.22	14.17	0.09
20	0.17	3.51	0.13	9.22	0.21	15.21	0.08
21	0.19	5.17	0.13	10.60	0.21	16.26	0.06
22	0.22	6.11	0.12	12.22	0.21	17.24	
23	0.04	0.23	0.25	6.54	0.11	13.23	0.21
24	0.02	1.07	0.27	7.35	0.10	14.12	0.22
25	0.00	1.48	0.29	8.13	0.08	14.54	0.23
26	-0.01	2.28	0.31	8.50	0.07	15.35	0.23
27	-0.01	3.09	0.32	9.29	0.06	16.16	0.24
28	-0.01	3.49	0.32	10.07	0.06	16.57	0.23
29	-0.01	4.30	0.32	10.46	0.06	17.38	0.23
30	0.01	5.14	0.31	11.29	0.06	18.24	0.22
31	0.03	6.02	0.29	12.12	0.06	19.13	

NO	VELOCITA	ORA	VELOCITA	ORA	VELOCITA	ORA	VELOCITA	ORA
	-3.77	1.59	4.00	7.60	-5.37	14.35	1.32	20.39
CA		4.40		11.04		18.57		23.24
	-3.37	2.55	3.47	8.38	-5.25	15.45	1.56	21.54
CA		5.29		11.57		19.37		
	-3.11	3.31	2.82	9.24	-5.23	16.29	1.90	23.18
CA		0.37		6.32		12.53		20.16
	-2.86	4.45	2.15	10.25	-5.18	17.15		
CA		1.55		7.52		13.22		21.24
	2.43	0.09	-2.79	6.03	1.53	11.42	-5.14	18.02
CA		3.02		9.04		14.26		21.60
	2.99	1.15	-2.92	7.19	1.11	12.43	-5.03	19.17
CA		4.22		10.34		15.02		22.31
	3.57	2.06	-3.23	8.26	0.84	14.07	-5.06	19.60
CA		5.24		12.02		15.42		23.29
	4.01	3.15	-3.63	9.22	0.69	15.13	-5.06	20.38
CA		6.12		13.07		16.53		24.24
	4.44	3.46	-4.06	10.07	0.64	16.03	-4.92	21.42
CA		7.19		14.16		17.29		
	4.62	4.38	-4.41	10.43	0.63	16.42	-4.85	22.12
CA		0.45		7.48		15.01		18.04
	4.62	5.25	-4.56	11.42	0.57	17.40	-4.58	23.10
CA		1.34		8.40		16.06		18.60
	4.59	5.35	-4.74	12.06	0.58	18.01	-4.36	23.36
CA		2.22		9.27		16.57		19.58
	4.33	6.10	-4.72	12.57	0.56	18.47		
CA		2.38		9.39		17.13		20.22
	-3.96	0.31	3.95	6.40	-4.59	13.46	0.57	19.32
CA		3.25		10.15		17.53		21.24
	-3.45	1.29	3.50	7.08	-4.55	14.05	0.60	20.20
CA		4.14		10.46		18.31		22.07
	-3.08	1.60	2.98	7.37	-4.39	14.56	0.71	20.48
CA		4.37		11.12		19.10		23.13
	-2.64	2.42	2.43	8.13	-4.19	15.50	0.87	21.33
CA		5.38		11.32		19.53		
	-2.23	3.35	1.84	8.58	-4.16	16.21	1.11	22.47
CA		0.19		6.23		12.18		20.15
	-1.96	4.43	1.23	9.58	-4.03	17.28	1.50	24.19
CA		1.46		7.31		12.33		21.10
	-1.90	5.60	0.73	10.51	-4.13	18.11		
CA		3.05		8.42		12.56		21.39
	2.08	1.02	-2.09	7.15	0.38	11.46	-4.30	18.56
CA		4.27		10.26		13.40		22.38
	2.68	2.19	-2.53	8.20	0.26	13.01	-4.55	19.41
CA		5.24		12.04		14.42		23.06
	3.37	2.50	-3.12	9.10	0.37	14.28	-4.73	20.52
CA		6.04		13.10		15.48		24.04
	3.94	3.43	-3.78	9.48	0.66	15.25	-5.07	21.30
CA		7.02		13.35		17.12		
	4.48	4.02	-4.28	10.49	1.01	16.05	-5.32	22.05
CA		0.60		7.54		14.23		18.18
	4.82	4.48	-4.85	11.16	1.33	17.07	-5.35	23.07
CA		1.26		8.13		15.07		19.11
	4.96	5.33	-5.15	12.10	1.65	17.35	-5.35	23.39
CA		2.21		8.58		15.53		19.59
	4.86	6.19	-5.44	12.35	1.83	18.34		
CA		2.48		9.43		16.14		20.45
	-5.04	0.43	4.67	6.36	-5.45	13.31	1.97	19.06
CA		3.46		10.28		17.09		21.35
	-4.70	1.20	4.18	7.26	-5.23	14.30	1.94	20.16
CA		4.18		10.42		17.40		22.32
	-4.16	2.04	3.48	8.20	-5.14	15.04	1.91	21.08
CA		5.27		11.27		18.49		23.44

Figure 2 shows a page of predictions for tidal current which differs slightly from the one on tidal predictions in that a line including the times of slack water (i.e. the times when the speed is zero) is given immediately below each line showing the prediction of extremes.

BIBLIOGRAPHY

CANO, M., GIORGI, M., TROTTI, L. : Rete Mareografica Italiana. Dati e caratteristiche dei Mareografi. Istituto Idrografico della Marina, Genova, 1962.

CARTWRIGHT, D. E., CATTON, D. B. : Analyse de Fourier des observations de marées. *Revue Hydrographique Internationale,* vol. XL, n° 1, janvier 1963.

CIRIANI, T.A., FRISIANI, A. L. : Tabulation of solutions of the cubic equation $z^3 + Az\text{-}A = 0$. *Calcolo,* Ottobre-Dicembre, 1965.

DOODSON, A. T. : The analysis of tidal observations. *Philosophical Transactions of the Royal Society of London,* Series A, Vol. 227, 1928.

GIORGI, M., STOCCHINO, C. : Le costanti armoniche di marea per il porto di Genova. Atti dell'Associazione Geofisica Italiana, 1966.

KUNZ, K. S. : Numerical Analysis. McGraw Hill, 1957.

LEGRAS, J. : Précis d'analyse numérique, Dunod, 1963.

McCORMICK, J. M., SALVADORI, M. G. : Numerical methods in Fortran. Prentice Hall, 1964.

MIYAZAKI, M. : A method for the harmonic analysis of tides. *The Oceanographical Magazine,* Vol. 10, No. 1, 1958.

MIYAZAKI, M., UENO, T. : Tidal analysis and prediction by using the electronic computer. *The Oceanographical Magazine,* Vol. 12, No. 1, 1960.

MURRAY, M. T. : Tide analysis with an electronic digital computer. *Cahiers Océanographiques,* vol. 15, n° 10, 1963.

MURRAY, M. T. : Méthode générale d'analyse des hauteurs horaires de la marée. *Revue Hydrographique Internationale,* vol. XLI, n° 2, juillet 1964.

SCARBOROUGH, J. B. : Numerical mathematical analysis. Johnson Hopkins, 1958.

SCHUREMAN, P. : Manual of harmonic analysis and prediction of tides. U. S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 98, 1964.

IBM Manuals

Fortran - IBM « General Information Manual ».

Fortran IV - IBM 7090/7094 IBSYS Operating System - Version 13.